FLUCTUATIONS AND WAVE SCATTERING IN A PLASMA SUBJECT TO STRONG ELECTRIC AND MAGNETIC FIELDS

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We have investigated fluctuations and scattering of electromagnetic waves in a partially ionized plasma subject to strong electric and magnetic fields. It is shown that in the case of a strong magnetic field the correlation functions for the fluctuations exhibit narrow peaks at frequencies corresponding to magnetoacoustic waves. As the magnitude of the external electric field E_0 approaches a critical value E_c there is a sharp rise in fluctuations whose wave vectors lie in the region characterized by $\chi < \pi/2$ (in contrast with the case considered in[2], a weak magnetic field, in which the sharp rise in the fluctuations occurs only when $\chi \ll 1$; χ is the angle between the wave vector of the fluctuation and the mean directed electron velocity $\mathbf{v}_0$). When electromagnetic waves propagate in the plasma the spectrum of the scattered radiation exhibits narrow peaks displaced from the main line by the frequency of the magnetoacoustic wave (magnetoacoustic satellites). When $E_0 \rightarrow E_c$ there is a sharp rise in the intensity of the satellites for scattering in the cone $\theta' > \theta$ and the integrated scattering cross-section increases as $(E_c - E_0)^{-1}$ (in contrast with a weak magnetic field in which the sharp rise in satellite intensity in scattering occurs only in a rigorously defined direction and in which the integrated scattering cross-section increases logarithmically; $\theta(\theta')$, is the angle between the wave vector of the incident (scattered) wave and $\mathbf{v}_0$). Thus, in the case of a strong magnetic field the effects of critical fluctuations and critical opalescence are much more pronounced than in the case of a weak field.

1. INTRODUCTION

In the present work we investigate fluctuations and scattering of electromagnetic waves in a partially ionized plasma to which are applied fixed and uniform electric and magnetic fields $\mathbf{E}_0$ and $\mathbf{H}_0$. Electrons in this plasma, under the effect of the fields, will execute motion with respect to the ions and the neutral particles; collisions between electrons and neutrals lead to an increase in the mean velocity associated with the random electron motion, that is to say, the electron component of the plasma is heated. As a result, as has been shown by Davydov[1] (cf. also[2]), these collisions lead to the establishment of a stationary nonMaxwellian electron velocity distribution which is characterized by a very high mean energy (effective temperature) T_e and a nonvanishing mean directed velocity $\mathbf{v}_0$.

It will be shown that the nature of the fluctuations and the associated scattering processes in such a plasma are very sensitive to the magnitude of the external magnetic field. For a modest magnetic field ($V_A/c \ll aq$; V_A is the Alfven velocity, a is the electron Debye radius and q is the wave vector associated with the fluctuations) the correlation functions for the fluctuations and the scattering cross sections exhibit the same structure as in the absence of a magnetic field, and specifically[3]; 1) the correlation functions exhibit sharp peaks at $\omega = qV_s(1 + a^2q^2)^{-1/2}$ (ω is the fluctuation frequency, V_s is the ion-acoustic velocity) corresponding to the possibility of ion-acoustic propagation in the plasma; 2) when $E_0 \rightarrow E_c$, where E_c is a critical field there is a sharp increase in the intensity of fluctuations characterized by wave vectors which are parallel to $\mathbf{v}_0$ (critical fluctuations); 3) when electromagnetic waves propagate in the plasma the spectrum of scattered radiation exhibits sharp peaks which are separated from the primary line by a frequency $\Delta\omega = 2kV_s \sin(\theta/2)$ where $\mathbf{k}$ is the wave vector of the incident wave and θ is the scattering angle (ion-acoustic satellites); 4) the existence of critical fluctuations when $E_0 \rightarrow E_c$ leads to a sharp rise in the intensity of waves scattered in s specific direction; the integrated scattering cross-section for electromagnetic wave scattering increases logarithmically as $E_0 \rightarrow E_c$.

In the case of a strong magnetic field ($V_A/c \gg aq$), as will be shown in the present work, we find 1) the correlation functions exhibit sharp peaks at $\omega = qV_s(1 + a^2q^2)^{1/2} \cos\chi$ (χ is the angle between the vectors $\mathbf{q}$ and $\mathbf{v}_0$) corresponding to the possibility of magnetoacoustic wave propagation in the plasma; 2) when $E_0 \rightarrow E_c$ there is a sharp rise in the intensity of fluctuations characterized by wave vectors that lie in the halfspace $\chi < \pi/2$; 3) when electromagnetic waves propagate in the plasma, the spectrum of scattered radiation exhibits sharp peaks which are separated from the primary line by a frequency $\Delta\omega = kV_s(\cos\theta - \cos\theta')$, where $\theta(\theta')$ is the angle between the vector $\mathbf{v}_0$ and the wave vector of the incident (scattered) wave (magnetoacoustic satellite); 4) when $E_0 \rightarrow E_c$ there is a sharp rise in the intensity (critical opalescence) which is observed for all scattered waves characterized by $\theta' > \theta$; the integrated scattering cross-section goes as $(E_c - E_0)^{-1}$ when $E_0 \rightarrow E_c$.

Thus, in the presence of a strong magnetic field effects such as critical fluctuations and critical opalescence are found to be more pronounced than in the case of a weak field. In particular, the total intensity of the scattered radiation as E_0 approaches E_c is found to be significantly higher in the first case than in the second.

2. CRITICAL FLUCTUATIONS

In investigating high-frequency fluctuations ($\omega \gg \nu_e$, ν_i; $\nu_{e,i}$ is the frequency of collisions between electrons and ions with neutral particles) we can use the conventional relationships for the correlation functions of the charge density $\langle \rho^2 \rangle_{q,\omega}$:[4]

$$\langle \rho^2 \rangle_{\mathbf{q},\omega} = 2\pi |\varepsilon_l(\mathbf{q},\omega)|^{-2} \sum_\alpha e_\alpha^2 \int F^{(\alpha)} \delta(\omega - \mathbf{qv})\, d\mathbf{v}, \tag{1}$$

where $F^{(\alpha)}$ and e_α are respectively the distribution function and the charge on the particle of species α (α = e, i) and $\epsilon_l = 1 + 4\pi \sum_\alpha \kappa_\alpha$ is the longitudinal (with respect to the wave vector $\mathbf{q}$) dielectric constant of the plasma while κ_α is the longitudinal electric permittivity of the α-component of the plasma; in the presence of a strong magnetic field ($\omega_i \gg \omega$, $\omega_i = |e_i| H_0 | m_i c$ is the ion-cyclotron frequency m_i is the ion mass) this quantity is given by

$$\varkappa_\alpha = \frac{e_\alpha^2}{m_\alpha q^2} \int \frac{q_\parallel \partial F^{(\alpha)} / \partial v_\parallel}{\omega - q_\parallel v_\parallel} d\mathbf{v},$$

where $q_\parallel$ and $v_\parallel$ are the components of the wave vector $\mathbf{q}$ and velocity $\mathbf{v}$ that are parallel to $\mathbf{H}_0$ (m_α is the mass of the particle of species α). In writing $F^{(e)}$ in these expressions we shall use the distribution function given by Davydov[1]:

$$F^{(e)}(\mathbf{v}) = F_0(v) + v^{-1}\mathbf{v}\mathbf{F}_1(v),$$

$$F_0 = C \exp\left\{-\frac{m_e^2 v^4}{4T_e^2}\right\}, \quad \mathbf{F}_1 = -\mathbf{v}_0 \frac{\partial F_0}{\partial v},$$

$$C = \frac{n_0}{\pi \Gamma(3/4)} \left(\frac{m_e}{2T_e}\right)^{3/2},$$

where n_0 is the equilibrium electron density.

In the case of a strong magnetic field ($\omega_e \gg \nu_e$, $w_e = eH_0/m_e c$ is the electron-cyclotron frequency) the effective electron temperature T_e and the mean directed electron velocity v_0 are given by

$$T_e = \left(\frac{m_0}{3m_e}\right)^{1/2} eE_0 l |\cos\beta|, \quad \mathbf{v}_0 = \frac{eE_0 \cos\beta}{m_e \nu_e} \frac{\mathbf{H}_0}{H_0},$$

whereas for a modest magnetic field ($\omega_e \ll \nu_e$) these quantities are given by

$$T_e = \left(\frac{m_e}{3m_e}\right)^{1/2} eE_0 l, \quad \mathbf{v}_0 = \frac{e\mathbf{E}_0}{m_e \nu_e},$$

where m_0 is the mass of the neutral particle, l is the electron mean-free path and β is the angle between the vectors E_0 and H_0 (this angle is assumed to be fairly close to $\pi/2$, $|\pi/2 - \beta| \gg \nu_e/\omega_e$). The ion velocity distribution is assumed to be a Maxwellian (at temperature T_i).

We note that although the form of the expression in (1) is similar to that obtained for a collisionless plasma, the quantity $\langle \rho^2 \rangle$ (as well as the other correlation functions in the high frequency region) depends on the form of the collision integral inasmuch as the equilibrium electron distribution function $F^{(e)}$ depends on the form of the collision integral (in this connection see[2]).

As is well known, the poles of the correlation function determine the complex characteristic frequencies of the system. Setting the denominator in (1) equal to zero and writing

$$T_i \ll m_0 q^{-2} \omega^2 \ll T_e,$$

we obtain the frequency ω and growth rate γ for the magnetoacoustic waves:

$$\omega^2 = \frac{q^2 V_s^2 \cos^2\chi}{1 + a^2 q^2}, \tag{2}$$

$$\frac{\gamma}{|\omega|} = \left(\frac{\pi}{\Gamma(1/4)}\right)^2 \left(\frac{3m_e}{2m_0}\right)^{1/2} \frac{[\xi(E_0) - \operatorname{sign}\cos\chi]}{1 + a^2 q^2},$$

$$\xi(E_0) = \frac{2^{3/4}}{\sqrt{3\pi}} \left(\frac{zm_0}{m_i}\right)^{1/2} \left\{1 + \frac{z\Gamma(3/4)}{\sqrt{\pi}} \left(\frac{m_0}{m_e}\right)^{1/2} \times \left(\frac{T_e}{T_i}\right)^{3/2} \exp\left[-\frac{z\Gamma(3/4) T_e}{\Gamma(1/4)(1 + a^2 q^2) T_i}\right]\right\}, \tag{3}$$

$$V_s^2 = \frac{2zT_e \Gamma(3/4)}{m_i \Gamma(1/4)}, \quad a^2 = \frac{T_e \Gamma(3/4)}{2\pi e^2 n_0 \Gamma(1/4)}, \tag{4}$$

where ze is the ion charge.

It will be evident that if $2^{3/2} zm_0 / 3\pi m_i > 1$ then for any values of the external magnetic field we find $\xi > 1$ and the plasma waves are damped. However, if $2^{3/2} zm_0 / 3\pi m_i < 1$ then in a sufficiently strong electric field E_0 ($E_0 > E_c$) $\xi < 1$ and waves characterized by wave vectors lying in the halfspace $\chi < \pi/2$ can grow. The appropriate critical value of the field can be obtained by using Eqs. (3) and (4):

$$E_c = E_c^0 \quad (\omega_e \ll \nu_e),$$

$$E_c = E_c^0 |\cos\beta|^{-1} \quad (\omega_e \gg \nu_e),$$

$$E_c^0 = \left(\frac{3}{4}\right)^{1/2} \frac{\Gamma(1/4) T_i}{\Gamma(3/4) zel} \left(\frac{m_e}{m_i}\right)^{1/2} \left\{\ln\frac{m_i}{m_e} + 3\ln\left(\frac{\pi^{2/3}}{2^{1/2}\Gamma^{2/3}(3/4) z^{1/3}} \ln\frac{m_i}{m_e}\right)\right\}. \tag{5}$$

Now, making use of Eq. (1) and assuming that the ions are highly magnetized ($\omega_i \gg \omega$) we can write $\langle \rho^2 \rangle$ in the form

$$\langle \rho^2 \rangle_{q,\omega} = \frac{\sqrt{z}\, a^2 q^5 V_s T_e}{2^{5/4} \sqrt{3} (1 + a^2 q^2)^2 (\xi - \operatorname{sign}\cos\chi)} \delta\left(\omega^2 - \frac{q^2 V_s^2 \cos^2\chi}{1 + a^2 q^2}\right). \tag{6a}$$

Similar relations can be used in the case of a very strong magnetic field to find the correlation function for the high-frequency $qV_s |\cos\chi| / (1 + a^2 q^2)^{1/2} \gg \nu_{e,i}$ fluctuations in the electron and ion density:

$$e^2 \langle \delta n_e^2 \rangle_{q,\omega} = (1 + a^2 q^2)^{-2} (ze)^2 \langle \delta n_i^2 \rangle_{q,\omega} = -(1 + a^2 q^2)^{-1} ze^2 \langle \delta n_e \delta n_i \rangle_{q,\omega}$$

$$= \frac{\sqrt{z}\, q V_s T_e \delta(\omega^2 - q^2 V_s^2 \cos^2\chi / (1 + a^2 q^2))}{2^{5/4} \sqrt{3}\, a^2 |\cos^3\chi| (\xi - \operatorname{sign}\cos\chi)} \tag{7a}$$

It is evident that when $E_0 \to E_c$ the correlation functions increase sharply (critical fluctuations). The existence of critical fluctuations is associated with a growing instability of the plasma when $E_\nu \geq E_c$. It will be evident that Eqs. (1), (6) and (7), which are obtained in a linear analysis, apply only in the stable region $E_0 < E_c$. In the present work we have not attempted to investigate fluctuations when $E_0 \geq E_c$ since the investigation of this regime a requires nonlinear analysis.

Attention is directed to the marked difference in the nature of the critical fluctuations in the case of a strong magnetic field and in the case of a weak magnetic field. In the case in which the ions are weakly magnetized ($\omega_i \ll \omega$), from[2]

$$\langle\rho^2\rangle_{\mathbf{q},\,\omega} = \frac{\sqrt{z}\,a^2q^5V_sT_e\delta(\omega^2 - q^2V_s^2/(1+a^2q^2))}{2^{5/4}\sqrt{3}\,(1+a^2q^2)^2\mu(\cos\chi_c - \cos\chi)}, \tag{6b}$$

$$e^2\langle\delta n_e^2\rangle_{\mathbf{q},\,\omega} = \frac{\sqrt{z}\,qV_sT_e\delta(\omega^2 - q^2V_s^2/(1+a^2q^2))}{2^{5/4}\sqrt{3}\,a^2\,\mu(\cos\chi_c - \cos\chi)}, \tag{7b}$$

$$\mu = 1 - \frac{2^{3/4}z^{3/2}\Gamma(3/4)}{\pi\sqrt{3}\,(1+a^2q^2)^{1/2}}\left(\frac{m_0}{m_e}\right)^{1/2}\left(\frac{T_e}{T_i}\right)^{3/2}$$
$$\times\exp\left\{-\frac{z\Gamma(3/4)T_e}{\Gamma(1/4)(1+a^2q^2)T_i}\right\}\operatorname{sign}\cos\chi,$$
$$\cos\chi_c = 2^{3/4}(zm_0/m_i)^{1/2}\mu^{-1}[3\pi(1+a^2q^2)]^{-1/2}.$$

When $E_0 \to E_c$ there is a sharp increase in the intensity of the fluctuations for wave vectors that lie parallel to $\mathbf{v}_0$. In the case of highly magnetized ions, in accordance with Eqs. (6a) and (7a) when $E_0 \to E_c$ there is a sharp rise in the intensity of fluctuations whose wave vectors lie in the halfspace $\chi < \pi/2$. Thus, the effect of critical fluctuations in the case of a strong magnetic field is much more pronounced than in a weak field.

3. CRITICAL OPALESCENCE

We now wish to consider combination scattering of transverse electromagnetic waves on the magnetoacoustic fluctuations of a plasma located in external electric and magnetic fields. As is well known, the refractive index for transverse electromagnetic waves $n = kc/\omega$ in such a plasma is given by

$$n_\pm^2 = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A},$$

where $A = \epsilon_1\sin^2\theta + \epsilon_3\cos^2\theta$, $B = -\epsilon_1\epsilon_3(1 + \cos^2\theta) - (\epsilon_1^2 - \epsilon_2^2)\sin^2\theta$, $C = \epsilon_3(\epsilon_1^2 - \epsilon_2^2)$; $\epsilon_{1,2,3}$ are the components of the high-frequency dielectric tensor for the plasma in the magnetic field

$$\varepsilon_1 = 1 - \frac{\Omega_e^2}{\omega^2 - \omega_e^2}, \quad \varepsilon_2 = -\frac{\Omega_e^2\,\omega_e}{\omega(\omega^2 - \omega_e^2)}$$
$$\varepsilon_3 = 1 - \frac{\Omega_e^2}{\omega^2};$$

where $\Omega_e = (4\pi e^2 n_0/m_e)^{1/2}$ is the electron plasma frequency, θ is the angle between the wave vector $\mathbf{k}$ and the mean directed electron velocity $\mathbf{v}_0$, which is related to the external fields $\mathbf{E}_0$ and $\mathbf{H}_0$ by the expression[5]

$$\mathbf{v}_0 = \frac{e}{m_e\nu_e}\left(1 + \frac{\omega_e^2}{\nu_e^2}\right)^{-1}\left(\mathbf{E}_0 + \frac{\omega_e}{\nu_e}\frac{[\mathbf{E}_0\mathbf{H}_0]}{H_0} + \frac{\omega_e^2}{\nu_e^2}\frac{\mathbf{H}_0(\mathbf{E}_0\mathbf{H}_0)}{H_0^2}\right).$$

Making use of the well known expressions for the differential scattering cross-section for electromagnetic waves[4] and assuming $\Delta\omega \ll \omega$ we find

$$d\Sigma = \frac{1}{2\pi}\left(\frac{e^2}{m_ec^2}\right)^2\left(\frac{\omega}{\Omega_e}\right)^4 N^{\lambda\lambda'}\langle\delta n_e^2\rangle_{\mathbf{q},\,\Delta\omega}d\omega'\,do', \tag{8}$$

where $\Delta\omega = \omega - \omega'$ and $\mathbf{q} = \mathbf{k} - \mathbf{k}'$; ω and ω' and $\mathbf{k}$ and $\mathbf{k}'$ are the frequencies and wave vectors for the incident and scattered waves;

$$N^{\lambda\lambda'} = n_{\lambda'}'^3|\mathbf{e}_{\lambda'}'(\hat{\varepsilon} - 1)\mathbf{e}_\lambda|^2\{n_\lambda(|\mathbf{e}_\lambda|^2 - k^{-2}|\mathbf{k}\mathbf{e}_\lambda|^2)(\mathbf{e}_{\lambda'}'^{*}\hat{\varepsilon}\mathbf{e}_{\lambda'}')\}^{-1},$$

where $\mathbf{e}_\lambda$, $\mathbf{e}_{\lambda}'(\lambda, \lambda' = \pm)$ are the polarization vectors for the incident and scattered waves

$$\mathbf{e}_{\lambda'}' = \left(\cos\varphi - \frac{i\varepsilon_2}{n_{\lambda'}'^2 - \varepsilon_1}\sin\varphi;\ \sin\varphi + \frac{i\varepsilon_2}{n_{\lambda'}'^2 - \varepsilon_1}\cos\varphi;\ \frac{n_{\lambda'}'^2\sin\theta'\cos\theta}{n_{\lambda'}'^2\sin^2\theta' - \varepsilon_3}\right)$$

$$\left(\mathbf{e}_\lambda = {}'\,1;\ \frac{i\varepsilon_2}{n_\lambda^2 - \varepsilon_1};\ \frac{n_\lambda^2\sin\theta\cos\theta}{n_\lambda^2\sin^2\theta - \varepsilon_3}\right).$$

and $n' = n(\omega', \theta')$; φ is the angle between the planes $(\mathbf{k}, \mathbf{v}_0)$ and $(\mathbf{k}', \mathbf{v}_0)$; $\hat{\epsilon}$ is the dielectric tensor for the plasma in the magnetic field; $\langle\delta n_e^2\rangle$ is the correlation function for the electron density, which is given by Eq. (7); and do' is the element of solid angle for the vector $\mathbf{k}'$.

We see that in principle in the presence of a magnetic field the plasma is capable of supporting four scattering processes for transverse waves (depending on the polarization of the incident and scattered waves): $t_+ \to t_+$, $t_- \to t_-$, $t_+ \to t_-$ and $t_- \to t_+$. If there is an arbitrary relation between frequencies ω, ω_e, and Ω_e these processes (taking account of acoustic fluctuations) can be forbidden since the frequency of the acoustic wave characterized by wave vector $\mathbf{q}$ can be small compared with $\Delta\omega$. In order to be definite we shall consider the case of high-frequency incident radiation $\omega \gg \omega_e$, Ω_e. In this case all four scattering processes are allowed.

Substituting Eqs. (7a) or (7b) in Eq. (8) and limiting ourselves to longwave incident radiation $ak \ll 1$, for the case of highly magnetized ions we find

$$d\Sigma = \frac{\Gamma(1/4)\sqrt{z}n_0N^{\lambda\lambda'}}{2^{9/4}\sqrt{3}\Gamma(3/4)\cos^4\chi(\xi - \operatorname{sign}\cos\chi)}\left(\frac{\omega}{\Omega_e}\right)^4\left(\frac{e^2}{m_ec^2}\right)^2$$
$$\times\{\delta[\Delta\omega - kV_s(\cos\theta - \cos\theta')] + \delta[\Delta\omega + kV_s(\cos\theta - \cos\theta')]\}d\omega'\,do' \tag{9a}$$

whereas for weakly magnetized ions

$$d\Sigma = \frac{\Gamma(1/4)\sqrt{z}n_0N^{\lambda\lambda'}}{2^{9/4}\sqrt{3}\Gamma(3/4)\mu(\cos\chi_c - \cos\chi)}\left(\frac{\omega}{\Omega_e}\right)^4\left(\frac{e^2}{m_ec^2}\right)^2$$
$$\times\left\{\delta\left(\Delta\omega - 2kV_s\sin\frac{\vartheta}{2}\right) + \delta\left(\Delta\omega + 2kV_s\sin\frac{\vartheta}{2}\right)\right\}d\omega'\,do', \tag{9b}$$

where $\cos\chi = [2\sin(\vartheta/2)]^{-1}(\cos\theta - \cos\theta')$ and ϑ is the scattering angle (the angle between the vectors $\mathbf{k}$ and $\mathbf{k}'$). We see that the spectrum of scattered radiation exhibits two satellites of the same intensity, the frequency in the first case being given by $\omega' = \omega \pm kV_s(\cos\theta - \cos\theta')$ and in the second case by $\omega' = \omega \pm 2kV_s\sin(\vartheta/2)$.

Carrying out the integration over ω' in Eq. (9) we can now determine the intensity of scattered radiation per unit solid angle:

$$\frac{d\Sigma}{do'} = \frac{\Gamma(1/4)\sqrt{z}n_0N^{\lambda\lambda'}}{2^{9/4}\sqrt{3}\Gamma(3/4)\cos^4\chi(\xi - \operatorname{sign}\cos\chi)}\left(\frac{\omega}{\Omega_e}\right)^4\left(\frac{e^2}{m_ec^2}\right)^2, \tag{10a}$$

$$\frac{d\Sigma}{do'} = \frac{\Gamma(1/4)\sqrt{z}n_0N^{\lambda\lambda'}}{2^{9/4}\sqrt{3}\Gamma(3/4)\mu(\cos\chi_c - \cos\chi)}\left(\frac{\omega}{\Omega_e}\right)^4\left(\frac{e^2}{m_ec^2}\right)^2. \tag{10b}$$

It is evident from these expressions that the angular distribution of the scattered radiation depends sensitively on the magnitude of the external electric field. When $E_0 \ll E_c$ this distribution is essentially isotropic. When E_0 is increased in the case of highly magnetized ions there is a sharp rise in the intensity of the radiation scattered in the angular region $\theta' > \theta$. However, in the case of weakly magnetized ions there is a sharp rise in the intensity of waves scattered in a specific direction, that is to say at angles close to θ_c', for which

$$\theta_c' = \arccos(\cos\theta - 2\sin\tfrac{1}{2}\vartheta).$$

The sharp rise in the intensity of the scattered electromagnetic radiation as $E_0 \to E_c$ is associated with a plasma instability that appears when $E_0 \to E_c$.

The nature of this effect is very similar to the familiar effect known as critical opalescence.

We wish to direct attention to the important distinction between critical opalescence in the case in which the ions are highly magnetized and the case in which they are weakly magnetized. In the second case critical opalescence occurs only for waves that are scattered at angles $\theta' \approx \theta'_c$ whereas in the first case this effect is observed for wave scattering for all angles that satisfy the condition $\theta' > \theta$. For this reason as E_0 approaches E_c the integrated scattering cross-section for electromagnetic waves will be significantly larger when the ions are highly magnetized as compared with the case in which they are weakly magnetized. The point here is that integration of Eq. (10) over do' shows that for the highly magnetized ions the integrated scattering cross-section is proportional to $(E_c - E_0)^{-1}$ as $E_0 \to E_c$ whereas in the case of weakly magnetized ions this cross-section is proportional to $\ln|1 - E_0/E_c|$.

In conclusion we note that for arbitrary scattering angles the cross-sections for all four processes $t \to t$ are of the same order of magnitude. However, for small scattering angles ($\vartheta \ll 1$) the cross-sections for the processes and $t_+ \to t_-$ and $t_- \to t_+$ are small (proportional to ϑ^4).

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