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COLLISIONLESS DRIFT INSTABILITY IN A HIGH-FREQUENCY ELECTRIC FIELD

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We consider the drift instability in an inhomogeneous collisionless plasma located in a strong constant magnetic field $\mathbf{H}_0$ and a high-frequency electric field. It is shown that if the electric field is of the appropriate amplitude and frequency the application of such a field can cause an expansion of the stability region and a reduction of the growth rate.

In recent years a number of authors have considered both the experimental[1,2] and theoretical[3-6] aspects of the effect of external high-frequency fields on low-frequency oscillations in a plasma. In the present work we consider the stability, with respect to drift waves, of a collisionless, weakly inhomogeneous plasma located in a strong fixed magnetic field $\mathbf{H}_0$ and subject to a high-frequency electric field characterized by the components

$$\tilde{E}_z = \tilde{E}_{z0}\cos\Omega t, \quad \tilde{H}_y = \frac{c}{\Omega}\frac{\partial \tilde{E}_{z0}}{\partial x}\sin\Omega t$$

(The z-axis is directed along $\mathbf{H}_0$, the x-axis is in the direction of the plasma inhomogeneity, and Ω is the frequency of the high-frequency field).

In the unperturbed (stationary) state, in contrast with[4], we find that the high-frequency field exerts the following average force[7] on the plasma particles:

$$\mathbf{F}_\alpha = -m_\alpha \nabla \Phi_\alpha, \quad \Phi_\alpha = \frac{e^2 \tilde{E}_{z0}^2}{4m_\alpha^2\Omega^2},$$

where α is taken to be e or i respectively for the electrons or ions. Here it is assumed that the plasma is at rest in the laboratory reference system and that the effect of the average high-frequency force reduces to an additional redistribution of the plasma pressure. In particular, if the fixed magnetic field is uniform a state is possible in which the plasma is confined exclusively by the pressure associated with the high-frequency field.

Since the average high-frequency force is exerted primarily on the electrons, in the stationary state one expects a separation of charges and the appearance of an associated electrostatic field $E_x = -\partial\Psi/\partial x$, owing to which the ions are also confined.

In the present work the notion of a weakly inhomogeneous plasma means that the scale length of the inhomogeneity in density L_n, temperature L_T and high-frequency field δ_s (skin depth) is large compared with the Larmor radii of the particles.

By virtue of the assumptions made above, in describing the unperturbed state of the plasma we can make use of a local Maxwellian distribution function of the form

$$f_{0\alpha} = \left(\frac{m_\alpha}{2\pi T_\alpha(X_\alpha)}\right)^{3/2} n_{0\alpha}(X_\alpha)\cdot \exp\left[-\frac{{}^1\!/_2 m_\alpha(v_{\alpha\perp}^2 + v_{\alpha\parallel}^2) + m_\alpha\Phi_\alpha(x) + e_\alpha\Psi(x)}{T_\alpha(X_\alpha)}\right], \tag{1}$$

where

$$v_{\alpha\parallel} = v_z - \frac{e_\alpha \tilde{E}_{z0}(x)}{m_\alpha\Omega}\sin\Omega t, \quad X_\alpha = x + \frac{v_y}{\omega_{H\alpha}}, \quad \omega_{H\alpha} = \frac{e_\alpha H_0}{m_\alpha c}.$$

The density of charged particles $n_e = n_i = n$ is related to the parameter $n_{0\alpha}$ by

$$n = n_{0\alpha}\exp\left(-\frac{m_\alpha\Phi_\alpha + e_\alpha\Psi}{T_\alpha}\right). \tag{2}$$

The perturbed quantities are written in a form proportional to $\exp[i(\mathbf{k}\cdot\mathbf{r} - \omega t)]$ and we require that the following conditions be satisfied on ω and $k_z v_{T\alpha}$: $k_z\tilde{u}_{0e} \ll \Omega$, $\lambda_\perp \ll L_n$, L_T, δ_s where $\lambda_\perp = 2\pi/k_\perp$ is the wavelength of the perturbation in the direction transverse to $\mathbf{H}_0$, $v_{T\alpha} = \sqrt{2T_\alpha/m_\alpha}$ is the mean thermal velocity for particles of species α, $\tilde{u}_{0\alpha} = e_\alpha\tilde{E}_{z0}/m_\alpha\Omega$ is the peak oscillatory particle velocity in the high-frequency field. If these conditions are satisfied then, in particular, use can be made of the averaged high-frequency potential in describing the perturbed states; moreover, the problem can be solved in the geometric-optics approximation.

The perturbation distribution function $f_{1\alpha}$ satisfies the kinetic equation

$$\frac{\partial f_{1\alpha}}{\partial t} + (\mathbf{v}\nabla)f_{1\alpha} + (\mathbf{g}_\alpha^{ef} + [\mathbf{v}\boldsymbol{\omega}_{H\alpha}])\frac{\partial f_{1\alpha}}{\partial \mathbf{v}} + \left(\delta\mathbf{g}_\alpha - \frac{e_\alpha}{m_\alpha}\nabla\psi\right)\frac{\partial f_{0\alpha}}{\partial\mathbf{v}} = 0. \tag{3}$$

Here, $\mathbf{g}_\alpha^{ef} = -\nabla(\Phi_\alpha + e_\alpha m_\alpha^{-1}\Psi)$ is the effective acceleration associated with the "gravity force" which takes account of all electrostatic forces that act on the particles in the unperturbed state: $\delta\mathbf{g}_\alpha = -\nabla\delta\Phi_\alpha$ and $\psi \equiv \delta\Psi$ are the perturbations in the high-frequency and electrostatic potentials respectively.

Solving Eq. (3) (for example, by integration over trajectories) and integrating over $d\mathbf{v}$ we obtain the following expression for the perturbation in the density of particles of species α:

$$\delta n_\alpha = \left(\frac{e_\alpha}{m_\alpha}\psi + \delta\Phi_\alpha\right) n \frac{m_\alpha}{T_\alpha} G_\alpha,$$

$$G_\alpha = \frac{T_\alpha}{m_\alpha n}\int d\mathbf{v}\int_0^\infty i\mathbf{k}\frac{\partial f_{0\alpha}}{\partial\mathbf{v}}(t-\tau)\exp\{i\mathbf{k}[\mathbf{r}_\alpha(t-\tau) - \mathbf{r}_\alpha(t)] + i\omega\tau\}\,d\tau. \tag{4}$$

In order to obtain the dispersion relation we must compute the perturbation of the high-frequency potential $\delta\Phi_e$ and associate it with δn_e. For this purpose we must consider the perturbation in the high-frequency motion of the plasma particles that arises as a result of the modulation of the high-frequency oscillations by the low-frequencies; this occurs at frequencies $\Omega \pm \omega \approx \Omega$ (the combination tones above the first harmonic can be neglected by virtue of the condition $\Omega \gg k_z\tilde{u}_{0e}$).

Following[3-6] we shall only consider electrostatic perturbations of the high-frequency field and write $\delta\tilde{\mathbf{E}}$

$= -\nabla\tilde{\psi}$ where $\tilde{\psi} = \tilde{\psi}_0 e^{i\Omega t}$. It can be shown that when $\lambda_\perp \ll \delta_S$ and the wavelengths of the perturbation in the z-direction is reasonably small the contribution of nonelectrostatic perturbations can be neglected. Nonelectrostatic perturbations will be investigated elsewhere.

The perturbed high-frequency oscillations can be described, by analogy with[4], using a system of "quasi-hydrodynamic" equations for a cold plasma (this procedure is valid by virtue of the condition $\Omega \gg k_z v_{Te}$)

$$i\Omega\delta\tilde{u}_\alpha + \frac{e_\alpha}{m_\alpha}\nabla\tilde{\psi} - [\delta\tilde{u}_\alpha \omega_{H\alpha}] = 0, \quad i\Omega\delta\tilde{n}_\alpha + \nabla(\delta n_\alpha \tilde{u}_{0\alpha} + n\delta\tilde{u}_\alpha) = 0, \quad (5)$$

where $\delta\tilde{u}_\alpha$ and $\delta\tilde{n}_\alpha \sim e^{i\Omega t}$ are the high-frequency perturbations in the velocity and density of particles of species α and the Poisson equation is written

$$\Delta\tilde{\psi} = 4\pi \sum_\alpha e_\alpha \delta\tilde{n}_\alpha. \quad (6)$$

Solving Eqs. (5) and (6) we find

$$\delta\tilde{E}_{z0} = -\frac{\partial\tilde{\psi}_0}{\partial z} = \frac{\omega_{pe}^2}{\Omega^2}\frac{k_z^2(\varepsilon_1 k_\perp^2 + \varepsilon_3 k_z^2)}{(\varepsilon_1 k_\perp^2 + \varepsilon_3 k_z^2)^2 - (\varkappa k_y \varepsilon_2)^2}\frac{\delta n_e}{n}\tilde{E}_{z0}, \quad (7)$$

where

$$\varepsilon_1 = 1 + \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega_{H\alpha}^2 - \Omega^2}, \quad \varepsilon_2 = \sum_\alpha \frac{\omega_{p\alpha}^2\,\omega_{H\alpha}}{\Omega(\omega_{H\alpha}^2 - \Omega^2)}, \quad \varepsilon_3 = 1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\Omega^2}$$

are the components of the dielectric tensor for the cold plasma

$$(\varepsilon_1 = \varepsilon_{xx} = \varepsilon_{yy}, \quad \varepsilon_2 = -i\varepsilon_{xy} = i\varepsilon_{yx}, \quad \varepsilon_3 = \varepsilon_{zz});$$
$$\omega_{p\alpha}^2 = \frac{4\pi n e^2}{m_\alpha}, \quad \varkappa = \frac{1}{n}\frac{dn}{dx}.$$

In finding $\delta\Phi_e$ we can make use of the following expression for the resulting averaged force that acts on the electrons in the perturbed state:

$$\mathbf{F}_e + \delta\mathbf{F}_e = -m\nabla(\Phi_e + \delta\Phi_e) = \frac{e}{2m}\{(\tilde{\mathbf{a}}\nabla)\tilde{\mathbf{E}} + [\tilde{\mathbf{a}}\,\mathrm{rot}\,\tilde{\mathbf{E}}]\}, \quad (8)$$

where $\tilde{\mathbf{a}} = \tilde{\mathbf{a}}_0 + \delta\tilde{\mathbf{a}}$ is the amplitude of the electron oscillation in the high-frequency field, $\tilde{\mathbf{E}} = \tilde{\mathbf{E}}_0 + \delta\tilde{\mathbf{E}}$ and $m \equiv m_e$. In the case being considered, we have $\mathbf{a}_0 = (0, 0, e\tilde{E}_{z0}/m\Omega^2)$ and $\delta\tilde{a}_z = e\delta\tilde{E}_{z0}/m\Omega^2$. Carrying out the indicated calculations we find

$$\delta\Phi_e = \frac{e^2\tilde{E}_{z0}\,\delta\tilde{E}_{z0}}{2m^2\Omega^2} = \frac{T_e}{m}F\frac{\delta n_e}{n},$$
$$F = \frac{\tilde{u}_{0e}^2}{\Omega^2}\frac{\omega_{pe}^2}{v_{Te}^2}\frac{k_z^2(k_\perp^2\varepsilon_1 + k_z^2\varepsilon_3)}{(k_\perp^2\varepsilon_1 + k_z^2\varepsilon_3)^2 - (\varkappa k_y \varepsilon_2)^2}. \quad (9)$$

We note that this result can also be obtained from a general expression for the high-frequency potential such as that given in[8,9].

In the particular case in which $\omega_{Hi} \ll \Omega \ll |\omega_{He}|$ the quantity F coincides with the quantity β given in[4]. The unlimited growth of F as the denominator approaches zero corresponds to a resonance between the external high-frequency field and the characteristic electrostatic oscillations of the plasma.

Taking account of Eq. (9) and writing $\delta\Phi_i = 0$, from Eq. (4) we find

$$\frac{\delta n_e}{n} = -\frac{e\psi}{T_e}\frac{G_e}{1 - FG_e}, \quad \frac{\delta n_i}{n} = \frac{e\psi}{T_i}. \quad (10)$$

The dispersion equation can now be obtained from the neutrality condition on the low-frequency oscillations $\delta n_e = \delta n_i$:

$$\frac{T_i}{T_e}G_e + G_i - FG_eG_i = 0. \quad (11)$$

We limit ourselves to the particular case

$$v_{Ti} \ll \omega / |k_z| \ll v_{Te}, \quad \omega \ll \omega_{Hi}, \quad k_\perp v_{Te} \ll |\omega_{He}|.$$

Carrying out conventional calculations (cf. for example[10]) we find that the quantity G_α is given by the following expression in the present case

$$G_e = -1 - \frac{i\sqrt{\pi}}{|k_z|v_{Te}}\left[\omega_e' - \omega^*\left(1 - \frac{\eta_e}{2}\right)\right],$$
$$G_i = A - 1 + \frac{T_i}{T_e}\frac{\omega^*}{\omega_i'}A\left(1 - \frac{\eta_i}{2}\delta\right), \quad (12)$$

where

$$A = e^{-s}I_0(s), \quad \delta = 2s\left[1 - \frac{I_1(s)}{I_0(s)}\right], \quad s = \frac{T_i}{m_i}\frac{k_\perp^2}{\omega_{Hi}^2};$$
$$\omega_\alpha' = \omega + \omega_{0\alpha}, \quad \omega_{0\alpha} = \frac{k_y g_\alpha^{ef}}{\omega_{H\alpha}}, \quad \omega^* = \frac{k_y \varkappa T_e}{m|\omega_{He}|}, \quad \eta_\alpha = \frac{d\ln T_\alpha}{d\ln n}.$$

We now solve Eq. (11) by successive approximations in the small parameter $\omega^*/k_z v_{Te}$. The frequency $\mathrm{Re}\,\omega \approx \omega$ is found from the zeroth approximation for the drift wave and the growth rate $\gamma = \mathrm{Im}\,\omega$ is found from the first approximation for the drift wave:

$$\omega = \omega^*(1 + F)\frac{\tau A(1 - \delta\eta_i/2)}{\tau + (1 - A)(1 + F)} - \omega_{0i}, \quad (13)$$

$$\gamma = \frac{\sqrt{\pi}\,\omega^{*2}}{|k_z|v_{Te}}\frac{\tau^2 A(1 - \delta\eta_i/2)}{[\tau + (1 - A)(1 + F)]^3}\Big\{(1 + \tau)(1 - A) + F[1 - (1 + \tau)A]$$
$$+ \tau A(1 + F)\frac{\delta\eta_i}{2} - \left(\xi + \frac{\eta_e}{2}\right)[\tau + (1 - A)(1 + F)]\Big\}, \quad (14)$$

where

$$\xi = \omega_0/\omega^*, \quad \omega_0 = \omega_{0e} - \omega_{0i}, \quad \tau = T_i/T_e.$$

In the limiting case $k_\perp v_{Ti}/\omega_{Hi} \ll 1$ (transverse wavelength much larger than the ion Larmor radius) we can write A = 1 and $\delta = 0$ in which case

$$\omega = \omega^*(1 + F) - \omega_{0i}, \quad (13a)$$
$$\gamma = -\sqrt{\pi}\frac{\omega^{*2}}{|k_z|v_{Te}}\left(F + \xi + \frac{\eta_e}{2}\right). \quad (14a)$$

It is evident from Eq. (14a) that in this case the plasma is stable with respect to the drift wave if $\eta_e > -2(F + \xi)$.

In the opposite limiting case of short wavelengths ($\lambda_\perp$ much shorter than the ion-Larmor radius but, by hypothesis, much larger than the electron Larmor radius) we shall limit our analysis, for reasons of simplicity, to the isothermal case $T_e = T_i$. We then find that the plasma remains stable in the region $2(1 - \xi) < \eta < 2$ ($\eta \equiv \eta_e = \eta_i$).

Under experimental conditions[1] $\Omega < \omega_{Hi}$; in this case, it is easy to show that for all wavelengths of perturbations that satisfy the condition $k_z^2/k_\perp^2 > m/m_i$ the quantity F is negative. Consequently, as is evident from Eq. (14a), this quantity acts as a destabilizing factor whereas ξ is always positive and hence provides an expansion of the stability region and reduces the growth rate associated with the instability. In order-of-magnitude terms we find

$$F \sim -\frac{\tilde{H}^2}{16\pi nT}, \quad \xi \sim \frac{L_n}{\delta_s}\frac{\tilde{H}^2}{16\pi nT},$$

so that when $L_n > \delta_s$ the stabilizing effect dominates in the skin depth.

It follows that for the conditions being treated here the effect of the high-frequency field on the stability of the plasma appears as a consequence of two independent mechanisms. One of these, investigated by Faĭnberg and Shapiro[4], is connected with the appearance of average high-frequency forces on the low-frequency plasma perturbations (δg-mechanism). The second mechanism arises owing to the existence of these forces even in the unperturbed state (g-mechanism). In both cases the stabilizing appears as a consequence of the increase in the frequency of the drift waves which leads to an increase in the Landau damping due to the electrons. In the case of the g-mechanism this increase in frequency can be associated with the Doppler shift (by an amount $k_y g_\alpha^{ef}/\omega_{H\alpha}$) due to the drift of charged particles with the velocity $g_\alpha^{ef}/\omega_{H\alpha}$; in this case it is important that the ions and electrons drift with different velocities (the expression for the growth rate only contains the velocity difference).

It is valid to neglect effects associated with the presence of the averaged high-frequency forces in the unperturbed state if the plasma is rarefied, in which case $\delta_s \gg L_n$ that is to say, the high-frequency field is essentially uniform over the cross-section of the plasma. In this case by virtue of the disappearance of the stabilizing g-mechanism the stability condition deteriorates and the results obtained here go over to the corresponding results obtained in[4]. In particular, in the short-wave limit ($k_\perp v_{Ti}/\omega_{Hi} \ll 1$) the stability region found here $2(1-\xi) < \eta < 2$ is found to disappear and an instability can occur, as was noted in[4], for all values of the parameter η (as in the case of the absence of the high-frequency field).

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