PROPAGATION OF SOUND IN NARROW CHANNELS IN A He^3—He^4 *SOLUTION*

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The effect of dissipative processes due to the presence of walls on the propagation of waves in narrow channels filled with a superfluid He^3—He^4 solution is considered. The dispersion of waves moving in such channels is studied as a function of the ratio between the penetration depth of a viscous wave and the cross section of the channel. The propagation velocities and absorption coefficients of first, second, and fourth sounds and the thermal wave are calculated and their dependence on the He^3 concentration is elucidated. Relations are obtained between the pressure, temperature, and concentration oscillation amplitudes in the various waves. All calculations are carried out for plane-parallel and for cylindrical capillaries.

THE propagation of waves in channels having sufficiently small dimensions is strongly influenced by dissipative processes connected with the presence of the walls. The character of the waves propagating in such channels, filled with a superfluid liquid, is determined by the ratio of the depth of penetration of the viscous wave $\lambda_V = (2\eta/\omega\rho_n)^{1/2}$ to the transverse dimension of the channel 2d. If $\lambda_V \ll d$, then ordinary first and second sound propagates in the channels. If $\lambda_V \gg d$, then the normal component of the liquid is slowed down and the wave propagating in the channels is fourth sound, which is a modification of first sound, while the second sound is modified into a rapidly damped thermal wave. On the other hand, when $\lambda_V \gg d$ the normal component of the liquid is partially stagnant, leading to strong dispersion of both first and second sound. In this region, both the velocity and the damping coefficient of the sound are determined by the ratio $\delta = d/\lambda_V$. The wave processes occurring in narrow channels with partial or total stagnation of the normal component of liquid He^4 has been investigated in a number of experimental and theoretical studies[1-10]. Propagation of fourth sound in an He^3—He^4 solution was studied in[11,12].

The purpose of the present paper is an investigation of wave dispersion as a function of the parameter δ, and an elucidation of the concentration dependence of the velocity and absorption of waves propagating in narrow channels filled with a superfluid He^3—He^4 solution. We confine ourselves to the hydrodynamic case, when the stagnation of the normal component is determined by the depth of penetration of the viscous wave. A hydrodynamic approach is justified at low sound frequencies and relatively high temperatures. At low temperatures, when the mean free path of the excitations is comparable with the characteristic transverse dimensions of the capillary, a kinetic analysis is necessary.

Since we are interested in effects connected with the presence of walls, we shall not take into account the dissipative terms that lead to volume damping of the sound. The only dissipative terms that will be taken into account are those connected with the first viscosity η and leading to stagnation of the normal component of the liquid in the capillary. In such an approximation, the linearized system of hydrodynamic equations for the He^3—He^4 solution becomes[13]

$$\dot{\rho} + \operatorname{div} \mathbf{J} = 0,$$

$$\frac{\partial J_i}{\partial t} + \nabla_i p = \eta \frac{\partial}{\partial x_k}\left(\frac{\partial v_{ni}}{\partial x_k} + \frac{\partial v_{nk}}{\partial x_i} - \frac{2}{3}\delta_{ik}\frac{\partial v_{nl}}{\partial x_l}\right),$$

$$\dot{\mathbf{v}}_s + \nabla\left(\mu - \frac{Z}{\rho}c\right) = 0,$$

$$(\dot{\sigma\rho}) + \operatorname{div}(\rho\sigma\mathbf{v}_n) = 0, \qquad (\dot{\rho c}) + \operatorname{div}(\rho c\mathbf{v}_n) = 0. \tag{1}$$

Here ρ is the density, σ the specific entropy, p the pressure, T the temperature, c the mass concentration of the He^3, η the viscosity coefficient, $\mathbf{v}_n$ and $\mathbf{v}_s$ the velocities of the normal and superfluid components, and J the total flux of liquid. Z and μ are expressed in terms of the chemical potentials μ_3 and μ_4 of the He^3 and He^4 in the solution in the following fashion: $Z = \rho(\mu_3 - \mu_4)$, $\mu = c\mu_3 + (1-c)\mu_4$, and $\nabla\mu = -\rho^{-1}\nabla p - \sigma\nabla T + Z\rho^{-1}\nabla c$.

The following conditions should be satisfied on the walls of the capillary: vanishing of the normal components of the total flux and of the impurity fluxes, vanishing of the tangential component of the velocity of the normal part of the liquid, and continuity of the heat flow. The heat flow through the boundary is expressed in terms of the temperature difference between the liquid and the solid boundary. These conditions can be written in the form

$$(\rho_s\mathbf{v}_s + \rho_n\mathbf{v}_n, \mathbf{n}) = 0, \quad [\mathbf{v}_n\mathbf{n}] = 0, \quad (\rho c\mathbf{v}_n + \mathbf{g}, \mathbf{n}) = 0,$$
$$\left(T\rho\sigma\mathbf{v}_n + \mathbf{q} - \frac{Z}{\rho}\mathbf{g}, \mathbf{n}\right) = -\varkappa_w(\nabla T_w, \mathbf{n}),$$
$$-\varkappa_w(\nabla T_w, \mathbf{n}) = \alpha(T - T_w), \tag{2}*$$

where T_W and κ_W are the temperature and thermal-conductivity coefficient of the wall, and α is the thermal resistance of the boundary between the solid body and the liquid helium. The fluxes g and q are expressed as follows:

$$-\mathbf{g} = \rho D\left(\nabla c + \frac{k_T}{T}\nabla T + \frac{k_p}{p}\nabla p\right),$$
$$-\mathbf{q} = T^2\left(\frac{\partial}{\partial T}\frac{Z}{\rho T} - \frac{k_T}{T}\frac{\partial}{\partial c}\frac{Z}{\rho T}\right)\mathbf{g} + \varkappa\nabla T, \tag{3}$$

where D, $k_T D$, and $k_p D$ are respectively the diffusion,

*$[\mathbf{V}_n\mathbf{n}] \equiv \mathbf{V}_n \times \mathbf{n}$.

thermal-diffusion, and barodiffusion coefficients.

The system of hydrodynamic equations (1) should be solved simultaneously with the equation for the heat conduction of the wall

$$C_w \frac{\partial T_w}{\partial t} = \varkappa_w \Delta T_w, \tag{4}$$

where C_W is the specific heat of the wall.

The deviations of the quantities from their equilibrium values will be sought in the form

$$v_n = L_1 \nabla Q_1 + L_2 \nabla Q_2 + u, \quad v_s = N_1 \nabla Q_1 + N_2 \nabla Q_2,$$
$$T' = M_1 Q_1 + M_2 Q_2. \tag{5}$$

Substitution of (5) in (1) leads to a system of equations for Q_i and U:

$$\Delta Q_i + k_i^2 Q_i = 0, \quad \operatorname{div} u = 0 \tag{6}$$

and to a connection between the amplitudes L_i, M_i, and N_i (i = 1, 2):

$$P_i = \frac{N_i}{L_i} = -\rho_n \left[\omega^2 \frac{\partial \rho}{\partial p} - k_i^2 \left(1 - \frac{\rho_s}{\rho_n}\beta\right)\right] \Big/ \rho_s \left[\omega^2 \frac{\partial \rho}{\partial p} - k_i^2 (1+\beta)\right],$$
$$D_i = \frac{M_i}{L_i} = \frac{i\omega\rho_n}{\rho\bar{\sigma}} \left[(1+\beta) - P_i\left(1 - \frac{\rho_s}{\rho_n}\beta\right)\right] - c^2 \frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right) \frac{i\rho_s k_i^2 (1 - P_i)}{\bar{\sigma}\omega\rho}, \tag{7}$$

where

$$\bar{\sigma} = \sigma - c\frac{\partial \sigma}{\partial c}, \quad \beta = \frac{c}{\rho}\frac{\partial \rho}{\partial c};$$

k_1 and k_2 are the wave vectors of the first and second sounds in the He^3–He^4 solution.

Solving the system (6) in the usual manner [8-9] we obtain from the boundary conditions (2) the dispersion equation for the waves propagating along the capillary. Since the dispersion of the waves is determined in the main by the stagnation of the normal component of the liquid, we neglect the heat transfer through the capillary walls in the boundary conditions (2). Taking into account the fact that in the case of interest to us the wavelength of the sound is much larger than the transverse dimensions of the capillary and the length λ_V of the viscous wave, the dispersion equation takes the form

$$k_\parallel^4 - k_\parallel^2 \left[k_{01}^2 + k_{02}^2\left(1 + \frac{\rho_s}{\rho_n}\beta^2\right) + ir\frac{\rho}{\rho_n}\left(\frac{\rho_n}{\rho}k_{01}^2 + \frac{\rho_s}{\rho}(1+\beta)k_{02}^\alpha\right)\right]$$
$$+\left(1 + ir\frac{\rho}{\rho_n}\right) k_{01}^2 k_{02}^2 = 0,$$
$$k_{01}^2 = \frac{\omega^2}{u_{01}^2} = \omega^2 \frac{\partial \rho}{\partial p}, \quad k_{02}^2 = \frac{\omega^2}{u_{02}^2} = \frac{\rho_n}{\rho_s}\omega^2 \left[\bar{\sigma}^2 \frac{\partial T}{\partial \sigma} + c^2 \frac{\partial}{\partial c}\left(\frac{Z}{\rho}\right)\right]^{-1}. \tag{8}$$

The dimensionless parameter r has different forms in the case of a plane-parallel capillary and a cylindrical capillary. For a plane-parallel capillary with width 2d we have

$$r = -i\frac{\rho_n}{\rho}\frac{\operatorname{tg} k_3 d}{k_3 d - \operatorname{tg} k_3 d}, \tag{9}$$

where the viscous-wave wave vector $k_3 = (i\omega\rho_n/\eta)^{1/3}$. For a cylindrical capillary with radius d we have

$$r = -i\frac{\rho_n}{\rho}\frac{2I_1(k_3 d)}{k_3 d\, I_0(k_3 d) - 2I_1(k_3 d)}. \tag{10}$$

The dispersion equation (8) is similar to the equation obtained by Pollack and Pellam [7] and by Adamenko and Kaganov [9] for pure He^4. In our case, however, the dispersion equation explicitly contains a parameter $\beta = (c/\rho)\partial\rho/\partial c$ which is connected with the He^3 concentration.

Equation (8) has two roots which can be written, accurate to terms proportional to the ratio u_2^2/u_1^2 (u_1 and u_2 are the velocities of first and second sound) in the form

$$k_{\parallel 1}^2 = \frac{\omega^2}{u_{01}^2}\left(1 + ir\frac{\rho}{\rho_n}\right)\left[1 + \frac{\rho_s}{\rho_n}\beta^2 + ir\frac{\rho_s}{\rho_n}(1+\beta)^2\right]^{-1},$$
$$k_{\parallel 2}^2 = \frac{\omega^2}{u_{02}^2}\left[1 + \frac{\rho_s}{\rho_n}\beta^2 + ir\frac{\rho_s}{\rho_n}(1+\beta)^2\right]. \tag{11}$$

The first root corresponds at small values of d/λ_V ($|r| \gg 1$) to fourth sound [11], and at large d/λ_V ($|r| \ll 1$) to first sound [15]. The second root corresponds to a thermal wave at low and intermediate values of d/λ_V, and to second sound at large d/λ_V ($|r| \ll 1$).

For the velocity of the first sound we obtain from (11) the expression

$$u_{1\delta} = u_{01}\left[1 + \frac{\rho_s}{\rho_n}\beta^2 - b\frac{\rho_n}{\rho}\left(1 - \frac{\rho_s}{\rho_n}\beta\right)^2\right]^{1/2} \cdot$$
$$\times\left\{1 + \frac{3}{8}\left[a\frac{\rho_n}{\rho}\left(1 - \frac{\rho_s}{\rho_n}\beta\right)^2\right]^2\left[1 + \frac{\rho_s}{\rho_n}\beta^2 - b\frac{\rho_n}{\rho}\left(1 - \frac{\rho_s}{\rho_n}\beta\right)^2\right]^{-2}\right\}. \tag{12}$$

The absorption of sound is determined by the imaginary part of the wave vector

$$\operatorname{Im} k_{\parallel 1} = \frac{\omega}{2u_{10}} a\frac{\rho_n}{\rho}\left(1 - \frac{\rho_s}{\rho_n}\beta\right)^2\left[1 + \frac{\rho_s}{\rho_n}\beta^2 - b\frac{\rho_n}{\rho}\left(1 - \frac{\rho_s}{\rho_n}\beta\right)^2\right]^{-3/2}$$
$$\times\left\{1 - \frac{5}{8}\left[a\frac{\rho_n}{\rho}\left(1 - \frac{\rho_s}{\rho_n}\beta\right)^2\right]^2\left[1 + \frac{\rho_s}{\rho_n}\beta^2 - b\frac{\rho_n}{\rho}\left(1 - \frac{\rho_s}{\rho_n}\beta\right)^2\right]^{-2}\right\}. \tag{13}$$

a and b are functions of the dimensionless parameter δ. For a plane-parallel capillary we have [9]

$$a = \frac{\operatorname{sh} 2\delta - \sin 2\delta}{4\delta(\cos^2\delta + \operatorname{sh}^2\delta)}, \quad b = \frac{\operatorname{sh} 2\delta + \sin 2\delta}{4\delta(\cos^2\delta + \operatorname{sh}^2\delta)}. \tag{14}$$

For a cylindrical capillary we have

$$a = \operatorname{Im}\frac{2I_1(k_3 d)}{k_3 d\, I_0(k_3 d)}, \quad b = \operatorname{Re}\frac{2I_1(k_3 d)}{k_3 d\, I_0(k_3 d)}. \tag{15}$$

From formulas (12)–(15) and the plots of Fig. 1 we see that the dispersion of the sound depends strongly both on the form of the capillary and on the He^3 concentration. The region of strong sound dispersion increases with increasing He^3 concentration. The sound damping due to the slippage of the normal component increases with increasing δ, reaching a maximum at $\delta \sim 1$, and then decreases with further increase of δ. The value of δ corresponding to the maximum sound damping depends both on the shape of the capillary and on the He^3

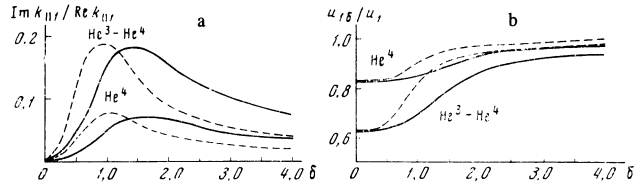

FIG. 1. Dependence of the quantities u_{10}/u_1 and $\operatorname{Im} k_{\parallel 1}/\operatorname{Re} k_{\parallel 1}$ on the parameter $\delta = d/\lambda_V$ at 1.8°K for pure He^4 and for a solution with concentration c = 8.38%. The solid curves correspond to a cylindrical capillary, and the dashed ones to a plane-parallel capillary.

concentration. At small values of δ, when complete stagnation of the normal component sets in, the velocity of the sound tends to a constant limit and becomes independent of δ (fourth sound). At fixed values of the He^3 concentration, d and ω, the damping of the sound increases with increasing temperature, and the velocity of the sound decreases.

For the velocity of the thermal-wave propagation we obtain

$$u_{2\delta} = \sqrt{2}\,u_{20}\left\{\left[\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^2 - 2\frac{\rho_s}{\rho}(1+\beta)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)m_2 + \left(\frac{\rho_s}{\rho}\right)^2(1+\beta)^4 m_1^2\right]^{1/2} + \left[\left(1+\frac{\rho_s}{\rho_n}\beta^2\right) + \frac{\rho_s}{\rho}m_2(1+\beta)^2\right]\right\}^{-1/2}. \quad (16)$$

For the imaginary part of the wave vector $k_{\|2}$ we have

$$\operatorname{Im} k_{\|2} = \frac{\omega}{\sqrt{2}\,u_{20}}\left\{\left[\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^2 - 2\frac{\rho_s}{\rho}(1+\beta)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)m_2 + \left(\frac{\rho_s}{\rho}\right)^2(1+\beta)^4 m_1^2\right]^{1/2} - \left[\left(1+\frac{\rho_s}{\rho_n}\beta^2\right) + \frac{\rho_s}{\rho}m_2(1+\beta)^2\right]\right\}^{1/2}, \quad (17)$$

where we have introduced, for convenience, the symbol

$$r = \frac{\rho_n}{\rho}(m_1 + im_2) = \frac{\rho_n}{\rho}\frac{a + i[a^2 - b(1-b)]}{a^2 + (1-b)^2}. \quad (18)$$

As seen from (16) and (17) and from the diagrams of Fig. 2, the velocity of the thermal wave decreases with decreasing δ and the thermal wave attenuates strongly. Unlike first sound, the region of propagation of second sound increases with increasing concentration.

The first sound in the capillary and the thermal wave can be regarded as superpositions of three types of oscillations: ordinary first sound, second sound, and a viscous wave. The contribution of each is determined by the boundary conditions and depends on $\delta = d/\lambda_V$. The contribution of first and second sound to the modified first sound can be readily determined by equating $v_{SZ}(d)$ to zero. The ratio of the amplitudes of the oscillations v_S, due to first and second sounds, turns out to be

$$\frac{N_1}{N_2} = \frac{u_1^2}{u_2^2}\left[1+\frac{\rho_s}{\rho_n}\beta^2 - b\frac{\rho_n}{\rho}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\right]\left[b\frac{\rho_n}{\rho}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\right]^{-1}. \quad (19)$$

We see from (19) that the contribution of second sound to the modified first sound is always small and decreases with increasing δ.

It is also easy to obtain the ratios of the amplitudes of the oscillations of the pressure p′, temperature T′, concentration c′ to the amplitude of the oscillations of the velocity of the superfluid part of the liquid

$$\frac{p'}{v_s} = \frac{\rho u_{1\delta}}{1+\beta}K_1^{-1},$$
$$\frac{T'}{v_s} = -\frac{\rho_s}{\rho_n}\sigma\frac{\partial T}{\partial\sigma}\left[b\frac{\rho_n}{\rho}\left(1-\frac{\rho_s}{\rho_n}\beta\right)+\beta\right]\frac{1}{u_{1\delta}(1+\beta)}K_1,$$
$$\frac{c'}{v_s} = -\frac{\rho_s}{\rho_n}\frac{c}{u_{1\delta}}\left[b\frac{\rho_n}{\rho}\left(1-\frac{\rho_s}{\rho_n}\beta\right)+\beta\right]\frac{1}{1+\beta}K_1, \quad (20)$$

where

$$K_1 = 1 + i\operatorname{Im} k_{\|1}/\operatorname{Re} k_{\|1}.$$

It is seen from (20) that the pressure, the temperature, as well as the concentration oscillate in the modified-first-sound wave. However, the amplitude of the pressure oscillations always exceeds the amplitude of

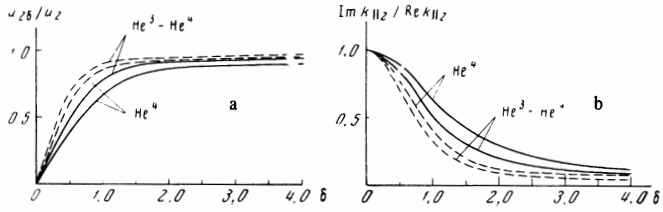

FIG. 2. Dependence of the quantities $u_2\delta/u_2$ and $\operatorname{Im} k_{\|2}/\operatorname{Re} k_{\|2}$ on the parameter $\delta = d/\lambda_V$ at 1.8°K for pure He^4 and for a solution with concentration c = 8.38%. The solid curves correspond to a cylindrical capillary, and the dashed ones to a plane-parallel capillary.

the temperature oscillations. If $\delta \to 0$, then it follows from (14) and (15) that $b \to 1$, and we obtain relations between the amplitudes of the oscillations in the fourth-sound wave.[12] In the other limiting case when $\delta \to \infty$, we have $b \to 0$ and relations are obtained between the oscillation amplitudes in the ordinary first sound[14].

The amplitude of the oscillations of the normal part of the liquid varies over the cross section of the channel, reaching a maximum value on the channel axis. It is therefore convenient to use the ratio of the normal velocity amplitude, averaged over the channel cross section, to the superfluid velocity amplitude:

$$\frac{\bar v_n}{v_s} = \left(1-\frac{\rho_s}{\rho_n}\beta\right)\frac{1}{1+\beta}(1-b+ia). \quad (21)$$

In the limit as $\delta \to 0$ we obtain that the normal velocity also tends to zero, since $b \to 1$ and $a \to 0$.

In the case of the thermal wave we have

$$\frac{N_1}{N_2} = -ir\frac{\rho_s}{\rho_n}(1+\beta)^2\left[1+\frac{\rho_s}{\rho_n}\beta^2 + ir\frac{\rho_s}{\rho_n}(1+\beta)^2\right]^{-1}.$$

With increasing $\delta = d/\lambda_V$ this ratio tends to zero, and the contribution of the first sound to the thermal wave decreases. In the second limiting case, when $\delta \to 0$ ($|r| \to \infty$), we have $N_1/N_2 = -1$. Consequently, if the normal component is stagnant, then the superfluid component likewise does not oscillate, leading to a rapid damping of the thermal wave.

For the oscillation amplitudes p′, T′ and c′ we have

$$\frac{p'}{v_s} = -\frac{\rho_s\rho}{\rho_n}u_{2\delta}\left(\beta - m_2\frac{\rho_n}{\rho}(1+\beta) + i\frac{\rho_n}{\rho}m_1(1+\beta)\right)K_2^{-1},$$
$$\frac{T'}{v_s} = -\frac{\rho_s}{\rho_n}\sigma\frac{\partial T}{\partial\sigma}\left[\left(1-\frac{\rho_s}{\rho_n}\beta\right)u_{2\delta}\right]^{-1}K_2,$$
$$\frac{c'}{v_s} = -\frac{\rho_s}{\rho_n}c\left[\left(1-\frac{\rho_s}{\rho_n}\beta\right)u_{2\delta}\right]^{-1}K_2, \quad (22)$$

where

$$K_2 = 1 + i\operatorname{Im} k_{\|2}/\operatorname{Re} k_{\|2}.$$

It follows from these formulas that in the thermal wave the temperature and concentration oscillation amplitudes are always larger than the pressure oscillation amplitude.

For the ratio of the velocity-oscillation amplitudes of the normal and superfluid parts of the liquid we obtain

$$\frac{\bar v_n}{v_s} = -\frac{\rho_s}{\rho_n}\frac{1+\beta}{1-\rho_s\beta/\rho_n}. \quad (23)$$

Since the ratio ρ_S/ρ_n for the solutions is smaller than for pure He^4, and since $\beta < 0$, the condition (23), which

is necessary for the propagation of the thermal wave (second sound), is easier to satisfy for solutions in the case of partial stagnation of the normal component. This explains why the region of propagation of the second sound increases with increasing He^3 concentration (Fig. 2). In capillaries whose transverse dimensions are much smaller than the depth of penetration of the viscous wave ($\delta \ll 1$), the normal component of the liquid is almost completely stagnant. In such a situation, fourth sound (which is a modification of first sound), and a damped thermal wave propagate in the superfluid liquid[8]. From the boundary conditions (2) it is possible to calculate, by the same method as used in[8] for pure He^4, the velocity and the absorption coefficient of the fourth sound in the He^3—He^4 solutions.

The square of the velocity of the fourth sound is

$$u_4^2 = \frac{\rho_s}{\rho} u_1^2 (1+\beta)^2 \left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1} + \frac{\rho_n}{\rho} u_2^2 \left(1+\frac{\rho_s}{\rho_n}\beta^2\right). \quad (24)$$

A similar expression is obtained for the velocity of sound[11] directly from the hydrodynamic equations (1), if we assume in them $v_n = 0$. The experimental data[12] on the velocity of fourth sound are in good agreement with those calculated by formula (24).

The absorption of sound is due both to the slippage of the normal component of the liquid, and to heat transfer through the capillary walls. The coefficient of sound absorption connected with the viscous dissipation mechanism (slippage of the normal component) is

$$\Upsilon_{\text{vis}} = \frac{d^2}{2gu_4} \frac{\omega^2 \rho_s \rho_n}{\eta \rho^2} \left[\frac{u_{01}^2(1+\beta)(1-\rho_s\beta/\rho_n) - u_{02}^2}{u_4^2}\right]^2. \quad (25)$$

For a plane-parallel capillary $g = 3$ and for a cylindrical one $g = 8$.

It is interesting to note that the obtained sound-absorption coefficient is inversely proportional to the viscosity coefficient η. This is connected with the fact that when η increases v_n decreases, and consequently the energy loss connected with the oscillation of the normal component in the capillary decreases.

The fourth-sound absorption coefficient connected with the heat transfer through the capillary walls is

$$\Upsilon_{\text{th}} = \frac{2}{fu_4}\frac{\sigma}{\sigma}\frac{\rho_n}{\rho d C_{\text{He}}}\frac{1}{u_4^2}\left[u_{02}^2 + u_{01}^2\frac{\rho_s}{\rho_n}\beta(1+\beta)\right]$$
$$\times\left[\frac{1}{\alpha}+\sqrt{\frac{1}{2\omega C_w \varkappa_w}}\right]\left[\frac{1}{\alpha^2}+\left(\frac{1}{\alpha}+\sqrt{\frac{2}{\omega C_w \varkappa_w}}\right)^2\right]^{-1},$$
$$\bar{\sigma} = \sigma - k_T c \frac{\partial}{\partial c}\left(\frac{Z}{\rho T}\right) \quad (26)$$

For a plane-parallel capillary $f = 2$, for a cylindrical one $f = 1$.

In the limiting case when $\delta \ll 1$, the thermal wave is strongly damped, and the damping is proportional to the viscosity coefficient η and is inversely proportional to the square of the capillary radius. We present an expression for the square of the wave vector of the thermal wave in the case of a plane-parallel capillary:

$$k_{\|2}^2 = i\frac{3\omega\eta u_4^2}{d^2\rho_n u_1^2 u_2^2} + \frac{\rho_s\rho_n}{\rho^2}\frac{\omega^2}{u_4^2}\left[u_{01}^2(1+\beta)\left(1-\frac{\rho_s}{\rho_n}\beta\right) - u_{02}^2\right]^2\frac{1}{u_1^2u_2^2}$$
$$-\frac{3\rho_s^2\eta}{\rho^2\rho_n d^3}\frac{\bar{\sigma}}{\sigma}(1+\beta)\left[\left(\frac{1}{\alpha}+\frac{1}{\sqrt{i\omega\varkappa_w C_w}}\right)C_{\text{He}}u_{02}^2\right]^{-1}. \quad (27)$$

In channels whose width satisfies the condition $d \gg \lambda_V$, ordinary first and second sound propagate, and their velocities is altered by the presence of dissipation on the channel walls. We present expressions for the velocity and wave-absorption coefficient in plane-parallel capillaries. All the formulas remain valid also for cylindrical capillaries, if we replace in them the width of the gap 2d by the rates of the capillary.

The velocity of first sound is equal to

$$u_1' = u_1\left[1-\frac{1}{2\rho d}\sqrt{\frac{\eta\rho_n}{2\omega}}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\right.$$
$$\left.-\frac{1}{2C_{\text{He}}\omega}\frac{\rho_s}{\rho_n}\frac{\bar{\sigma}}{\sigma}\beta^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}R_1\right]. \quad (28)$$

The imaginary part of the wave vector, which determines the sound absorption, is

$$\operatorname{Im} k_1 = \frac{\omega}{u_1}\left[\frac{1}{2\rho d}\sqrt{\frac{\eta\rho_n}{2\omega}}\left(1-\frac{\rho_s}{\rho_n}\beta\right)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\right.$$
$$\left.+\frac{\rho_s\bar{\sigma}}{C_{\text{He}}\omega\rho_n\sigma}\beta^2\left[1+\frac{\rho_s}{\rho_n}\beta^2\right]^{-1}R_2\right], \quad (29)$$

where we have introduced the notation

$$R_1 = \frac{1}{d}\sqrt{\frac{2}{\omega C_w \varkappa_w}}\left[\frac{1}{\alpha^2}+\left(\frac{1}{\alpha}+\sqrt{\frac{2}{\omega C_w\varkappa_w}}\right)^2\right]^{-1},$$

$$R_2 = \frac{1}{d}\left(\frac{1}{\alpha}+\sqrt{\frac{1}{2\omega C_w\varkappa_w}}\right)\left[\frac{1}{\alpha^2}+\left(\frac{1}{\alpha}+\sqrt{\frac{2}{\omega C_w\varkappa_w}}\right)^2\right]^{-1}.$$

As seen from the presented formulas, unlike the case of pure He^4, the first-sound velocity and the absorption coefficient of solutions contain terms proportional to β^2, connected with heat transfer through the boundary. This is due to the fact that the first-sound wave in the He^3—He^4 solution contains temperature oscillations whose amplitude is proportional to β (20).

The velocity of second-sound propagation is

$$u_2' = u_2\left[1-\frac{\rho_s}{2\rho d}\sqrt{\frac{\eta}{2\rho_n\omega}}(1+\beta)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\right.$$
$$\left.-\frac{\bar{\sigma}}{2\omega C_{\text{He}}\sigma}\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}R_1\right]. \quad (30)$$

The imaginary part of the wave vector of the second sound is given by

$$\operatorname{Im} k_2 = \frac{\omega}{u_2}\left[\frac{\rho_s}{2\rho d}\sqrt{\frac{\eta}{2\rho_n\omega}}(1+\beta)^2\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}\right.$$
$$\left.+\frac{\bar{\sigma}}{\omega C_{\text{He}}\sigma}\left(1+\frac{\rho_s}{\rho_n}\beta^2\right)^{-1}R_2\right]. \quad (31)$$

The absorption of second sound consists of two parts due to the viscous losses and the heat-conduction losses, and depends explicitly on the He^3 concentration. The absorption coefficient depends on the thermal resistance of the boundary α^{-1} and of the solid $(2/\omega C_w \kappa_w)^{1/2}$. If the solid wall has a large thermal conductivity, then the absorption is determined principally by the thermal resistance of the boundary. To the contrary, in the case of low thermal conductivity of the solid wall, the absorp-

tion is determined by the thermal resistance of the wall. The absorption of the sound by the walls is significant only at small sound frequencies. At high frequencies, on the other hand, the volume of absorption, which depends on the frequency like ω^2, greatly exceeds the surface effects. If we put $\beta = 0$ in formulas (28)–(31), then we obtain the well known result[2-4] for He^4.

Experiments aimed at the study of the propagation of waves in narrow channels are performed in filters made of pressed fine powder. The geometry of the channel in such filters is very complicated and differs greatly from that considered by us. As shown by the experiment[6,7,10], the irregularity of the geometry of the channels leads to an effective decrease of the diameter of the channels and to an increase of the path traversed by the wave. To clarify the influence of the geometry of the channels on the propagation of the sound, it would be of interest to perform experiments on solutions with different He^3 concentration and to compare the differences of the sound velocities for different concentrations with the calculated ones.

A study of the propagation of the waves under conditions of total or partial deceleration of the normal component of the He^3–He^4 solution makes it possible to investigate the behavior of the He^3 atoms in narrow channels, to measure the ratio ρ_s/ρ, to determine the energy spectrum of the impurities and the dependence of the effective mass of He^3 on the concentration, and to study the dissipation mechanisms under such conditions. The entire calculation in our paper is performed under the assumption that the He^3 atoms contribute only to the normal component of the liquid. Experiments on the propagation of sound make it possible to verify this assumption in a wide range of concentrations at temperatures.

In the construction of the plots of Figs. 1 and 2 we use the experimental data for ρ, ρ_n, and β from[15-18].

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