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PERTURBATION OF A MEDIUM BY A FIELD OF A STRONG ELECTROMAGNETIC WAVE

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A dispersion equation is obtained, describing the spectrum of low-frequency perturbations in a transparent medium (in a plasma and in a liquid), through which an electromagnetic pump wave passes. It is shown that at wave amplitudes exceeding a definite critical value the concept of weakly-coupled acoustic and electromagnetic perturbations loses its meaning. Under these conditions it follows from the solution of the dispersion equation that the initial perturbations of the density increase aperiodically in time.

INTRODUCTION

RECENTLY, principally in connection with the development of laser and microwave techniques, more interest has been evinced in the properties of various media acted upon by strong electromagnetic field. One of the manifestations of this action is the radical change in the dispersion laws for elementary excitations in the medium. Moreover, a strong electromagnetic field may give rise to new branches of elementary excitations, and also cause the state of the medium to become unstable. Effects of this kind have been investigated theoretically for an electron-ion plasma situated in a homogeneous high frequency electric field (see the review [1]).

In this paper we consider the dispersion law for density perturbations in a transparent homogeneous unbounded medium—a plasma or an arbitrary liquid—through which a plane electromagnetic linearly-polarized pump wave with specified amplitude passes. It is shown that the customary nonlinear-optics notion of weak parametric coupling between acoustic and electromagnetic waves, which is the basis of the theory of stimulated Mandel'shtam-Brillouin scattering (SMBS) (see [2,3]), is valid for a pump-wave amplitude lower than a definite critical value, which exceeds the SMBS threshold. On the other hand, if the amplitude exceeds this critical value, then the coupling between the density and field perturbations in the medium cannot be regarded as weak, and in this case density perturbations that increase aperiodically with time are possible.

We begin the analysis with the case of an electron-ion plasma, which we describe by means of the kinetic equation without a collision integral. Further, using phenomenological relations, the dispersion equation obtained for the plasma from the microscopic theory is generalized to include the case of an arbitrary liquid medium. From an analysis of the solution of the dispersion equation in the weak parametric coupling approximation we obtain the conditions under which such an approximation is valid. Finally, we investigate the solution of the dispersion equation in the approximation in which the coupling between the density and field perturbations is not assumed to be weak.

1. PLASMA

A. Ground state. We assume that in the ground state there exists in the plasma a pump wave whose electric field varies like

$$\mathbf{E}^{(0)}(\mathbf{r}, t) = \mathbf{E}_0 \cos(\omega_0 t - \mathbf{k}_0\mathbf{r}). \quad (1.1)$$

To determine the particle distribution function in the ground state $F_\alpha(\mathbf{r}, \mathbf{v}_\alpha, t)$ we use the kinetic equation

$$\frac{\partial F_\alpha}{\partial t} + \mathbf{v}_\alpha \frac{\partial F_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\left(\mathbf{E}^{(0)} + \frac{1}{c}[\mathbf{v}_\alpha \mathbf{B}^{(0)}]\right)\frac{\partial F_\alpha}{\partial \mathbf{v}_\alpha} = 0, \quad (1.2)^*$$

where the index α indicates the species of the particle; e_α and m_α are respectively the charge and mass of the particles; $\mathbf{B}^{(0)}$ is the magnetic field of the pump wave, which is connected with the electric field (1.1) by the equation

$$\operatorname{rot} \mathbf{E}^{(0)} = -\frac{1}{c}\frac{\partial \mathbf{B}^{(0)}}{\partial t}.$$

We assume that the pump wave is turned on adiabatically at the instant of time $t = -\infty$, and perturbs weakly the stationary and homogeneous distribution function $F_{0,\alpha}(v_\alpha)$. Using perturbation theory relative to $\mathbf{E}_0$ and confining ourselves to the linear and quadratic terms, we obtain from (1.2) the particle distribution function in the ground state:

$$F_\alpha(\mathbf{r}, \mathbf{v}_\alpha, t) = F_{0,\alpha}(v_\alpha) + \Delta F_\alpha(\mathbf{v}_\alpha) - \frac{e_\alpha}{m_\alpha}\frac{a_{ij}^{(\alpha)}}{\Omega_0}\frac{\partial F_{0,\alpha}}{\partial v_{\alpha,i}} E_{0j}\sin(\omega_0 t - \mathbf{k}_0\mathbf{r}), \quad (1.3)$$

where

$$\Delta F_\alpha(\mathbf{v}_\alpha) = \frac{1}{4}\left(\frac{e_\alpha}{m_\alpha}\right)^2 E_{0j}E_{0n}\frac{1}{\omega_0}\left\{\frac{\partial}{\partial v_{\alpha,n}}\left(\frac{a_{ij}^{(\alpha)}}{\Omega_0}\frac{\partial F_{0,\alpha}}{\partial v_{\alpha,i}}\right) + \frac{\partial}{\partial v_{\alpha,i}}\left(\frac{a_{ij}^{(\alpha)}}{\Omega_0}\frac{k_{0m}v_{\alpha,n}}{\Omega_0}\frac{\partial F_{0,\alpha}}{\partial v_{\alpha,m}}\right)\right\}; \quad (1.4)$$

$$a_{ij}^{(\alpha)} \equiv a_{ij}^{(\alpha)}(\omega_0, \mathbf{k}_0) = \omega_0^{-1}(\Omega_0\delta_{ij} - k_{0i}v_{\alpha,j}), \quad \Omega_0 = \omega_0 - \mathbf{k}_0\mathbf{v}_\alpha. \quad (1.5)$$

In the derivation of (1.3) we had discarded terms proportional to E_0^2 but varying at double the frequency

*$[\mathbf{v}_\alpha \mathbf{B}^{(0)}] \equiv \mathbf{v}_\alpha \times \mathbf{B}^{(0)}$.

(in particular, such terms result from allowance for the second harmonic of the pump wave). As will be shown later, allowance for these terms is unnecessary for the problem of interest to us.

It is obvious that the frequency ω_0 of the pump wave and the wave vector $\mathbf{k}_0$ are not independent, but satisfy definite dispersion equations. As usual, by obtaining the current linear in $\mathbf{E}_0$ with the aid of (1.3) and substituting it in Maxwell's equations, we obtain the dispersion laws for the pump wave:[4]

$$\varepsilon^l(\omega_0, \mathbf{k}_0) = 0, \ [\mathbf{k}_0\mathbf{E}_0] = 0$$

In the case of a longitudinal wave and

$$c^2k_0^2/\omega_0^2 - \varepsilon^{tr}(\omega_0, \mathbf{k}_0) = 0, \ \ \mathbf{k}_0\mathbf{E}_0 = 0$$

in the case of a transverse wave; here

$$\varepsilon^l(\omega, \mathbf{k}) = 1 + \sum_\alpha \frac{4\pi e_\alpha^2}{m_\alpha k^2\omega}\int d\mathbf{v}_\alpha \frac{\mathbf{k}\mathbf{v}_\alpha}{\omega - \mathbf{k}\mathbf{v}_\alpha}\left(\mathbf{k}\frac{\partial F_{0,\alpha}}{\partial \mathbf{v}_\alpha}\right), \qquad (1.6)$$

$$\varepsilon^{tr}(\omega, \mathbf{k}) = 1 + \sum_\alpha \frac{2\pi e_\alpha^2}{m_\alpha\omega}\int \frac{d\mathbf{v}_\alpha}{\omega - \mathbf{k}\mathbf{v}_\alpha}\left[\mathbf{v}_\alpha\frac{\partial F_{0,\alpha}}{\partial \mathbf{v}_\alpha} - \frac{\mathbf{k}\mathbf{v}_\alpha}{k^2}\left(\mathbf{k}\frac{\partial F_{0,\alpha}}{\partial \mathbf{v}_\alpha}\right)\right] \qquad (1.7)$$

B. Dispersion equation. We consider small perturbations of the ground-state distribution function (1.3) $\delta f_\alpha(\mathbf{r}, \mathbf{v}_\alpha, t)$. The linearized kinetic equation for these perturbations is given by

$$\frac{\partial \delta f_\alpha}{\partial t} + \mathbf{v}_\alpha\frac{\partial \delta f_\alpha}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha}\left(\delta\mathbf{E} + \frac{1}{c}[\mathbf{v}_\alpha\delta\mathbf{B}]\right)\frac{\partial F_\alpha}{\partial \mathbf{v}_\alpha} + \frac{e_\alpha}{m_\alpha}\left(\mathbf{E}^{(0)} + \frac{1}{c}[\mathbf{v}_\alpha\mathbf{B}^{(0)}]\right)\frac{\partial \delta f_\alpha}{\partial \mathbf{v}_\alpha} = 0,$$

where $\delta\mathbf{E}$ and $\delta\mathbf{B}$ are the electric and magnetic fields of the perturbations. Going over to Fourier transforms in the coordinates and in the time for the fields and the distribution functions, we write this equation in the form

$$\delta f_\alpha(\omega, \mathbf{k}, \mathbf{v}_\alpha) = -i\frac{e_\alpha}{m_\alpha}\frac{1}{\Omega}\int d\mathbf{k}'\,d\omega'\left\{\alpha_{ij}^{(\alpha)}(\omega - \omega', \mathbf{k} - \mathbf{k}')\cdot\frac{\partial}{\partial v_{i,\alpha}}F_\alpha(\omega', \mathbf{k}', \mathbf{v}_\alpha)\,\delta E_j(\omega - \omega', \mathbf{k} - \mathbf{k}') + \alpha_{ij}^{(\alpha)}(\omega', \mathbf{k}')\,\delta f_\alpha(\omega - \omega', \mathbf{k} - \mathbf{k}', \mathbf{v}_\alpha)E_j^{(0)}(\omega', \mathbf{k}')\right\}, \qquad (1.8)$$

$$\Omega = \omega - \mathbf{k}\mathbf{v}_\alpha.$$

Using formula (1.8) as a recurrence relation, we can easily find $\delta f_\alpha(\omega, \mathbf{k}, \mathbf{v}_\alpha)$ with any degree of accuracy in powers of the amplitude of the pump wave. We confine ourselves to the linear and quadratic terms and, using (1.1) and (1.3), write with the aid of Maxwell's equations for the Fourier component of the perturbation field intensity the relation

$$T_{ij}(\omega, \mathbf{k})\,\delta E_j(\omega, \mathbf{k}) = {}^1\!/_2 E_{0n}[S_{inj}(\omega, \mathbf{k}, \omega_0, \mathbf{k}_0)\,\delta E_j(\omega - \omega_0, \mathbf{k} - \mathbf{k}_0) + S_{inj}(\omega, \mathbf{k}, -\omega_0, -\mathbf{k}_0)\,\delta E_j(\omega + \omega_0, \mathbf{k} + \mathbf{k}_0)], \qquad (1.9)$$

where we use the following notation

$$T_{ij}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\left(\frac{c^2k^2}{\omega^2} - \varepsilon^{tr}(\omega, \mathbf{k})\right) + \frac{k_ik_j}{k^2}\varepsilon^l(\omega, \mathbf{k}) + \Delta\varepsilon_{ij}(\omega, \mathbf{k}) + {}^1\!/_4 E_{0n}E_{0m}[\varepsilon_{imnj}(\omega, \mathbf{k}, \omega - \omega_0, \mathbf{k} - \mathbf{k}_0, \omega, \mathbf{k}) + \varepsilon_{imjn}(\omega, \mathbf{k}, \omega - \omega_0, \mathbf{k} - \mathbf{k}_0, -\omega_0, -\mathbf{k}_0) + \Delta\varepsilon_{ii}(\omega, \mathbf{k}) + {}^1\!/_4 E_{0n}E_{0m}[\varepsilon_{imnj}(\omega, \mathbf{k}, \omega - \omega_0, \mathbf{k} - \mathbf{k}_0, \omega, \mathbf{k}) \qquad (1.10)$$

$$S_{inj}(\omega, \mathbf{k}, \omega_0, \mathbf{k}_0) = \varepsilon_{inj}(\omega, \mathbf{k}, \omega_0, \mathbf{k}_0) + \varepsilon_{ijn}(\omega, \mathbf{k}, \omega - \omega_0, \mathbf{k} - \mathbf{k}_0) \qquad (1.11)$$

$$\Delta\varepsilon_{ij}(\omega, \mathbf{k}) = -\frac{4\pi}{\omega}\sum_\alpha \frac{e_\alpha^2}{m_\alpha}\int d\mathbf{v}_\alpha \frac{v_{\alpha,i}}{\Omega}\cdot \alpha_{nj}^{(\alpha)}(\omega, \mathbf{k})\frac{\partial \Delta F_\alpha(\mathbf{v}_\alpha)}{\partial v_{\alpha,n}}. \qquad (1.12)$$

The quantities ϵ_{ijn} and ϵ_{ijnm} are tensors of the nonlinear dielectric constants of the plasma. The expressions for them, in terms of the distribution function $F_{0\alpha}$ are well known (see for example [5]) and are not presented here, for brevity.

Equation (1.9) relates the perturbation of a field of frequency ω and wave vector $\mathbf{k}$ with the perturbations having frequencies $\omega \pm \omega_0$ and wave vectors $\mathbf{k} \pm \mathbf{k}_0$. This is a reflection of the fact that in the ground state the plasma contains a pump wave that is periodic in time and in space. Replacing the quantities ω and $\mathbf{k}$ in (1.9) by $\omega - \omega_0$ and $\mathbf{k} - \mathbf{k}_0$ (and also by $\omega + \omega_0$ and $\mathbf{k} + \mathbf{k}_0$), and discarding terms containing $\delta\mathbf{E}(\omega - 2\omega_0, \mathbf{k} - 2\mathbf{k}_0)$ (or $\delta\mathbf{E}(\omega + 2\omega_0, \mathbf{k} + 2\mathbf{k}_0)$), we obtain the dispersion equation

$$\mathrm{Det}\,\{T_{ij}(\omega, \mathbf{k}) - {}^1\!/_4 E_{0n}E_{0s}[S_{inp}(\omega, \mathbf{k}, \omega_0, \mathbf{k}_0)\cdot T_{pq}^{-1}(\omega - \omega_0, \mathbf{k} - \mathbf{k}_0)\cdot S_{qsj}(\omega - \omega_0, \mathbf{k} - \mathbf{k}_0, -\omega_0, -\mathbf{k}_0) + S_{inp}(\omega, \mathbf{k}, -\omega_0, -\mathbf{k}_0)T_{pq}^{-1}(\omega + \omega_0, \mathbf{k} + \mathbf{k}_0)\cdot S_{qsj}(\omega + \omega_0, \mathbf{k} + \mathbf{k}_0, \omega_0, \mathbf{k}_0)]\} = 0. \qquad (1.13)$$

The possibility of neglecting the terms containing $\delta\mathbf{E}(\omega \pm 2\omega_0, \mathbf{k} \pm 2\mathbf{k}_0)$ (these are precisely the terms that appear in (1.9) when account is taken, in the ground state, of terms that vary at double the frequency) as connected with the fact that we confine ourselves in the dispersion equation only to terms that are quadratic in $\mathbf{E}_0$. On the other hand, allowance for these terms leads to the appearance of terms proportional to E_0^4 in the dispersion equation.

C. Low-frequency perturbations. In the general case the expressions for the tensors T_{ij} and S_{inp} are rather complicated.[5] To simplify them, we make a number of assumptions. We assume that the phase velocity of the pump wave is much larger than the thermal velocity of the particles ($\omega_0 \gg |\mathbf{k}_0\cdot\mathbf{v}_\alpha|$, $|\mathbf{k}\cdot\mathbf{v}_\alpha|$), and the perturbation frequency is much smaller than the frequency of the pump wave ($\omega_0 \gg |\omega|$). In addition, we take into consideration only the electronic terms of the tensors of the nonlinear dielectric constant, meaning that we neglect the oscillations of the ions in the field of the pump wave. We then have

$$S_{inm}(\omega, \mathbf{k}, \omega_0, \mathbf{k}_0) = -S_{mni}(\omega - \omega_0, \mathbf{k} - \mathbf{k}_0, -\omega_0, -\mathbf{k}_0) = -\frac{ie}{m\omega_0^2}\delta_{nm}k_i\delta\varepsilon_e^l(\omega, \mathbf{k}), \qquad (1.14)$$

$$T_{ij}(\omega \pm \omega_0, \mathbf{k} \pm \mathbf{k}_0) = \left\{1 - \frac{1}{4}\frac{k^2}{\omega_0^2}\delta\varepsilon_e^l(\omega, \mathbf{k})\left[\frac{((\mathbf{k} \pm \mathbf{k}_0)\mathbf{v}_E)^2}{(\mathbf{k} \pm \mathbf{k}_0)^2}\times\frac{1}{\varepsilon^l(\omega \pm \omega_0, \mathbf{k} \pm \mathbf{k}_0)} + \frac{[(\mathbf{k} \pm \mathbf{k}_0)\mathbf{v}_E]^2}{(\mathbf{k} \pm \mathbf{k}_0)^2}\frac{1}{\varepsilon^{tr}(\omega \pm \omega_0, \mathbf{k} \pm \mathbf{k}_0) - c^2(\mathbf{k} \pm \mathbf{k}_0)^2/(\omega \pm \omega_0)^2}\right]\right\}^{-1} \times\left\{\frac{(\mathbf{k} \pm \mathbf{k}_0)_i(\mathbf{k} \pm \mathbf{k}_0)_j}{(\mathbf{k} \pm \mathbf{k}_0)^2}\frac{1}{\varepsilon^l(\omega \pm \omega_0, \mathbf{k} \pm \mathbf{k}_0)} + \frac{\delta_{ij} - (\mathbf{k} \pm \mathbf{k}_0)_i(\mathbf{k} \pm \mathbf{k}_0)_j/(\mathbf{k} \pm \mathbf{k}_0)^2}{\varepsilon^{tr}(\omega \pm \omega_0, \mathbf{k} \pm \mathbf{k}_0) - c^2(\mathbf{k} \pm \mathbf{k}_0)^2/(\omega \pm \omega_0)^2}\right\} \qquad (1.15)$$

$$T_{ij}(\omega, \mathbf{k}) = \left(\delta_{ij} - \frac{k_ik_j}{k^2}\right)\left(\frac{c^2k^2}{\omega^2} - \varepsilon^{tr}(\omega, \mathbf{k})\right) + \frac{k_ik_j}{k^2}\varepsilon^l(\omega, \mathbf{k}) + \frac{1}{2}\frac{\omega_{Le}^2}{\omega^2}\int dv_e\frac{F_{0,e}}{\Omega^2}\left[(k_j\mathbf{k}\mathbf{v}_e - k^2v_{e,j})\frac{\mathbf{k}\mathbf{v}_E}{\Omega}\left(2v_{E,i} + 3\frac{\mathbf{k}\mathbf{v}_E}{\Omega}v_{e,i}\right) + \frac{(\mathbf{k}\mathbf{v}_E)^2}{\Omega}(\delta_{ij}\mathbf{k}\mathbf{v}_e - k_iv_{e,j})\right], \qquad (1.16)$$

where $\omega_{L\alpha}^2 = 4\pi N_\alpha e_\alpha^2/m_\alpha$ is the Langmuir frequency of particles of species α, $\mathbf{v}_E = e\mathbf{E}_0/m\omega_0$ is the amplitude of the velocity of the oscillations of the electron in the field of the pump wave, and

$$\delta\varepsilon_\alpha^l(\omega,\mathbf{k}) = \frac{4\pi e_\alpha^2}{m_\alpha k^2\omega}\int d\mathbf{v}_\alpha \frac{\mathbf{k}\mathbf{v}_\alpha}{\omega-\mathbf{k}\mathbf{v}_\alpha}\left(\mathbf{k}\frac{\partial F_{0,\alpha}}{\partial \mathbf{v}_\alpha}\right). \quad (1.17)$$

We are interested in perturbations whose phase velocity is larger than the thermal velocity of the ions, and whose frequencies are smaller than the ion Langmuir frequency ($\delta\epsilon_i^l \gg 1$). Using (1.14)–(1.16), we get from (1.13)

$$0 = \varepsilon^l(\omega,\mathbf{k}) + \delta\varepsilon_e^l(\omega,\mathbf{k})\,\delta\varepsilon_i^l(\omega,\mathbf{k})\frac{k^2}{4\omega_0^2}\left\{\frac{((\mathbf{k}-\mathbf{k}_0)\mathbf{v}_E)^2}{(\mathbf{k}-\mathbf{k}_0)^2}\right.$$
$$\times\frac{1}{\varepsilon^l(\omega-\omega_0,\mathbf{k}-\mathbf{k}_0)} + \frac{((\mathbf{k}+\mathbf{k}_0)\mathbf{v}_E)^2}{(\mathbf{k}+\mathbf{k}_0)^2}\frac{1}{\varepsilon^l(\omega+\omega_0,\mathbf{k}+\mathbf{k}_0)}$$
$$+\frac{[(\mathbf{k}-\mathbf{k}_0)\mathbf{v}_E]^2}{(\mathbf{k}-\mathbf{k}_0)^2}\frac{1}{\varepsilon^{tr}(\omega-\omega_0,\mathbf{k}-\mathbf{k}_0)-c^2(\mathbf{k}-\mathbf{k}_0)^2/(\omega-\omega_0)^2}$$
$$\left.+\frac{[(\mathbf{k}+\mathbf{k}_0)\mathbf{v}_E]^2}{(\mathbf{k}+\mathbf{k}_0)^2}\frac{1}{\varepsilon^{tr}(\omega+\omega_0,\mathbf{k}+\mathbf{k}_0)-c^2(\mathbf{k}+\mathbf{k}_0)^2/(\omega+\omega_0)^2}\right\} \quad (1.18)$$

In the limit when $\mathbf{k}_0 = 0$ and $\omega_0 \gg \omega_{Le}$, and also neglecting the perturbation of the magnetic field of the pump wave ($c = \infty$), it follows from (1.18) that

$$\varepsilon^l(\omega,\mathbf{k}) + \frac{1}{2}\frac{(\mathbf{k}\mathbf{v}_E)^2}{\omega_0^2}\delta\varepsilon_e^l(\omega,\mathbf{k})\,\delta\varepsilon_i^l(\omega,\mathbf{k}) = 0.$$

This equation was first obtained by Aliev and Silin[6] for the case of arbitrary intensities of the pump field (instead of the factor $(\mathbf{k}\cdot\mathbf{v}_E)^2/2\omega_0^2$ they contain the quantity $1 - J_0^2(\mathbf{k}\cdot\mathbf{v}_E/\omega_0)$, where J_0 is a Bessel function of zero order).

The distribution function of the particles in the ground state will be assumed Maxwellian. Then formula (1.17) allows us to write[4]

$$\delta\varepsilon_\alpha^l(\omega,k) = (kr_{D\alpha})^{-2}\left[1-\beta_\alpha\exp\left(-\frac{\beta_\alpha^2}{2}\right)\int\limits_{+i\infty}^{\beta_\alpha} d\tau\exp\left(\frac{\tau^2}{2}\right)\right],$$

where $r_{D\alpha} = (T_\alpha/4\pi N_\alpha e_\alpha^2)^{1/2}$ is the Debye radius of the particles species α, $\beta_\alpha = \omega/kv_{T\alpha}$, $v_{T\alpha} = \sqrt{T_\alpha/m_\alpha}$, and T_α is the temperature of the particles of species α. For perturbations whose phase velocity is smaller than the thermal velocity of the electrons with larger than the thermal velocity of the ions, we obtain from (1.18)

$$\omega^2 = \omega_{Li}^2\left\{(kr_{De})^2 + \frac{k^2}{4}\left[\frac{((\mathbf{k}-\mathbf{k}_0)\mathbf{v}_E)^2}{(\mathbf{k}-\mathbf{k}_0)^2}\frac{1}{(\omega-\omega_0)^2-\omega_p^2}\right.\right.$$
$$+\frac{((\mathbf{k}+\mathbf{k}_0)\mathbf{v}_E)^2}{(\mathbf{k}+\mathbf{k}_0)^2}\frac{1}{(\omega+\omega_0)^2-\omega_p^2}$$
$$+\frac{[(\mathbf{k}-\mathbf{k}_0)\mathbf{v}_E]^2}{(\mathbf{k}-\mathbf{k}_0)^2}\frac{1}{(\omega-\omega_0)^2-\omega_p^2-c^2(\mathbf{k}-\mathbf{k}_0)^2}$$
$$\left.\left.+\frac{[(\mathbf{k}+\mathbf{k}_0)\mathbf{v}_E]^2}{(\mathbf{k}+\mathbf{k}_0)^2}\frac{1}{(\omega+\omega_0)^2-\omega_p^2-c^2(\mathbf{k}+\mathbf{k}_0)^2}\right]\right\}. \quad (1.19)$$

In the derivation of (1.19) we have neglected the imaginary parts of the dielectric constants and assumed that $\omega_0 \gg k_0 v_{Te}$, and also used the notation $\omega_p^2 = \omega_{Le}^2 + \omega_{Li}^2$.

2. LIQUID

To obtain a dispersion equation describing the spectra of small perturbations in a liquid medium through which an electromagnetic wave passes, we shall use a method employed in macroscopic electrodynamics to describe scattering[7,8].

As the material equation relating the induction vector **D** with the electric field intensity vector **E** we use

$$D_i(\mathbf{r},t) = \int\limits_{-\infty}^{t} dt'\varepsilon_{ij}(t-t',t,\mathbf{r})E_j(t',\mathbf{r}), \quad (2.1)$$

where the dielectric tensor $\epsilon_{ij}(\tau, t, \mathbf{r})$ consists of two terms:

$$\varepsilon_{ij}(\tau,t,\mathbf{r}) = \varepsilon(\tau)\delta_{ij} + \delta\varepsilon_{ij}(\tau,t,\mathbf{r}); \quad (2.2)$$

$\epsilon(\tau)$ is the dielectric constant of a homogeneous isotropic and stationary liquid, and $\delta\epsilon_{ij}$ is a small addition that takes into account the perturbations in the liquid. We assume that the electric field can also be represented in the form of two terms:

$$\mathbf{E}(\mathbf{r},t) = \mathbf{E}_0\cos(\omega_0 t - \mathbf{k}_0\mathbf{r}) + \delta\mathbf{E}(\mathbf{r},t). \quad (2.3)$$

The first term describes the field of the pulse wave and the second the field of small perturbations in the liquid.

When account is taken of the largest terms in formulas (2.2) and (2.3) we get from the equations

$$\mathrm{rot\,rot}\,\mathbf{E} = -\frac{1}{c^2}\frac{\partial^2\mathbf{D}}{\partial t^2}, \quad \mathrm{div}\,\mathbf{D} = 0 \quad (2.4)$$

the dispersion laws for the pump wave; in a transparent liquid these are of the form

$$\omega_0^2\varepsilon(\omega_0) = c^2k_0^2, \ \varepsilon(\omega_0) = 0, \quad (2.5)$$

where

$$\varepsilon(\omega) = \int\limits_0^\infty d\tau\,\varepsilon(\tau)e^{i\omega\tau}.$$

Further, from (2.4) and formulas (2.2) and (2.3) we obtain the connection between the perturbations of the field and the perturbation of the dielectric constant, which takes in the linear approximation the form

$$\delta E_j(\omega,\mathbf{k}) = \frac{\omega_0^2}{k^2c^2}E_{0n}T_{ji}^{-1}(\omega,\mathbf{k})$$
$$\times\int dr dt\,e^{i\omega t-i\mathbf{k}\mathbf{r}}\delta\varepsilon_{in}(\omega_0,t,\mathbf{r})\cos(\omega_0 t-\mathbf{k}_0\mathbf{r}); \quad (2.6)$$

where $\delta\mathbf{E}(\omega, \mathbf{k})$ is the Fourier component of the perturbation field,

$$T_{ij}(\omega,\mathbf{k}) = \delta_{ij}\left(1-\frac{\omega^2}{c^2k^2}\varepsilon(\omega)\right) - \frac{k_ik_j}{k^2},$$
$$\delta\varepsilon_{ij}(\omega,t,\mathbf{r}) = \int\limits_0^\infty d\tau\,e^{i\omega\tau}\delta\varepsilon_{ij}(\tau,t,\mathbf{r}). \quad (2.7)$$

In the derivation of (2.6) we used the assumption that the function $\delta\epsilon_{ij}$ changes little when the argument t changes by an amount ω_0^{-1}. In addition, it is assumed that small perturbations do not influence the transparency of the liquid.

For simplicity we assume that the perturbations of the dielectric constant are of the scalar type[7] and are determined by the isothermal perturbations of the density of the liquid:

$$\delta\varepsilon_{ij}(\omega,t,\mathbf{r}) = \delta_{ij}\left(\frac{\partial\varepsilon(\omega)}{\partial\rho_0}\right)_T\delta\rho(\mathbf{r},t), \quad (2.8)$$

where ρ_0 is the density of the unperturbed liquid. Then, using relation (2.6), we obtain with the aid of (2.8) the connection between the perturbations of the field and the perturbations of the density:

$$\delta E_i(\omega, \mathbf{k}) = \frac{\omega_0^2}{2c^2} E_{0j} \left(\frac{\partial \varepsilon(\omega_0)}{\partial \rho_0} \right)_T T_{ij}^{-1}(\omega, \mathbf{k}) [\delta\rho(\omega + \omega_0, \mathbf{k} + \mathbf{k}_0) + \delta\rho(\omega - \omega_0, \mathbf{k} - \mathbf{k}_0)], \quad (2.9)$$

where $\delta\rho(\omega, \mathbf{k})$ is the Fourier component of the density perturbations.

Unlike in scattering theory, where it is assumed that the perturbations of the density did not depend on the parameters of the incident wave, we shall assume that $\delta\rho$ is connected with the pump field. To establish this connection, we use the continuity equation and the equations of motion of the liquid, with allowance for the time-averaged forces acting in an alternating electromagnetic field.[9] In the standard notation, these equations are given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial \mathbf{r}} \rho \mathbf{v} = 0, \quad (2.10)$$

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} \right) = -\frac{\partial p}{\partial \mathbf{r}} + \frac{1}{8\pi} \frac{\partial}{\partial \mathbf{r}} \left[\overline{E_i E_j} \rho \left(\frac{\partial \varepsilon_{ij}(\omega_0, t, \mathbf{r})}{\partial \rho} \right)_T \right] - \frac{\overline{E_i E_j}}{8\pi} \frac{\partial}{\partial \mathbf{r}} \varepsilon_{ij}(\omega_0, t, \mathbf{r}), \quad (2.11)$$

where the var in (2.11) denotes averaging with respect to time.

Linearizing (2.10) and (2.11) with respect to small perturbations of the field, of the dielectric constants, of the pressure, and of the density, and using formulas (2.6)–(2.8), we obtain the dispersion law for small isothermal perturbations in a transparent liquid medium:

$$\omega^2 = k^2 \Big\{ \left(\frac{\partial p}{\partial \rho} \right)_T - \frac{1}{8\pi} \rho E_0^2 \left(\frac{\partial^2 \varepsilon}{\partial \rho^2} \right)_T + \frac{1}{16\pi} \rho \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T^2 \Big[\frac{((\mathbf{k}+\mathbf{k}_0)\mathbf{E}_0)^2}{(\mathbf{k}+\mathbf{k}_0)^2 \varepsilon(\omega+\omega_0)} + \frac{((\mathbf{k}-\mathbf{k}_0)\mathbf{E}_0)^2}{(\mathbf{k}-\mathbf{k}_0)^2 \varepsilon(\omega-\omega_0)} + \frac{[(\mathbf{k}+\mathbf{k}_0)\mathbf{E}_0]^2}{(\mathbf{k}+\mathbf{k}_0)^2} \frac{1}{\varepsilon(\omega+\omega_0) - c^2(\mathbf{k}+\mathbf{k}_0)^2/(\omega+\omega_0)^2} + \frac{[(\mathbf{k}-\mathbf{k}_0)\mathbf{E}_0]^2}{(\mathbf{k}-\mathbf{k}_0)^2} \frac{1}{\varepsilon(\omega-\omega_0) - c^2(\mathbf{k}-\mathbf{k}_0)^2/(\omega-\omega_0)^2} \Big] \Big\},$$

$$\varepsilon(\omega_0) \equiv \varepsilon, \quad \rho \equiv \rho_0. \quad (2.12)$$

In the case of a plasma in which the electron temperature exceeds the ion temperatures ($T_e > T_i(|e|/|e_i|)$), we have for the quantities entering in (2.12)

$$\left(\frac{\partial p}{\partial \rho} \right)_T = \frac{T_e}{m_i} \frac{|e_i|}{|e|}, \quad \varepsilon(\omega) \approx 1 - \frac{4\pi N_e e^2}{m\omega^2}, \quad \rho \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T = \varepsilon - 1.$$

Using these expressions, we get from (2.12) the dispersion law (1.19).

3. WEAK COUPLING APPROXIMATION

We assume further that the pump wave has a transverse polarization and a frequency higher than the natural frequency of the longitudinal electric waves in the medium,[7] so that $\epsilon(\omega_0) \neq 0$ (for a plasma this means that $\omega_0 > \omega_p$). Using the condition $|\omega| \ll \omega_0$ assumed in the derivation of the dispersion equation, as well as formula (2.5), we rewrite (2.12) in the form

$$\left\{ \omega^2 - k^2 \left[\left(\frac{\partial p}{\partial \rho} \right)_T - \frac{1}{8\pi} \rho E_0^2 \left(\frac{\partial^2 \varepsilon}{\partial \rho^2} \right)_T \right] \right\} \left\{ -\omega \frac{\partial(\omega_0^2 \varepsilon)}{\partial \omega_0} - c^2(k^2 - 2\mathbf{k}\mathbf{k}_0) \right\} \times \left\{ \omega \frac{\partial(\varepsilon\omega_0^2)}{\partial \omega_0} - c^2(k^2 + 2\mathbf{k}\mathbf{k}_0) \right\} = \frac{\omega_0^2 c^2 k^4}{8\pi} \rho \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T^2 \left(\frac{(\mathbf{k}\mathbf{E}_0)^2}{k_0^2} - E_0^2 \right). \quad (3.1)$$

It is obvious that this equation describes the spectrum of coupled perturbations of the density and of the electromagnetic field in the medium. It is usually assumed that the amplitude of the pump wave is sufficiently small and that the coupling of the acoustic (ion-acoustic) waves with the electromagnetic ones is weak. In this case Eq. (3.1) can be solved by perturbation theory, assuming that in the zeroth approximation $\mathbf{E}_0 = 0$. We then obtain that for nonresonant wave vectors ($k^2 \neq \pm 2\mathbf{k}\cdot\mathbf{k}_0$) there exist in the liquid only acoustic waves with the usual dispersion law

$$\omega^2 = \omega_s^2 \equiv k^2 s^2, \quad (3.2)$$

where $s^2 = (\partial p/\partial \rho)_T$.

On the other hand, if the wave vectors of the pump wave and of the perturbation are such that the following relation is satisfied (the resonance condition)

$$k^2 = \pm \left(2\mathbf{k}\mathbf{k}_0 - \omega_s \frac{1}{c^2} \frac{\partial}{\partial \omega_0}(\omega_0^2 \varepsilon) \right), \quad (3.3)$$

then electromagnetic waves (the Stokes and anti-Stokes components of the Mandel'shtam-Brillouin scattering) besides the sound waves, exist and relation (3.3) determines their dispersion law

$$\varepsilon(\omega \pm \omega_0)(\omega \pm \omega_0)^2 = c^2(\mathbf{k} \pm \mathbf{k}_0)^2.$$

Allowance for the pump wave for nonresonant perturbations leads only to an insignificant change of the frequency given by formula (3.2). Conversely, for resonant wavelengths the pump wave gives rise to an instability, and the amplitudes of the sound waves and of the electromagnetic waves (of the Stokes component) increase with time. Assuming in (3.1) that $\omega = \omega_s + \Delta$ in the case when $\mathbf{k} \parallel \mathbf{k}_0$, we get

$$\Delta = \pm ikE_0 \rho \left(\frac{\partial \varepsilon}{\partial \rho} \right)_T \omega_0 \left(32\pi\rho\omega_s \frac{\partial(\omega_0^2 \varepsilon)}{\partial \omega_0} \right)^{-1/2}. \quad (3.4)$$

The maximum amplitude of the pump wave at which the weak-coupling approximation is applicable is determined by the inequality $|\Delta| \ll \omega_s$. Using (3.2) and (3.4) we obtain

$$E_0 \ll E_{0,\,\mathrm{cr}} \equiv \left| \frac{s\omega_s^{1/2}}{\omega_0 \rho (\partial\varepsilon/\partial\rho)_T} \left(32\pi\rho \frac{\partial}{\partial \omega_0}(\varepsilon\omega_0^2) \right)^{1/2} \right|. \quad (3.5)$$

At optical frequencies, the quantities which enter in formula (3.5) for most liquids are:[8] $s^2 = 10^{10}$ cm^2/sec^2, $\omega_0 = 2 \times 10^{15}$ sec^{-1}, $\epsilon = 3.5$, $\rho = 1$ g/cm^3, $\rho(\partial\epsilon/\partial\rho)_T \sim 1$, $\omega_0^2(\partial\epsilon/\partial\omega) \sim \omega_0\epsilon$, $\omega_s \approx 2 \times 10^{10}$ sec^{-1}. It follows from (3.5) that $E_{0,cr} \approx 2 \times 10^6$ V/cm, corresponding to an energy flux density $W \approx 10^{11}$ W/cm^2.

In the case of a plasma, we get from (3.5)

$$\frac{v_{E,\,\mathrm{cr}}^2}{v_{Te}^2} = 32 \frac{\omega_0^2}{\omega_p^2} \frac{1}{c} \sqrt{\frac{T_e}{m_i}}. \quad (3.6)$$

If we assume that $\omega_0/\omega_p \approx 10$ and $(\sqrt{T_e/m_i})/c \approx 10^{-5}$, then $v_{E,cr} \approx 0.1\, v_{Te}$.

To determine the minimum amplitude of the pump wave, starting with which instability sets in, it is necessary to take into account the dissipative processes that lead to the damping of the sound and electromagnetic waves. If in (2.11) we take account of the viscosity of the liquid and assume that the imaginary part of the dielectric constant $\epsilon'' = \mathrm{Im}\,\epsilon(\omega_0)$ differs from zero, then we get

$$E_0^2 > E_{0,\,\min}^2 = 16\pi \frac{\omega_s}{\rho(\partial\varepsilon/\partial\rho)_T^2} \eta\varepsilon'', \quad (3.7)$$

where η is the viscosity coefficient.

The minimum amplitude for the plasma can be obtained by taking into account the imaginary parts of the longitudinal and transverse dielectric constants ($\epsilon^{l\prime\prime}(\omega_s)$ and $\epsilon^{tr\prime\prime}(\omega_0)$) in (1.18):

$$\frac{v_E^2}{v_{Te}^2} > \frac{v_{E,\,\min}^2}{v_{Te}^2} = \frac{\omega_0^2}{\omega_p^2}\varepsilon^{l\prime\prime}(\omega_s)\varepsilon^{tr\prime\prime}(\omega_0). \quad (3.8)$$

Assuming that the damping of the electromagnetic wave is connected with particle collisions, and that of the ion-acoustic waves with Landau damping on the electron[4], we get

$$\frac{v_{E,\,\min}^2}{v_{Te}^2} = \sqrt{\frac{m}{m_i}}\frac{\nu_{\rm eff}}{\omega_0}, \quad (3.9)$$

where $\nu_{\rm eff}$ is the effective collision frequency, whose order of magnitude is equal to $(e^2N_e^{1/3}/T_e)^{3/2}\omega_p$.

We note that in solving (3.1) we have assumed that the perturbation wave vector is given and we obtained ω. By the same token, we have considered the problem of the initial perturbations in the medium. In the nonlinear optics it is customary to solve the boundary-value problem. This corresponds to determining the wave vector from the dispersion equation (3.1) at a given frequency ω. As a result we get when $\mathbf{k} \parallel \mathbf{k}_0$:

$$k = 2k_0 + \Delta k = 2k_0 \pm i\frac{\omega_0}{cs}\left(\frac{1}{32\pi}\right)^{1/2}\frac{1}{\sqrt{\rho}}\left(\rho\frac{\partial\varepsilon}{\partial\rho}\right)_T E_0. \quad (3.10)$$

The second term, taken with a minus sign describes the parametric amplification of the sound (see [3]). From the condition $|\Delta k| \ll k_0$ we obtain the critical field for the boundary-value problem

$$E_{0,\,\rm cr}^{\rm b} = 2\sqrt{32\pi}\,\frac{s\sqrt{\rho}}{\rho(\partial\varepsilon/\partial\rho)_T}. \quad (3.11)$$

This field is much larger than the critical field (3.5) for the initial problem (by approximately a factor $\sqrt{c/s}$). It follows therefore that the applicability of the approximation of weak parametric coupling of the waves is determined by the formulation of the problem, and can be determined only by considering the concrete conditions under which the experiment is performed.

4. STRONG COUPLING

To determine the spectrum of the perturbations when $E_0 > E_{0,\rm cr}$, we rewrite (2.12) in the form

$$\omega^2 = k^2\left\{\left(\frac{\partial p}{\partial\rho}\right)_T - \frac{1}{8\pi}\rho E_0^2\left(\frac{\partial^2\varepsilon}{\partial\rho^2}\right)_T + \frac{1}{8\pi}\omega_0^2\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)^2\frac{k^2}{k_0^2}\right.$$
$$\left.\times\frac{k_0^2E_0 - (\mathbf{k}\mathbf{E}_0)^2}{c^2[4(\mathbf{k}\mathbf{k}_0)^2 - k^4] - 4\mathbf{k}\mathbf{k}_0\omega\,\partial(\omega_0^2\varepsilon)/\partial\omega_0}\right\}. \quad (4.1)$$

In all cases of practical interest, the second term in the curly brackets of (4.1) is much smaller than the first and will be disregarded in what follows.

We consider the simplest and most interesting case, when $\mathbf{k} \parallel \mathbf{k}_0$. We assume first that the following inequality is satisfied

$$kc^2|4k_0^2 - k^2| \gg 4k_0\left|\omega\frac{\partial}{\partial\omega_0}(\varepsilon\omega_0^2)\right|. \quad (4.2)$$

In this case the solution of the dispersion equation (4.1) is given by

$$\omega^2 = k^2\left\{\left(\frac{\partial p}{\partial\rho}\right)_T - \frac{1}{8\pi}E_0^2\rho\frac{1}{\varepsilon}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2\frac{k_0^2}{k^2 - 4k_0^2}\right\}. \quad (4.3)$$

When $k \ll k_0$, the second term in (4.3), for all realistically attainable fields, is small compared with the first, and the dispersion law (4.3) coincides with formula (3.2). On the other hand, if $k \to 2k_0$, then the second term in (4.3) increases. However, at small pump-wave amplitudes, the inequality (4.2) is violated before the dispersion law (3.2) is altered. We are interested precisely in the opposite case, when the second term in (4.3) is larger than the first. In conjunction with the inequality (4.2), this condition is fulfilled for wave vectors satisfying the inequality

$$\frac{1}{8\pi}E_0^2\rho\frac{k_0^2(\partial\varepsilon/\partial\rho)_T^2}{\varepsilon(\partial p/\partial\rho)_T} \gg |k^2 - 4k_0^2| \gg \left[\frac{\partial}{\partial\omega_0}(\omega_0^2\varepsilon)\right]^{2/3}$$
$$\times\left[\frac{2}{\pi}\frac{k_0^4}{c^4}E_0^2\rho\frac{1}{\varepsilon}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2\right]^{1/3}. \quad (4.4)$$

It is obvious that the inequalities (4.4) can be fulfilled only for pump-wave amplitudes larger than a certain value. Accurate to a numerical factor on the order of unity, this value coincides with the critical field (3.5).

If the wave vectors of the perturbations and the field intensities are such that the inequalities (4.4) are satisfied, and at the same time $k > 2k_0$, then, as follows from (4.3), we get $\omega^2 < 0$. In other words, under these conditions the density perturbations have an aperiodic instability. The instability increment increases as k approaches $2k_0$, but at the resonance point $k = 2k_0$ itself, formula (4.3) is not valid, for then the inequality (4.2) is violated. To investigate the solution of (4.1) at the resonance, we put $\mathbf{k} \parallel \mathbf{k}_0$ and $k = 2k_0$; as the result we get

$$\omega^3 = k^2\left\{\left(\frac{\partial p}{\partial\rho}\right)_T\omega - \frac{E_0^2}{16\pi}\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2\frac{\omega_0^2}{\partial(\omega_0^2\varepsilon)/\partial\omega_0}\right\}. \quad (4.5)$$

The perturbations that increase most rapidly with time occur when the second term in (4.5) is larger than the first:

$$\left(\frac{\partial p}{\partial\rho}\right)_T|\omega| < \frac{E_0^2}{16\pi}\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2\frac{\omega_0^2}{\partial(\omega_0^2\varepsilon)/\partial\omega_0}, \quad (4.6)$$

and the unstable solution is of the form

$$\omega = \frac{1}{2}(1 + i\sqrt{3})(2k_0)^{2/3}E_0^{2/3}\left[\frac{1}{16\pi}\rho\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2\frac{\omega_0^2}{\partial(\omega_0^2\varepsilon)/\partial\omega_0}\right]^{1/3}. \quad (4.7)$$

It should be noted that expression (4.7) agrees with the general theory developed by Nishikawa[10] for strong parametric coupling in a system with two degrees of freedom. As applied to our formulation of the problem, a solution analogous to (4.7) was considered by L. V. Keldysh and N. N. Lavrinovich (private communication).

To determine the critical amplitude of the pump wave, starting with which instability sets in, we consider the inequality (4.6). Substituting (4.7) in (4.6) we find that the amplitude of the pump wave should be larger than the critical value (3.5). The time of instability development at the same numerical values as used for the liquid in the preceding section, is $\sim 3 \times 10^{-11}$ sec at $E_0 \approx E_{0,\rm cr}$.

As applied to a plasma, the dispersion law (4.3) was first obtained by Volkov[11] with the aid of the equations of two-fluid hydrodynamics. However, the maximum increment, which takes place under the condition $k = 2k_0$, was not considered by him. Let us estimate the order of magnitude of this increment with the aid of the formula (4.7):

$$\gamma_{\max} = \mathrm{Im}\,\omega \sim \omega_0\left(\frac{v_E}{c}\frac{\omega_{Li}}{\omega_0}\right)^{2/3}. \quad (4.8)$$

We note that the limitation $|\omega| \ll \omega_{Li}$, which was assumed by us in the derivation of the dispersion equation, together with the condition $v_E > v_{E,cr}$, where $v_{E,cr}$ is given by formula (3.6), leads to the inequality $\omega_p > (v_{Te}/c)^{1/3}\omega_0$.

CONCLUSIONS

Thus, the notion customarily used in nonlinear optics, namely that the acoustic and electromagnetic waves are two weakly-coupled branches of elementary excitations in a medium, is valid for pump-wave amplitudes lower than the critical value $E_{0,cr}$ (see formulas (3.5) and (3.6)). It is precisely under these conditions that one can speak of a growth of the sound-wave amplitudes with an increment (3.4).

On the other hand, if the amplitude of the pump wave is larger than $E_{0,cr}$, then the connection between the perturbations of the density and of the electromagnetic field can no longer be regarded as weak. In this case it follows from the solution of the dispersion equation that the perturbations with wave vectors in the interval

$$2k_0 \leqslant k \leqslant 2k_0\left[1 + \frac{1}{64\pi}E_0{}^2\rho\frac{1}{\varepsilon^2(\partial p/\partial\rho)_T}\left(\frac{\partial\varepsilon}{\partial\rho}\right)_T^2\right]$$

grow aperiodically with time.

The physical cause of this instability lies in the following. The electromagnetic wave scattered by the density perturbations, in conjunction with the pump wave, forms a field structure of the standing-wave type, and if the resultant striction force is sufficient to increase the initial perturbations of the density, then this increases the scattered field and leads to a further increase of the forces acting on the liquid and to a further increase of the density perturbations. The field of the scattered wave at a given point can then be sufficiently large because the waves emerging from a region of space having a length of order c/γ ($\gamma = \mathrm{Im}\,\omega$) add up coherently.

We note that the assumption used by us, namely that the density perturbations in the liquid are isothermal, is generally speaking not always valid. Actually, the perturbations should be regarded as adiabatic and it is necessary to take the temperature perturbations into account. However, inasmuch as the difference between the adiabatic and isothermal compressibilities of the liquid is small, we have assumed the perturbations to be isothermal, in order to simplify the derivations, and disregarded the temperature perturbations.

It is obvious that the linear theory considered here cannot answer the question concerning the fate of the unstable perturbations. It is possible, however, that it is precisely these instabilities, whose threshold exceeds the SMBS threshold, caused the anomalies of the SMBS observed in strong fields and the damage produced in bodies at the focus of a laser beam.

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