MAGNETORESISTANCE OF SEMIMETALS

V. G. PESCHANSKIĬ and M. Ya. AZBEL'

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences, Moscow State University

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The distribution of an electric current in bounded conductors is studied for an arbitrary nature of electron scattering from the sample boundary. Specular scattering, in particular, is studied in detail. In a strong magnetic field the electric current is concentrated near the conductor surface and the higher the degree of specularity of scattering of the charge carriers the higher is the inhomogeneity of the electric current in the sample. In semimetals, in which electrons are virtually specularly scattered by the sample surface, the static skin effect for currents leads to linear growth of the resistance in a magnetic field parallel to the Fermi surface. In principle, a study of the dependence of the resistance on the strength of a parallel magnetic field yields the specularity parameter for scattering of electrons by the surface of a semimetallic sample.

A large surface current that compensates for the classical magnetic moment produced by the electrons moving in closed orbits[1] is produced near the boundary of a conductor in a magnetic field. When the electrons collide with the surface of the sample, the electron orbit in the magnetic field becomes discontinuous, and the center of the orbit shifts jumpwise in a direction opposite to the direction of revolution of the electron inside the conductor (Fig. 1). The electrons are most "mobile" near the surface of the sample, since the center of the electron orbits in the interior of the conductor can be displaced in a magnetic field only with the aid of an electric field (all the electron orbits ϵ = const, p_z = const are assumed to be closed; ϵ is the energy and p_z the projection of the momentum of the electron in the direction of the magnetic field).

In transport phenomena occurring in the presence of a strong magnetic field ($r \ll l$; r—radius of curvature of the trajectory and l—electron free path), the kinetic coefficients are determined mainly by the dynamics of the electron motion, and are strongly dependent, in particular, on whether the electron motion in the given direction is finite or infinite. This is the cause of the strong anisotropy of the electric and thermal resistances of conductors in a strong magnetic field[2-4]. The different character of motion of the electrons inside the conductor and near its surface[1)] leads in a number of cases to a concentration of the electric and thermal fluxes in a surface layer on the order of the Larmor radius[5-7].

In a magnetic field, the more "mobile" electrons near the surface of the conductor and the electrons in its interior make different contributions to the total transverse electric conductivity. In accordance with this attribute, they can be divided into three groups:

1) Electrons reflected from the surface of the conductor and colliding with it within a time on the order of the period of motion in the magnetic field.

2) Electrons reflected from the surface of the conductor traversing with a probability $e^{-d/l}$ the total thickness of the sample d without experiencing collisions on their path.

3) Electrons that never collide with the surface of the conductor during the free path time; these are the electrons in the interior of the sample, which in practice do not drift towards its surface. In thick conductors ($d \gg l$) these include all the electrons whose distance to the surface of the sample exceeds the mean free path.

A strong magnetic field greatly decreases the transverse electric conductivity of the interior of the sample, because the electron can travel a distance of the order of the Larmor radius r in the direction of the electric field between two collisions. In addition, the magnetic field displaces the electron, with equal probability, both in the direction of the electric field and in the opposite direction. Therefore the transverse electric conductivity is proportional only to the square of r:

$$\sigma_\perp = \sigma_0 (r / l)^2,$$

where $\sigma_0 = \sigma_0(l)$ is the electric conductivity of the bulky sample in the absence of the magnetic field.

The situation is different near the surface of the conductor. When the electrons collide with the boundary of the sample they are displaced in a definite direction, there is no averaging over the random mo-

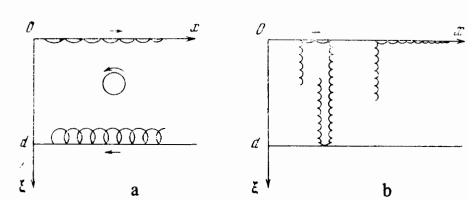

FIG. 1. Examples of trajectories of the motion of an electron in a magnetic field that is parallel (a) or inclined(b) to the surface of the plate.

[1)]The effects considered here are not connected in any way with the magnetic properties of the conductors. In the absence of an external electric field, in the quasiclassical approximation, the electric current vanishes at any point inside the conductor, and the so called surface current is offset by the motion of the "internal" electrons, while the Fermi distribution function of the electrons $f_0(\epsilon)$ satisfies the boundary conditions for any character of the scattering of the electrons by the sample boundary.

ments of the collisions, and the electric conductivity of the surface layer is at least proportional to r, i.e., it is much larger than in the interior of the sample.

A very important factor is the character of the reflection of the electrons from the surface of the conductor. If the scattering of the electrons by the sample boundary is diffuse, there is no correlation at all between the reflected and incident electrons. In this sense, diffuse scattering of the electron is equivalent to collision inside the volume, and the indicated three groups of electrons have in thin samples ($d \ll l$) different effective free paths, namely r, d, and l respectively. For electrons of the first group, the relative number of which is of the order of the ratio of the Larmor radius to the thickness of the sample r/d, a magnetic field of arbitrary magnitude can no longer be regarded as strong ($l_{eff} \approx r$), and the contribution made by them to the electric conductivity is equal to $(r/d)\sigma_0(l_{eff}) = \sigma_0 r^2/ld$. In thick conductors, the electrons of the second and third group have the same effective mean free path and it is meaningless to distinguish between them. The contribution of the electrons of the second group to the electric conductivity of thin samples was calculated by the authors in[6] and is given by the same expression as for an unbounded conductor, except that the role of the mean free path of the electrons is played by the thickness of the sample. As a result, the contribution of the electrons of the first and second groups to the total electric conductivity of a thin sample turns out to be of the same order of magnitude.

The total transverse electric conductivity $\sigma_\perp$ of conductors of any thickness has the following asymptotic form in strong magnetic fields ($r \ll l$, d):

$$\sigma_\perp = \sigma_0 (ar^2/ld + br^2/l^2). \qquad (1)$$

The factors a and b in (1) are quantities of the order of unity. The dependence of the electric conductivity on the magnetic field turns out to be the same as in bulky samples. However, the electric current flows differently at different depths in the sample. A purely static skin effect takes place, with the direct current being concentrated near the surface of the conductor.

Of course, the static skin effect for currents arises only at those magnetic-field orientations for which the resistance increases without limit with the field, i.e., when the numbers of the electrons and holes are equal or when the Fermi surface is open. In the opposite case, the distribution of the electric current in the sample is determined mainly by the Hall components of the electric conductivity tensor, which have the same asymptotic form for all depths in the conductor (the Hall field, as usual, is excluded from the condition that the total Hall current in the conductor vanish).

If the scattering of the electrons by the boundary of the sample is specular, as is apparently the case in semimetals (Bi, Sb), then the reflected and incident electrons turn out to be strictly correlated. When the electron is specularly scattered, its energy and the projections p_η and p_ζ of its momentum on the plane tangent to the surface of the sample at the point of incidence of the electron are conserved. Consequently, the projection of the momentum on the inward normal to the surface of the sample (this direction is chosen to be the ξ axis) is determined from the equation

$$\varepsilon(p_\zeta', p_\eta, p_\xi) = \varepsilon(p_\zeta, p_\eta, p_\xi);$$
$$v_\xi(p_\xi') > 0, \quad v_\xi(p_\xi) < 0. \qquad (2)$$

here $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the electron velocity.

If the Fermi surface is nonconvex or multifoliate, then equation (2) may have several solutions p_ξ', and it is then necessary to take into account the concrete mechanism of the scattering of the electron by the boundary of the sample. The Fermi surfaces of Bi and Sb, for which specular scattering can be expected, are well approximated by several ellipsoids, and these ellipsoids are so narrow that only in a small region of directions ξ (the region of normals n) can an electron be transferred, by virtue of Eq. (2), from one ellipsoid to another (the so called intervalley transition). Therefore in most cases equation (2) has a unique solution, and, knowing the momentum of the electron and its coordinates inside the conductor, it is possible to determine all the instants of the reflection of the electron from the boundary of the sample, experienced by the electron over the mean free path l. In this case only the mean free path is the dissipative length. As a result, even in metals with closed Fermi surfaces, owing to the specular scattering by the sample boundary, the electron may move on an open trajectory, and the electrons with open orbits make an appreciably larger contribution to the electric conductivity than the electrons whose orbits are closed in momentum space. It is perfectly clear that in the case of specular scattering a change takes place in the criterion whereby the sample is regarded as bulky, and the boundary effects turn out to be significant even in samples whose thickness d greatly exceeds the electron mean free path ($d \gg l$).

The situation is particularly simple in a magnetic field parallel to the surface of the plate. The electrons here are only from the first and third groups, with those of the first group moving on open trajectories. The motion of these electrons is infinite in the entire plane of the plate and is bounded only by the mean free path l, whereas the electrons of the third group (in the center of the sample) can drift only along the direction of the magnetic field.

The transverse electric conductivity of a plate in a parallel magnetic field is

$$\sigma_\perp = \sigma_0 (r/d + r^2/l^2), \qquad (3)$$

and when $d \ll l^2/r$ the electric conductivity $\sigma_\perp$ is determined only by the surface layer of the plate, in which practically the entire current flowing through the sample is concentrated. Factors on the order of unity in formula (3) have been omitted. The resistance is inversely proportional to the thickness of the current layer, i.e., to the Larmor radius, and the Kapitza law (ρ 11 H) holds true.

According to presently available experimental results, the resistance of thin metallic conductors (Cu, Au) in the absence of an external magnetic field behaves as if the scattering of the electrons by the boundary of the sample were specular[8]. This is apparently connected with the fact that when H = 0 the electric current is conducted in thin samples primarily by glancing electrons, i.e., electrons moving practically parallel to the surface of the sample. The collision between such electrons and the surface of the sample is accompanied by a small change of the electron momentum, and these collisions are practically specular. In a strong magnetic field, all the electrons trajectories become twisted, with the exception of the electrons at the limiting point of the Fermi surface, so that a characteristic feature in this case is a steep incidence of the electron on the surface of the sample, and the momentum loss upon collision is of the order of the momentum of the incident electron. Consequently, in a magnetic field, only carriers with small momentum are specularly scattered by the boundary of the sample, namely anomalously small groups of electrons which practically have no influence on the electric conductivity, or else the electrons in Bi and Sb.

We shall develop below a theory of electric conductivity of bounded conductors in a magnetic field. Although the character of the scattering of the electrons by the boundary of the sample is assumed to be arbitrary, nevertheless principal attention will be paid to the case of greatest interest, that of specular scattering.

1. COMPLETE SYSTEM OF EQUATIONS OF THE PROBLEM

In order to determine the electric current

$$\mathbf{j}(\mathbf{r}) = e\int \mathbf{v} f(\mathbf{r},\mathbf{p})\, d^3p, \tag{4}$$

it is necessary to solve the kinetic equation for the electron distribution function $f(\mathbf{r}, \mathbf{p})$:

$$\mathbf{v}\frac{\partial f}{\partial \mathbf{r}} + e\mathbf{E}\frac{\partial f}{\partial \mathbf{p}} = \hat{W}f, \tag{5}$$

which must be supplemented by the equation for the electric field—the condition for the electron neutrality of the conductor:

$$\int e[f(\mathbf{r},\mathbf{p}) - f_0(\varepsilon)]\, d^3p = 0. \tag{6}$$

The latter follows from Maxwell's equations and is connected with the fact that the plasma frequency $\omega_0 = (4\pi\sigma_0/t_0)^{1/2}$ greatly exceeds the actually realized electron collision frequencies $1/t_0$ even for such poor conductors as Bi and Sb.

We seek the distribution function f in the form

$$f = f_0(\varepsilon) - e\psi(\mathbf{r},\ \mathbf{p})\,\partial f_0/\partial\varepsilon.$$

Here $f_0(\epsilon)$ is the equilibrium Fermi distribution function of the electrons, e is the electron charge, and $\hat{W}f$ is the collision integral, which we shall assume, for convenience in calculation, to be the operator of multiplication of the non-equilibrium addition to the distribution function of the electrons by the collision frequency:

$$\hat{W}f = (f_0 - f)/t_0. \tag{7}$$

Linearizing the kinetic equation (5) with respect to the weak electric field $\mathbf{E}$, we obtain for the function ψ the equation

$$\mathbf{v}\frac{\partial\psi}{\partial\mathbf{r}} + \frac{\partial\psi}{\partial t} + \frac{\psi}{t_0} = \mathbf{E}\mathbf{v}. \tag{8}$$

We have retained all the symbols of[6], and as before, t is the time of motion of the electron along the orbit ϵ = const and p_z = const in the magnetic field.

At $t_0 = \infty$, equation (8), naturally, coincides with equation (6) of[6]. In the case of a finite electron mean free path, the general solution of (8) is

$$\psi(\mathbf{r},\mathbf{p}) = e^{-t/t_0}U(\mathbf{r}-\mathbf{r}(t)) + \int_{\lambda(\mathbf{r},\,t,\,p_z)}^{t} e^{(t_1-t)/t_0}\mathbf{v}(t_1)E(\mathbf{r}+\mathbf{r}(t_1,p_z)-\mathbf{r}(t,p_z))\,dt_1, \tag{9}$$

where

$$\mathbf{r}(t,p_z) = \int^{t}\mathbf{v}(t,p_z)\,dt,$$

and U is an arbitrary function of its argument. Since the moment of reflection of the electron λ from the surface of the sample at the point $\mathbf{r}_s$ is also a function of $\mathbf{r} - \mathbf{r}(t)$

$$\mathbf{r}-\mathbf{r}(t) = \mathbf{r}_s - \int^{\lambda}\mathbf{v}(t_1)\,dt_1, \tag{10}$$

it follows that it is more convenient for us to represent U in the form

$$U(\mathbf{r}-\mathbf{r}(t)) = \exp\left\{\frac{\lambda(\mathbf{r};t,p_z)}{t_0}\right\}F(\mathbf{r}-\mathbf{r}(t)).$$

The function F describes the distribution over the momenta of the reflected electrons on the surface of the conductor:

$$\psi^s|_{v_\xi>0} = F(\mathbf{r}_s-\mathbf{r}(t)), \tag{11}$$

and the form of this function depends significantly on the character of the scattering of the electrons by the boundary of the conductor. For example, in diffuse scattering all the directions of electron reflection are equally probable, and on the Fermi surface F is a constant quantity, which can be readily determined from the condition that the normal component of the electric current outside the current contacts vanish on the surface of the sample:

$$j_\xi^s = \langle v_\xi F\rangle_+ + \langle v_\xi\psi^s\rangle_-. \tag{12}$$

We have introduced here the symbol

$$\langle g\rangle \equiv \int_{\varepsilon(\mathbf{p})=\varepsilon_F} e^2\left|\frac{\partial\varepsilon}{\partial\mathbf{p}}\right|^{-1} g\,dS_p \equiv \frac{e^3H}{c}\int\int_{\varepsilon(\mathbf{p})=\varepsilon_F} g\,dt\,dp_z,$$

where dS_p is an area element on the Fermi surface, ϵ_F is the Fermi energy. In the expression for $\langle g\rangle_+$, the integration is carried out only over that part of the Fermi surface, where $v_\xi > 0$ and $\langle g\rangle_- \equiv \langle g\rangle - \langle g\rangle_+$.

In the kinetic equations, for an arbitrary electron scattering, it is customary to introduce the specularity parameter q—the probability that the electron will be specularly reflected from the sample surface, so that q = 0 corresponds to a diffuse scattering and q = 1 to a pure specular scattering. Then the functions ψ for the reflected and incident electrons on the surface of the sample are connected by the following relation (see (2)):

$$\psi^s(t',p_z')|_{v_\xi>0} = q\psi^s(t,p_z)|_{v_\xi<0} + \chi. \tag{13}$$

Using (12), we can express the change of the chemical potential of the reflected electrons $\chi(\mathbf{r}_s)$ in terms of the function $\psi^s(t, p_z)$ of the incident electrons:

$$\chi(\mathbf{r}_s) = (1-q)\frac{\langle v_\xi\psi^s(t,p_z)\rangle_-}{\langle v_\xi\rangle_-}. \tag{14}$$

The fundamental equations for the determination of the electrostatic potential

$$\varphi(\mathbf{r}) = -\int E(\mathbf{r})d\mathbf{r} \tag{15}$$

and of the function $F(\mathbf{r} - \mathbf{r}(t))$ at $q \neq 0$ are the condition (6) for the electron neutrality of the conductor and the condition (13), which can be written in the form

$$\psi^s(t', p_z') = q\psi^s(t, p_z) + (1 - q)\frac{\langle v_\xi \psi^s(t, p_z)\rangle_-}{\langle v_\xi\rangle_-}. \tag{16}$$

Along the trajectory of the electron motion in the magnetic field, which is a characteristic of the kinetic equation (8) at $t_0 = \infty$, the function F is a constant quantity:

$$F(\mathbf{r} - \mathbf{r}(t, p_z)) = F(\mathbf{r}_i - \mathbf{r}(\lambda_i, p_z)) = F_i \tag{17}$$

(λ_i is the moment of the reflection of the electron from the sample surface at the point $\mathbf{r}_i$). Upon colliding with the surface of the sample, the electron goes over in general to another orbit p_z = const, and then the change of the function F can be determined with the aid of equation (16), which is a recurrence relation connecting F_i with the value of the function F_{i+1} at the earlier instant of collisions $\lambda_{i+1} < \lambda_i$:

$$F_i = \hat{R}_i F_{i+1} + A_i, \tag{18}$$

where

$$A_i = -q\int_{\lambda_{i+1}}^{\lambda_i} \exp\left\{\frac{t_1 - \lambda_i}{t_0}\right\} d\varphi(\mathbf{r}_i + \mathbf{r}(t_1) - \mathbf{r}(\lambda_i))$$
$$-\frac{1-q}{\langle v_\xi\rangle_-}\left\langle v_\xi(t)\int_{\lambda(\mathbf{r}_i, t)}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} d\varphi(\mathbf{r}_i + \mathbf{r}(t_1) - \mathbf{r}(t))\right\rangle_-; \tag{19}$$

$$\hat{R}_i g \equiv q\exp\left\{\frac{\lambda_{i+1} - \lambda_i}{t_0}\right\} g$$
$$+\frac{1-q}{\langle v_\xi\rangle_-}\left\langle v_\xi(t)\exp\left\{\frac{\lambda(\mathbf{r}_i, t) - t}{t_0}\right\} g(\mathbf{r}_i - \mathbf{r}(t))\right\rangle_-. \tag{20}$$

If the time between two collisions of the electron with the surface of the sample $\delta\lambda_i = \lambda_i - \lambda_{i+1}$ is much larger than the free path time of the electron, as is possible in thick samples, then the function F can be determined, with a high degree of accuracy, in terms of the well known distribution function of the electrons in an unbounded sample. In this case the electron distribution function inside the conductor is

$$\psi(\mathbf{r}; t, p_z) = \int_{\lambda}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} \mathbf{v}(t_1)\mathbf{E}(\mathbf{r} + \mathbf{r}(t_1, p_z) - \mathbf{r}(t, p_z))dt_1$$
$$+ q\int_{-\infty}^{\lambda} \exp\left\{\frac{t_1 - t}{t_0}\right\} \mathbf{v}(t_1 + \delta, p_z - \delta p_z)\mathbf{E}(\mathbf{r} + \mathbf{r}(t_1 + \delta, p_z - \delta p_z)$$
$$- \mathbf{r}(t, p_z))dt_1 + \frac{1-q}{\langle v_\xi\rangle_-}\left\langle v_\xi(\lambda)\int_{-\infty}^{\lambda} \exp\left\{\frac{t_1 - t}{t_0}\right\} \mathbf{v}(t_1 + \delta, p_z - \delta p_z)\right.$$
$$\left.\times \mathbf{E}(\mathbf{r}_s + \mathbf{r}(t_1 + \delta, p_z - \delta p_z) - \mathbf{r}(\lambda + \delta, p_z - \delta p_z))dt_1\right\rangle_-. \tag{21}$$

In (21), the integration variable t_1 coincides with the true time of motion of the electron. The phase shifts δ and δp_z are the coordinates, in terms of the variables ϵ, p_z, and t, of the discontinuity of the electron momentum vector upon colliding with the surface of the sample at the instant of time λ:

$$\delta\mathbf{P} = \{p_\xi'(\lambda) - p_\xi(\lambda)\}\mathbf{n}, \tag{22}$$

so that $\delta p_z = \{p_\xi'(\lambda) - p_\xi(\lambda)\}$ cos α is the angle between the direction of the magnetic field and the inward normal n to the surface of the sample; the quantity δ describes the projection of the vector $\delta\mathbf{P}$ in the xy plane and vanishes if $\sin\alpha = 0$.

To determine the function F for thin samples ($d \ll l$) it is necessary to use the recurrence relation (18) many times:

$$F_n = \prod_{i=0}^{m-1} \hat{R}_{n+i} F_{n+m} + A_n + \sum_{j=1}^{m-1} \prod_{i=0}^{j-1} R_{n+i} A_{n+j}, \quad m > 1. \tag{23}$$

Since the norm of the operator $\hat{R}_i = \exp\{-\delta\lambda_i/t_0\} < 1$, then as $m \to \infty$ the first term in (23) tends to 0, and the function F_n can be represented in the form of a well-converging series:

$$F_n = A_n + \sum_{j=1}^{\infty} \prod_{i=0}^{j-1} R_{n+i} A_{n+j}. \tag{24}$$

The equation for the determination of the electrostatic potential—the condition for the electron neutrality of the conductor—now assumes the form

$$\left\langle \exp\left\{\frac{\lambda(\mathbf{r}; t) - t}{t_0}\right\}\left\{A_1 + \sum_{j=2}^{\infty}\prod_{i=1}^{j-1} \hat{R}_i A_j\right\}\right\rangle$$
$$-\left\langle \int_{\lambda(\mathbf{r}; t)}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} d\varphi(\mathbf{r} + \mathbf{r}(t_1) - \mathbf{r}(t))\right\rangle = 0. \tag{25}$$

The obtained system of equations (25), (24), (19), and (20) solves in principle the problem of determining the electric-current distribution in the sample

$$\mathbf{j}(\mathbf{r}) = -\left\langle \mathbf{v}(t)\int_{\lambda(\mathbf{r}, t)}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} d\varphi(\mathbf{r} + \mathbf{r}(t_1) - \mathbf{r}(t))\right\rangle$$
$$+\left\langle \mathbf{v}(t)\exp\left\{\frac{\lambda(\mathbf{r}; t) - t}{t_0}\right\}\left\{A_1 + \sum_{j=2}^{\infty}\prod_{i=1}^{j-1} \hat{R}_i A_j\right\}\right\rangle. \tag{26}$$

The main difficulty in this problem arises in the summation of the series (24). If the electron collides frequently with the surface of the sample and executes a non-periodic motion (for example, in a magnetic field inclined to the surface of the thin plate), then it is possible to obtain only a qualitative solution of the problem. In all other cases it is possible to obtain an explicit expression for the distribution of the electric current in the sample.

In the case of diffuse scattering of electrons by the sample surface, the function F is a constant quantity for all the reflected electrons, and, as noted above, it can be readily determined from the condition (12):

$$F = \chi = \left\langle v_\xi(t)\int_{\lambda}^{t} \exp\left\{\frac{t' - t}{t_0}\right\} \mathbf{v}(t')\mathbf{E}(\mathbf{r}_s + \mathbf{r}(t') - \mathbf{r}(t))dt'\right\rangle_-$$
$$\times\left\langle v_\xi(t)\left[1 - \exp\left\{\frac{\lambda(\mathbf{r}_s, t) - t}{t_0}\right\}\right]\right\rangle_-^{-1}. \tag{27}$$

When q = 0, the electron neutrality condition is

$$\left\langle \exp\left\{\frac{\lambda(\mathbf{r}; t) - t}{t_0}\right\}\right\rangle\left\langle v_\xi(t)\int_{\lambda}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} d\varphi(\mathbf{r}_s + \mathbf{r}(t_1) - \mathbf{r}(t))\right\rangle_-$$
$$\times\left\langle v_\xi(t)\exp\left\{\frac{\lambda(\mathbf{r}_s; t) - t}{t_0}\right\}\right\rangle_-^{-1}$$
$$+\left\langle \int_{\lambda(\mathbf{r}; t)}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} d\varphi(\mathbf{r} + \mathbf{r}(t_1) - \mathbf{r}(t))\right\rangle = 0. \tag{28}$$

As $t_0 \to \infty$, equations (27) and (28), naturally, go over into the corresponding expressions in [6].

Allowance for the finite mean free path of the electron does not lead to an essential change of the results obtained in[6]. For any ratio between the sample thickness and the electron mean free path, the resistance of conductors with symmetrical current contacts either increases quadratically with the magnetic field, or reaches saturation in strong magnetic fields.

The problem is greatly simplified also at q = 1, when the operator $\hat{R}_i$ is the operator of multiplication by $\exp\{(\lambda_{i+1} - \lambda_i)/t_0\}$, and the series (24) can be readily summed for an arbitrary sample geometry and for an arbitrary electron dispersion law. In this case the electron distribution function is

$$\psi(\mathbf{r}; t, p_z) = \int_{\lambda_1}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} \mathbf{v}(t_1, p_z) \mathbf{E}(\mathbf{r} + \mathbf{r}(t_1, p_z) - \mathbf{r}(t, p_z))\, dt_1 + \sum_{i=1}^{\infty} \int_{\lambda_{i+1}}^{\lambda_i} \exp\left\{\frac{t_1 - t}{t_0}\right\} \mathbf{v}(t_1 + \delta_i, p_{zi}) \times \mathbf{E}(\mathbf{r}_i + \mathbf{r}(t_1 + \delta_i, p_{zi}) - \mathbf{r}(\lambda_i + \delta_i, p_{zi}))\, dt_1. \quad (29)$$

We note that this result can also be obtained by using the Chambers method[9], if in the electron distribution function

$$f(\mathbf{r}, \mathbf{p}) = f_0(\varepsilon - \Delta\varepsilon) \approx f_0(\varepsilon) - \Delta\varepsilon \partial f_0 / \partial\varepsilon$$

$\Delta\epsilon$ is taken to mean the entire energy acquired by the charge in the electric field by the instant of time t:

$$\Delta\varepsilon(t, p_z; \mathbf{r}) = e \int_{\lambda_1}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} \mathbf{v}(t_1, p_z) \times \mathbf{E}(\mathbf{r} + \mathbf{r}(t_1, p_z) - \mathbf{r}(t, p_z))\, dt_1 + e \sum_{i=1}^{\infty} \int_{\lambda_{i+1}}^{\lambda_i} \exp\left\{\frac{t_1 - t}{t_0}\right\} \mathbf{v}(t_1 + \delta_i, p_{zi}) \times \mathbf{E}(\mathbf{r}_i + \mathbf{r}(t_1 + \delta_i, p_{zi}) - \mathbf{r}(\lambda_i + \delta_i, p_{zi}))\, dt_1. \quad (30)$$

In each specular collision with the sample surface at the instant λ_i, the electron energy is conserved, thus ensuring continuity of the function $\Delta\epsilon(t) = e\psi(t, p_z; \mathbf{r})$. Whereas in diffuse scattering the condition (12) is the equation for the determination of the function F, in purely specular scattering the condition (12) that there be no electric current flow through the surface of the sample is satisfied automatically. Since we have, by

$$v_\xi \frac{eH}{c}\, d\varepsilon\, dt\, dp_z = v_\xi\, dp_\zeta\, dp_\eta\, dp_\xi = \operatorname{sign} v_\xi\, d\varepsilon\, dp_\eta\, dp_\zeta,$$

virtue of the conservation of the quantities ϵ, p_η, and p_ζ in collisions between the electron and the surface of the sample and by virtue of the continuity of the electron distribution function ψ,

$$j_\xi^s = \langle v_\xi \psi^s \rangle_- + \langle v_\xi' \psi^s \rangle_+ = (\operatorname{sign} v_\xi + \operatorname{sign} v_\xi') \langle v_\xi \psi^s \rangle_- \equiv 0.$$

If the electron collides periodically with the surface of the sample, then the problem can be exactly solved also in the case of an arbitrary character of electron scattering ($0 < q < 1$). In this case all the operators $\hat{R}_i$ are identical, the series (24) is a geometrical progression and can be easily summed. We shall illustrate the proposed method of calculating the electric-current distribution in the sample by means of several examples.

2. RESISTANCE OF A PLATE IN A MAGNETIC FIELD

We consider first the simplest case when the magnetic field is parallel to the surface of the plate. In strong magnetic fields ($r \ll l$, d) the surfaces of the plates are not correlated with each other, since the electrons can collide with only one of them. It therefore suffices to consider the distribution of the electric current

$$j_\beta(\xi) = \langle v_\beta(t) \exp\left\{\frac{\lambda(\xi; t) - t}{t_0}\right\} F(\xi - \xi(t)) \rangle + \langle v_\beta(t) \int_{\lambda(\xi; t)}^{t} \exp\left\{\frac{t_1 - t}{t_0}\right\} \mathbf{v}(t_1) \mathbf{E}(\xi + \xi(t_1) - \xi(t))\, dt_1 \rangle, \quad \beta = (x, \eta), \quad (31)$$

near one of the surfaces of the plates, say the surface $\xi = 0$. In the interior of the sample, the electrons do not collide with the surface, and therefore $\lambda = -\infty$, the electric current is determined only by the second term in (31), and is homogeneous over the entire interior of the sample (a homogeneous electric field, as in an unbounded conductor, satisfies the electroneutrality condition (6)).

Near the surface, the most important role in (31) is played by the first term, and the problem reduces to a determination of the function $F(\xi - \xi(t))$. In a magnetic field parallel to the plate surface, for specular reflection of the electron, the momentum projections p_z and p_x are conserved. Consequently, the function $p_x(t)$ is continuous on the entire open trajectory of the electron. From the equations of motion of the electron

$$x(t) = x_0 - cp_y(t) / eH, \quad \xi(t) \equiv y(t) = y_0 + cp_x(t) / eH, \quad (32)$$

where x_0, y_0 are the coordinates of the center of its orbit, it follows that the electron, starting from the surface of the plate with a certain value of the momentum $p_x = eHy_0/c$, returns to the surface with the same value of the momentum p_x. If equation (2) has a unique solution, then after a time interval $\delta\lambda_i = \lambda_i - \lambda_{i+1}$ the electron returns to the initial state, i.e., its motion is periodic. As a result, the function F is constant along the entire open trajectory of the electron:

$$F(-\xi(\lambda_{i+1})) = F(-\xi(\lambda_i)) = F(-\xi(\lambda)).$$

In this case it is more convenient to use directly the relations (12) and (13) and to obtain an expression for $F(-\xi(\lambda))$:

$$F(\xi - \xi(t)) = F(-\xi(\lambda)) = \left[\chi + q \int_{\lambda_2}^{\lambda_1} \exp\left\{\frac{t_1 - \lambda_1}{t_0}\right\} \mathbf{v}(t_1) \mathbf{E}(\xi(t_1) - \xi(\lambda_1))\, dt_1\right] \times \left[1 - q \exp\left\{\frac{\lambda_2 - \lambda_1}{t_0}\right\}\right]^{-1}, \quad (33)$$

where

$$\chi = \left\langle v_\xi(t) \int_{\lambda(0;t)}^{t} \exp\left\{\frac{t_1 - t}{t}\right\} \mathbf{v}(t_1) \mathbf{E}(\xi(t_1) - \xi(t)) \times \frac{1 - q}{1 - q \exp\{[\lambda(0; t) - t]/t_0\}}\, dt_1 \right\rangle_- \times \left\langle v_\xi(t) \frac{1 - \exp\{[\lambda(0; t) - t]/t_0\}}{1 - q \exp\{[\lambda(0; t) - t]/t_0\}} \right\rangle^{-1}. \quad (34)$$

Using the electron neutrality condition, we can express the field $E_\xi(\xi)$ in terms of the homogeneous electric fields E_x and E_η (the η axis is always the projection of the direction of the magnetic field on the surface of the sample; in the case of a parallel magnetic field the η axis coincides with the z axis). However, there is no necessity for determining the inhomogeneous electric field. Since open electron orbits are possible only in the surface layer of thickness $\Delta\xi = 2cp_\perp^{max}/eH \approx 2r_{max}$, it is sufficient to retain in expression (31) for the electric current only the terms proportional to the electron mean free path $l = vt_0$. It is easy to show that these terms do not contain the inhomogeneous field E_ξ:

$$\int_{\lambda(0;t)}^{t} \exp\left\{\frac{t_1-t}{t_0}\right\} E_\xi(\xi(t_1)-\xi(t))v_\xi(t_1)dt_1$$

$$\approx \frac{1}{t_0}\int_{\lambda(0;t)}^{t}\Phi(\xi(t_1)-\xi(t))\exp\left\{\frac{t_1-t}{t_0}\right\}dt_1 + \frac{\lambda(0;t)-t}{t_0}\Phi(0),$$

$$\int_{\lambda_2}^{\lambda_1}\exp\left\{\frac{t_1-\lambda_1}{t_0}\right\}E_\xi(\xi(t_1)-\xi(\lambda_1))v_\xi(t_1)dt_1$$

$$\approx \frac{\lambda_2-\lambda_1}{t_0}\Phi(0)+\frac{1}{t_0}\int_{\lambda_2}^{\lambda_1}\Phi(\xi(t_1)-\xi(\lambda_1))dt_1,$$

where $\Phi(\xi)$ is the electric potential in the direction of the ξ axis:

$$\varphi(\mathbf{r}) = -E_\beta x_\beta + \Phi(\xi).$$

We see therefore that in the expression (34) for the electric current the terms containing the inhomogeneous electric field E_ξ are proportional to r, and not to l, and are immaterial for the determination of the asymptotic total electric conductivity of the plate.

The distribution of the electric current near the surface of the plate, in the main approximation in $\gamma = r/l$, is given by

$$j_\alpha(\xi) \approx \langle v_\alpha(t)F(\xi-\xi(t))\rangle = \left\langle \frac{v_\alpha(t)[\chi + qE_\beta(x_\beta(\lambda_1)-x_\beta(\lambda_2))]}{1-q\exp\{(\lambda_2-\lambda_1)/t_0\}}\right\rangle. \tag{35}$$

The electric conductivity $\sigma_{xx}(\xi)$ in the surface layer, as seen from formula (35), is quite large and increases with increasing degree of specularity of the scattering of the electron by the sample surface:

$$\sigma_{xx}(\xi) = \left\langle \frac{v_x(t)[x(\lambda_1)-x(\lambda_2)]}{1-q\exp\{(\lambda_2-\lambda_1)/t_0\}}\right\rangle \sim \sigma_0\frac{\gamma}{1-q+\gamma}. \tag{36}$$

The total transverse electric conductivity of the plate

$$\sigma_\perp = \sigma_0\frac{\gamma}{1-q+\gamma}\frac{r}{d} + \sigma_0\gamma^2 = \sigma_0\gamma^2\left[1+\frac{l/d}{q_1+\gamma}\right] \tag{37}$$

is determined mainly by a surface layer with thickness on the order of r, if

$$q_1 \lesssim \gamma \ll l/d,$$

where $q_1 = 1 - q$ is the parameter of the diffuseness of the scattering of the electrons by the conductor surface (factors of the order of unity were omitted from formula (37)).

The weak diffuseness of the electron scattering, which apparently really exists even in Bi, limits somewhat the region of magnetic fields at which the Kapitza law holds true. The transverse resistance of the plate

$$\rho = \rho_\infty\left[1+\frac{l/d}{q_1+\gamma}\right]^{-1} = \begin{cases} Aq_1dH^2 + B(d/l)H, & d \ll l^2/r \\ \rho_\infty, & d \gg l^2/r \end{cases} \tag{38}$$

increases linearly with the magnetic field if $q_1 l \ll r \ll l^2/d$. The constants A and B are determined by the dynamic characteristics of the electrons (they do not depend on the mean free path), and ρ_∞ is the resistivity of the unbounded conductor. A further increase of the magnetic field again leads to a quadratic growth of the resistance with increasing magnetic field when $r \ll lq_1$, but the size effect still remains (ρ ll d).

In thick samples and in a strong magnetic field, there are two regions of quadratic growth of the resistance, in the interval between which the resistance increases linearly with the magnetic field:

$$\rho = \begin{cases} \rho_\infty & l^2/d \ll r \ll l \\ B(d/l)H, & lq_1 \ll r \ll l^2/d. \\ Aq_1dH^2, & r \ll lq_1 \end{cases} \tag{39}$$

In thin plates ($d \ll l$), the first inequality in formula (39) is not realizable, and the resistance is never equal to ρ_∞. The linear dependence of the resistance on the magnetic field in the intermediate region of the magnetic fields ($r \sim l$), discovered by Borovik[10], goes over in strong magnetic fields into the Kapitza law.

In a magnetic field inclined to the surface of the plate, the electrons move in general along non-periodic trajectories. In the case of specular reflection of the electron, its momentum projection p_z changes jumpwise

$$\delta p_z = n_z\delta p, \tag{22a}$$

and the electron goes over to another orbit p_z = const after each collision with the surface of the sample (Fig. 2).

From the condition for the continuity of the function $p_\eta(t)$:

$$p_\eta(t) = p_z\cos\theta + p_y(t)\sin\theta$$

we can determine the jump of the center of the orbit:

$$\delta p_y = -\delta p_z \operatorname{ctg}\theta = -\frac{eH}{c}\delta x_0 \tag{40}$$

(θ is the angle of inclination of the magnetic field to the surface of the sample). It is important to note that in each reflection of the electron from the surface $\xi = 0$, the jump of the center or orbit δx_0 has the same direction, since δp_z has the same sign. Near the surface $\xi = d$, the momentum jump δp_z has an opposite sign, since the inward normals to the surfaces of the plate

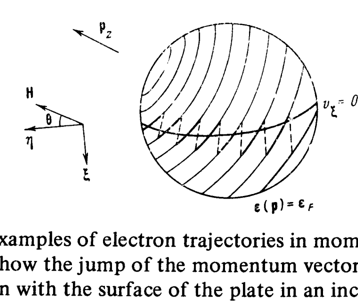

FIG. 2. Examples of electron trajectories in momentum space. The dashed lines show the jump of the momentum vector **δp** upon collision of the electron with the surface of the plate in an inclined magnetic field.

are antiparallel. Consequently, just as in a parallel magnetic field, near the surfaces $\xi = 0$ and $\xi = d$ the centers of the electron orbits drift in opposite directions. However, the total displacement of the center of the electron orbit during the free-path time turns out to be much smaller than l:

$$\delta x^{\text{tot}} = -\frac{c}{eH}\delta p_y^{\text{tot}} = \frac{c}{eH}\delta p_z^{\text{tot}} \operatorname{ctg}\theta < 2r\operatorname{ctg}\theta. \qquad (41)$$

Consequently, the electric conductivity of a surface layer of thickness of the order r in an inclined field is smaller by a factor l/r than at $\theta = 0$ and $q = 1$, since $l_{\text{eff}} \sim r$. In (41) it is assumed that $\theta \gg r/l$, and in the opposite case $\delta_x^{\text{tot}} = \min\{l, 2r\cot\theta\}$.

In a thin plate the electron may collide with both surfaces of the plate, and the center of its orbit may shift in opposite directions. Therefore the electron drift in the interior of the sample in the direction of the x axis turns out to be practically negligible, and the electric conductivity of the interior layers of the plate coincides in order of magnitude with the electric conductivity of a bulky sample.

In the plates ($d \gg l$) the surfaces $\xi = 0$ and $\xi = d$ are not correlated with each other. An electron near the surface $\xi = 0$ has two alternatives: it can either be reflected once or several times from the sample boundary and go off to the interior of the conductor, or it can experience a large number of specular collisions and approach the line $v_\xi = 0$ on the Fermi surface (Fig. 2). In both cases the displacement of the center of the orbit in the x-axis direction is of the order of r. In the case of diffuse scattering of the electrons by the sample surface, although there is no correlation between the incident and reflected electrons, nevertheless the centers of the electron orbits are also shifted by a distance on the order of r, leading to a large electric conductivity of a surface layer with thickness of the order of r. In an inclined magnetic field, the asymptotic value of the plate resistance in a strong magnetic field depends little on the scattering specularity parameter q. We therefore confine ourselves to the case of pure specular reflection ($q = 1$).

It is easy to show that in an inclined magnetic field there is likewise no need to determine the inhomogeneous electric field. Allowance for the specular collisions of the electrons in the expression for the electric current

$$\begin{aligned} j_\alpha(\xi) = &\Big\langle v_\alpha(t)e^{-t/t_0}\Big\{\int_{\lambda_1}^{t} e^{t_1/t_0} d\Phi(\xi + \xi(t_1) - \xi(t)) \\ &+ \sum_{i=1}^{\infty}\int_{\lambda_{i+1}}^{\lambda} e^{t_1/t_0} d\Phi(\xi(t_1+\delta_i, p_{zi}) - \xi(\lambda_i + \delta_i, p_{zi}))\Big\}\Big\rangle \\ &+ \Big\langle v_\alpha(t, p_z)e^{-t/t_0}\Big\{\int_{\lambda_1}^{t} e^{t_1/t_0} v_\beta(t_1, p_z)\,dt_1 \\ &+ \sum_{i=1}^{\infty}\int_{\lambda_{i+1}}^{\lambda_i} e^{t_1/t_0} v_\beta(t_1+\delta_i, p_{zi})\,dt_1\Big\}\Big\rangle E_\beta \end{aligned} \qquad (42)$$

is significant only in the component σ_{xx} of the electric conductivity tensor

$$\begin{aligned} \sigma_{xx}(\xi) = &\frac{c}{eH}\Big\langle v_x(t)e^{-t/t}\sum_{i=1}^{\infty} e^{\lambda_i/t_0}\delta p_y(\lambda_i)\Big\rangle \\ &+ \frac{c}{eH}\Big\langle v_x(t, p_z)e^{-t/t_0}\frac{1}{t_0}\Big\{\int_{\lambda_i}^{t} e^{t_1/t_0} p_y(t_1, p_z)\,dt_1 \\ &+ \sum_{i=1}^{\infty}\int_{\lambda_{i+1}}^{\lambda_i} e^{t_1/t_0} p_y(t_1+\delta_i, p_{zi})\,dt_1\Big\}\Big\rangle. \end{aligned} \qquad (43)$$

Here $\delta p(\lambda_i)$ is the discontinuity of the electron momentum upon reflection from the surface of the sample at the instant of time λ_i. In formula (43) we used the relation

$$v_x(t_1) = -\frac{c}{eH}\frac{\partial p_y(t_1)}{\partial t_1}.$$

In a parallel magnetic field $\delta p_y(\lambda_i)$ is a constant quantity and

$$\frac{c}{eH}\sum_{i=1}^{\infty} e^{\lambda_i/t_0}\delta p_y(\lambda_i) \approx \frac{c}{eH}\frac{\delta p_y(\lambda_1)}{1-\exp\left\{\frac{(\lambda_2-\lambda)}{t_0}\right\}} \approx l,$$

while in an inclined magnetic field

$$\left|\frac{c}{eH}\sum_{i=1}^{\infty} e^{\lambda_i/t_0}\delta p_y(\lambda_i)\right| < \frac{c}{eH}\left|\sum_{i=1}^{\infty}\delta p_y(\lambda_i)\right| < 2r\operatorname{ctg}\theta.$$

In the first term of (43), the summation is only over the states (t, p_z) of the electrons colliding with the surface of the sample, for which not all values of t are admissible near the surface of the conductor (this is clearly seen from Fig. 2, the phase on the electron orbit changes jumpwise upon collision). Therefore the mean value

$$\overline{Sv_x} = T^{-1}\int_0^T v_x(t)S(\xi - \xi(t))\,dt$$

does not vanish so long as $\xi < r$. Here $S(\xi - \xi(t))$ is the Heaviside function, namely,

$$S(x) = \begin{cases} 1, & x > 0 \\ 0, & x < 0' \end{cases}$$

and T is the period of motion of the electron in the magnetic field.

In the center of the sample $\overline{Sv}_x = \overline{v}_x = 0$, and the transverse electric conductivity is proportional to r^2.

A large electric current ($\sigma_{xx} \sim r$) is concentrated in a surface layer with thickness of the order of r, but the static skin effect in a magnetic field inclined to the surface of the plate does not lead to a dependence of the resistance on the magnetic field different from that in an unbounded conductor.

3. RESISTANCE OF A WIRE IN A MAGNETIC FIELD

The distribution of the electric current in a semimetallic wire

$$\begin{aligned} j_x(y,z) = &\sigma_{xx}(y,z)E_x + \Big\langle v_x(t, p_z)e^{-t/t_0} \\ &\times\Big\{\int_{\lambda_1}^{t} e^{t_1/t_0} d\varphi(y + y(t_1, p_z) - y(t, p_z);\, z + z(t_1, p_z) - z(t, p_z)) \\ &+ \sum_{i=1}^{\infty}\int_{\lambda_{i+1}}^{\lambda_i} e^{t_1/t_0} d\varphi(y + y(t_1+\delta_i, p_{zi}) - y(t, p_z); \\ &z + z(t_1+\delta_i, p_{zi}) - z(t, p_z))\Big\}\Big\rangle, \end{aligned} \qquad (44)$$

just as in a plate, does not depend significantly on the inhomogeneous electric field. It is therefore sufficient to investigate the first term in formula (44). Here $\varphi(y, z)$ is the electrostatic potential in the plane of the cross section of the wire, the magnetic field is perpendicular to the wire axis (x axis), and the scattering of the electrons by the surface of the sample is assumed to be purely specular.

We can represent the component of the electric conductivity tensor $\sigma_{xx}(y, z)$ by means of formula (43), except that the λ_i will now be functions of y and z. It suffices for us to investigate the term connected with the jumps of the projection of the momentum $\delta p_y(\lambda_i)$ upon collision of the electron with the surface of the wire:

$$\sigma_{xx}(y, z) \approx \frac{c}{eH}\left\langle v_x(t)\, e^{-t/t_0} \sum_{i=1} e^{\lambda_i/t_0} \delta p_y(\lambda_i) \right\rangle + \sigma_0 \left(\frac{r}{l}\right)^2. \quad (45)$$

For convenience in the calculations, we shall assume that the wire is a right circular cylinder with diameter d. It is seen from Fig. 3 that the quantity $\delta p_y(\lambda_i)$ is of constant sign, since the projections of the normal zone on the y axis at the wire-surface points connected by the electron-motion trajectory have equal signs.

In a thin wire ($d \ll l$), the electron can execute during the free-path time at least l/d collisions with the surface of the sample, i.e., it can move in the direction of the wire axis through a distance on the order of lr/d. The only exceptions are the regions $|y| \lesssim r$, where the electron does not drift at all along the wire axis, since $\delta p_y \approx 0$ at small values of $|y|$. A more accurate estimate shows that

$$\frac{c}{eH}\sum_{i=1}^{\infty} e^{\lambda_i/t_0}\delta p_y(\lambda_i) \approx \frac{lr}{d}\frac{y}{\sqrt{d^2-4y^2}}; \qquad \left|\frac{d}{2}-y\right| \gtrsim r. \quad (46)$$

i.e., the drift of the electrons increases with increasing distance from the axial cross section of the wire y = 0, and is maximal in the vicinity of the points $y = \pm d/2$. Practically the entire current flowing through the sample is concentrated in a small region, of the order of r^2 near the points $y = \pm d/2$. In Fig. 4 these are the doubly-hatched regions. The transverse resistance of the wire increases quadratically with increasing strong magnetic field, as does the resistance of an unbounded sample, much less than ρ_∞:

$$\rho = \rho_\infty (d/l)^2, \quad d < l.$$

In diffuse scattering of the electrons from the surface of the wire, we have $\rho = \rho_\infty d/l$. In thick wires ($d \gg l$), the electric conductivity of a small vicinity (with area on the order of r^2) of the points $y = \pm d/2$ is the same as in the absence of the magnetic field, and the electric conductivity near the entire surface of the wire is $\sigma \sim \sigma_0 r/l$. However, the total electric conductivity is determined essentially by the interior of the wire and $\rho = \rho_\infty$.

If the surface of the conductor has jags, then a large electric current is concentrated near the corners of these jags (in a cross section region on the order of r^2), and when the distance between the jags is smaller than or equal to the Larmor radius, the transverse resistance increases linearly with the strong magnetic field.

Thus, the distribution of the electric current in the interior of a conducting sample with equal numbers of electrons and holes ($n_1 = n_2$) depends strongly on the character of the scattering of the electron by the sample surface. In the case of specular scattering of electrons, which is possible only in semimetals, the transverse resistance can increase linearly with increasing strong magnetic field, owing to the static skin effect for the electric current.

The static skin effect in semimetals is most clearly pronounced if the surface of the sample has frequent roughnesses. In this case, the transverse resistance increases linearly with the field, until the Larmor radius is larger than or is of the order of the distance between the jags of the rough surface of the sample. In a magnetic field parallel to the surface of the plate, the state of the surface of the sample does not play an important role. Practically the entire current flowing through the plate is concentrated in a narrow surface layer with a thickness on the order of a Larmor radius r, and the resistance increases linearly with the magnetic field.

We have shown earlier that in a bounded sample the distribution of the electric current and of the heat flux are to a considerable degree analogous to those obtained in[7]. Using the ideas of[7], it is quite easy to show that the thermal resistance of semimetals (Bi and Sb) also increases linearly with the magnetic field if

$$r \ll l^2/d,$$

until the ratio r/l becomes comparable with the small quantity $q_1 = 1 - q$.

An investigation of the dependence of the electric resistance and of the thermal resistance of semimetals on the magnetic field makes it also possible to determine the parameter of the specularity of the scattering of electrons by the boundary of the sample. In metals, only electrons in anomalously small groups, which are completely responsible for the electric conductivity of the thin metallic sample ($d \lesssim l$), are specularly scattered by the surface of the sample, if

$$r \ll ln'/n,$$

where n' is the density of the electrons of the anomalously small groups, and $n = n_1 + n_2$. In this region of magnetic fields, the transverse resistance increases linearly with the magnetic field, but in such strong fields the quantization of the energy levels of the elec-

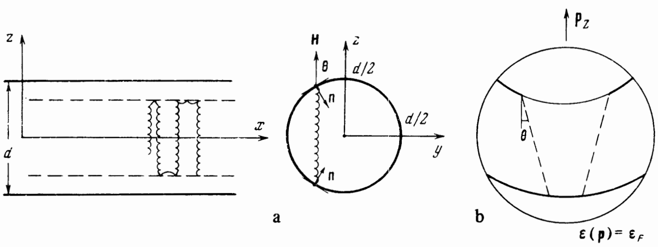

FIG. 3. Trajectories of electron motion in a magnetic field in a thin wire, in coordinate space (a) and in momentum space (b).

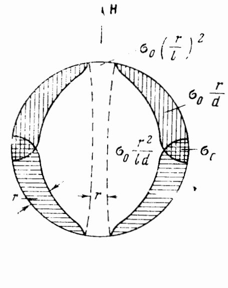

FIG. 4. Distribution of the electric current over the cross section of a thin wire. The entire current flowing through the sample is concentrated in the shaded regions, and an appreciable part of the current is concentrated in the doubly hatched region with area of the order of r^2.

trons becomes appreciable, and the quasiclassical analysis of the galvanomagnetic phenomena can hardly be regarded as valid.

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