SPIN WAVES AND THERMODYNAMIC PROPERTIES OF UNIAXIAL ANTIFERROMAGNETS

J. SÓLYOM[1)]

Institute for Theoretical Physics, Academy of Sciences, USSR

Submitted July 1, 1968

Zh. Eksp. Teor. Fiz. 55, 2355–2366 (December, 1968)

The Heisenberg model of antiferromagnets with an easy axis of magnetization is examined using perturbation theory. The temperature dependence of the spin wave energy at low temperatures and the spin wave damping at low temperatures, as well as in the vicinity of the Néel point are calculated. The free energy, sublattice magnetization, and magnetic susceptibility are also investigated.

1. INTRODUCTION

THE energy spectrum of spin waves and thermodynamic quantities of Heisenberg antiferromagnetics have up until now been investigated predominantly in the Holstein-Primakoff approximation[1] or by the method of time Green functions.[2] However, as is well known, the applicability of the results obtained by these methods is limited, and hence in[3] the spin waves and thermodynamic properties of ideal Heisenberg antiferromagnets were investigated in the framework of perturbation theory proposed for ferromagnets by Vaks, Larkin, and Pikin.[4]

Actually, in crystalline substances there is always some kind of anisotropy, and therefore it is of interest to investigate the effect of magnetic anisotropy on the spin wave spectrum and magnetic properties. In this paper we investigate a magnetically anisotropic antiferromagnet of the "easy-axis" type. After a short exposition of a diagrammatic technique, which differs somewhat from the technique used in[3], we shall determine the spin wave spectrum and its temperature renormalization. In addition, we investigate the damping of spin waves both at low temperatures and in the vicinity of the Néel point and obtain the condition for the existence of long-wavelength spin waves in this temperature interval. In particular, from these expressions the damping of spin waves in an isotropic antiferromagnet is obtained. In Sec. 4, we consider the free energy, sublattice magnetization, and magnetic susceptibility.

2. CORRELATION FUNCTIONS AND SPIN WAVE SPECTRUM

We shall investigate a Heisenberg antiferromagnet with magnetic anisotropy of the "easy-axis" type and suppose that the Hamiltonian of the system contains, besides the exchange term, an anisotropic part consisting of both single- and double-ion terms, i.e.,

$$\mathscr{H} = -\mu H\Big(\sum_f S_f^z + \sum_g S_g^z\Big) - \sum_{f,g} V(f-g)\mathbf{S}_f\mathbf{S}_g - \delta\sum_{f,g}' V(f-g)S_f^zS_g^z - K\Big[\sum_f (S_f^z)^2 + \sum_g (S_g^z)^2\Big]. \quad (2.1)$$

where f and g indicate sites of the first and second sublattices, respectively, and δ and K are anisotropy constants.

In order to use perturbation theory to calculate the correlation functions and free energy, we divide the Hamiltonian into two parts:

$$\mathscr{H} = \mathscr{H}_0 + \mathscr{H}_1, \quad (2.2)$$

where

$$\mathscr{H}_0 = \sum_{f,g} V(f-g)(1+\delta)\langle S_f^z\rangle_0\langle S_g^z\rangle_0 + K\Big(\sum_f \langle S_f^z\rangle_0^2 + \sum_g \langle S_g^z\rangle_0^2\Big) - \frac{1}{\beta}\sum_f y_f S_f^z - \frac{1}{\beta}\sum_g y_g S_g^z, \quad (2.3)$$

$$y_f = \beta\Big[\mu H + \sum_g V(f-g)(1+\delta)\langle S_g^z\rangle_0 + 2K\langle S_f^z\rangle_0\Big], \quad (2.4)$$

$$y_g = \beta\Big[\mu H + \sum_f V(f-g)(1+\delta)\langle S_f^z\rangle_0 + 2K\langle S_g^z\rangle_0\Big]; \quad (2.5)$$

$$\mathscr{H}_1 = -\sum_{f,g} V(f-g)(1+\delta)(S_f^z - \langle S_f^z\rangle_0)(S_g^z - \langle S_g^z\rangle_0) - \sum_{f,g} V(f-g)(S_f^+S_g^- + S_f^-S_g^+) - K\Big[\sum_f (S_f^z - \langle S_f^z\rangle_0)^2 + \sum_g (S_g^z - \langle S_g^z\rangle_0)^2\Big]; \quad (2.6)$$

$\mathscr{H}_0$ approximates the molecular field, $\mathscr{H}_1$ is the perturbation. As in[3], we define $\langle S_f^z\rangle_0$ and $\langle S_g^z\rangle_0$ in self-consistent fashion with the aid of $\mathscr{H}_0$ and fix their values:

$$\langle S_f^z\rangle_0 = b(y_f) = (S+1/2)\,\mathrm{cth}\,(S+1/2)y_f - 1/2\,\mathrm{cth}\,1/2 y_f,$$

$$\langle S_g^z\rangle_0 = b(y_g) = (S+1/2)\,\mathrm{cth}\,(S+1/2)y_g - 1/2\,\mathrm{cth}\,1/2 y_g. \quad (2.7)$$

Note that in the absence of an external magnetic field $y_f = -y_g$, and consequently $\langle S_f^z\rangle_0 = -\langle S_g^z\rangle_0$.

To determine the spin-wave spectrum we shall investigate the Fourier component of the Green function of the spin operators

$$K_f(k, i\omega_n) = \frac{1}{2\beta}\int_{-\beta}^{\beta} e^{i\omega_n t}\,dt \sum_{f_1} e^{-ik(f_1-f_2)}\langle \hat{T}\{S_{f_1}^+(t)S_{f_2}^-(0)\}\rangle, \quad (2.8)$$

where $\beta = 1/T$, $i\omega_n = i2\pi Tn$ are the imaginary frequencies of the temperature technique,[5] and $S(t) = e^{\mathscr{H}t}Se^{-\mathscr{H}t}$. An analogous expression can be written down for the second sublattice.

In the zeroth approximation the Green functions have the form

$$K_f^{(0)}(i\omega_n) = G_f^{(0)}(i\omega_n)\langle S_f^z\rangle_0 = \frac{\langle S_f^z\rangle_0}{y_f - i\beta\omega_n},$$

1)Permanent address: Central Institute for Physics Research, Budapest, Hungary.

$$K_g^{(0)}(i\omega_n) = G_g^{(0)}(i\omega_n)\langle S_g^z\rangle_0 = \frac{\langle S_g^z\rangle_0}{y_g - i\beta\omega_n}. \tag{2.9}$$

The Green function of the first sublattice $K_f^{(0)}(i\omega_n)$ is represented in the diagrams by a solid line, and the Green function of the second sublattice $K_g^{(0)}(i\omega_n)$ by a broken line. A wavy line corresponds to the interaction $V(f-g)$ and its Fourier component V_k. In this it is necessary to consider that the interaction coupling the z components of the spins contains the factor $(1+\delta)$. A circle represents single-ion anisotropic interaction.

In the first approximation the Green function can be represented as the sum of staircase diagrams (Fig. 1).

This sum will be represented as a solid line with two arrows. The equation for this quantity is given graphically in Fig. 2. Analytically, it has the form

$$K_f(k, i\omega_n) = K_f^{(0)}(i\omega_n) + K_f^{(0)}(i\omega_n)\beta V_k K_g^{(0)}(i\omega_n)\beta V_k K_f(k, i\omega_n), \tag{2.10}$$

$$K_f(k, i\omega_n) = \frac{K_f^{(0)}(i\omega_n)}{1-(\beta V_k)^2 K_f^{(0)}(i\omega_n) K_g^{(0)}(i\omega_n)}$$

$$= \frac{K_f^{(0)}(i\omega_n)}{1-(\beta V_k)^2\langle S_f^z\rangle_0\langle S_g^z\rangle_0 G_f^{(0)}(i\omega_n) G_g^{(0)}(i\omega_n)}. \tag{2.11}$$

The energy of the spin waves is determined by the poles of (2.11) after the analytical extension $i\omega_n \to \omega + i\delta$. In this approximation we obtain

$$\varepsilon_k = \langle S_f^z\rangle_0 |V_0| \sqrt{(1+\alpha+\delta)^2 - \gamma_k^2}, \tag{2.12}$$

where

$V_0 = \sum_f V(f-g) = \sum_g V(f-g)$, $\alpha = 2K/|V_0|$, $\gamma_k = V_k/V_0$. A

From this, it is seen that the energy of the spin waves is positive, if $\alpha + \delta > 0$. In the opposite case, if $\alpha + \delta < 0$, we have an antiferromagnet with magnetic anisotropy of the "easy-axis" type. We shall not consider this case here.

In the second approximation, in expanding over $1/S$ or $1/r_0^3$, where r_0 is the mean interaction radius, according to[3], it is necessary to take into account the irreducible parts containing one loop. However, at low temperatures, where it is possible to neglect the exponentially small terms of the type $\exp(-y_f)$, it is more convenient to keep the staircase structure of the correlation function and renormalize each part of the staircase. The graphical equations of the renormalized parts are sketched out in Fig. 3.

As is seen, double-ion anisotropy does not yield new diagrams compared with the case of an isotropic antiferromagnet. Its effect consists only in that the interaction coupling the peaks of S^z is multiplied by the factor $(1+\delta)$. At low temperatures the peaks of S^z always have one line going in and one line coming out, so that they are easily distinguished in these diagrams. Single-ion anisotropy gives a new diagram in the renormalization of the simple Green functions. In these diagrams, the circle corresponds to the factor 2K.

FIG. 1.

FIG. 2.

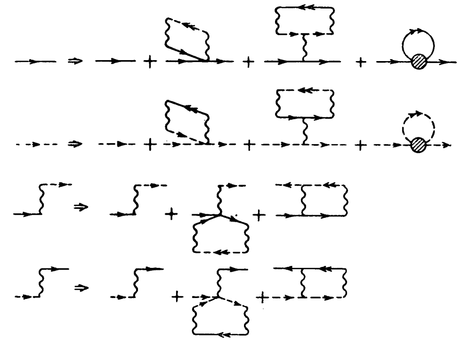

FIG. 3.

Indicating the renormalized quantities with a tilde and introducing the notation

$$\tilde{K}_f(i\omega_n) = K_f^{(0)}(i\omega_n)[1 + \Sigma_{ff} G_f^{(0)}(i\omega_n)],$$

$$\tilde{K}_g(i\omega_n) = K_g^{(0)}(i\omega_n)[1 + \Sigma_{gg} G_g^{(0)}(i\omega_n)],$$

$$(\tilde{V}_k)_{fg} = V_k[1 + \Sigma_{fg}],$$

$$(\tilde{V}_k)_{gf} = V_k[1 + \Sigma_{gf}], \tag{2.13}$$

we obtain the following expressions for these quantities:

$$\Sigma_{ff} = -\Sigma_{gg} = \frac{\beta}{2NS}\sum_q \left[\varepsilon_q \operatorname{cth}\frac{\varepsilon_q}{2T} - S|V_0|(1+\delta)\operatorname{cth}\frac{\varepsilon_0}{2T}\right],$$

$$\Sigma_{fg} = \Sigma_{gf} = \frac{1}{2NS}\sum_q \left[\operatorname{cth}\frac{\varepsilon_0}{2T} - \left(1 - \frac{V_q V_{k-q}}{V_0 V_k}\,\frac{1+\delta}{1+\alpha+\delta}\right)\frac{\varepsilon_0}{\varepsilon_q}\operatorname{cth}\frac{\varepsilon_q}{2T}\right]$$

$$\varepsilon_0 = S|V_0|(1+\alpha+\delta). \tag{2.14}$$

The correlation function in the staircase approximation has the form

$$K_f(k, i\omega_n) = \frac{\tilde{K}_f(i\omega_n)}{1 - \tilde{K}_f(i\omega_n)\beta(\tilde{V}_k)_{fg}\tilde{K}_g(i\omega_n)\beta(\tilde{V}_k)_{gf}}. \tag{2.15}$$

Substituting (2.13) and (2.14) into (2.15) and carrying out the analytical extension $i\omega_n \to \omega + i\delta$, we obtain for the spin-wave spectrum

$$\omega_k^2 = S^2V_0^2\left[1 + \frac{1}{2NS}\sum_q\left(\frac{1+\delta}{1+\alpha+\delta} - \frac{\varepsilon_q}{\varepsilon_0}\operatorname{cth}\frac{\varepsilon_q}{2T}\right)\right]^2 (1+\alpha+\delta)^2$$
$$- S^2V_k^2\left[1 + \frac{1}{2NS}\sum_q\left(1 - \frac{\varepsilon_q}{\varepsilon_0}\operatorname{cth}\frac{\varepsilon_q}{2T} + \right.\right.$$
$$\left.\left. + \frac{S^2V_q^2[(1+\delta)(1+\alpha+\delta)-1]}{\varepsilon_0\varepsilon_q}\operatorname{cth}\frac{\varepsilon_q}{2T}\right)\right]^2. \tag{2.16}$$

Here we have used the fact that in the case of nearest-neighbor interaction[6]

$$\sum_q V_{k-q} f(q) = \gamma_k \sum_q V_q f(q).$$

Because of the presence of a gap in the spin-wave spectrum, the temperature dependence of the energy has an exponential character at very low temperatures.

It is interesting to examine the temperature renormalization in this temperature region, where the dispersion of the spin waves, whose energy is approximately thermal ($\epsilon_q \sim T$), is already linear. At such temperatures

$$\frac{1}{N}\sum_q \frac{\varepsilon_q}{S|V_0|}\operatorname{cth}\frac{\varepsilon_q}{2T} \approx \frac{1}{N}\sum_q \frac{\varepsilon_q}{S|V_0|} + \frac{6}{\pi^2\eta^3}\zeta(4)\left(\frac{T}{S|V_0|}\right)^4 + \ldots,$$
$$\frac{1}{N}\sum_q \frac{S|V_0|}{\varepsilon_q}\operatorname{cth}\frac{\varepsilon_q}{2T} \approx \frac{1}{N}\sum_q \frac{S|V_0|}{\varepsilon_q} + \frac{1}{\pi^2\eta^3}\zeta(2)\left(\frac{T}{S|V_0|}\right)^2 + \ldots, \qquad (2.17)$$

where $\eta = 1/(3)^{1/2}(2)^{1/3}$ for the simple cubic and $\eta = 1/2$ for the body-centered cubic lattice. It is seen from this that, unlike the isotropic case, where the first temperature correction is proportional to T^4, there is a term proportional to T^2 in the spin-wave energy in the case of an anisotropic antiferromagnet. It should be noted, however, that the factor $(1 + \delta)(1 + \alpha + \delta) - 1$ stands in front of these terms, so that in fact the quadratic term is smaller than the quartic.

3. DAMPING OF THE SPIN WAVES

In this section we shall investigate the damping of the spin waves in two limiting cases, namely at low temperatures and near the Néel point. For simplicity, we consider in both cases the contribution of the simplest diagrams corresponding to the first and second approximations in the expansion in $1/r_0^3$ or $1/S$. In addition, we investigate the special case when $\alpha = 0$, so that the anisotropy of the system has only double-ion character.

At low temperatures we take into account the scattering of the spin waves by each other. In this model the total number of spin waves is conserved, and therefore the division of a spin wave into two or the combination of two spin waves into one is not considered. A diagram describing this process is given in Fig. 4, where the heavy lines describe the propagation of the spin waves, and the solid circle (the uppermost part) stands for the amplitude of this scattering. Again applying the staircase approximation, we see that the upper part has a different form in the renormalization of the individual parts of the staircase. The upper parts at the left and right for renormalization of the simple Green function of the first sublattice are given respectively in Fig. 5, a and b.

By connecting the tails of these upper parts in every possible way, we obtain the strictly energetic part, which consists of eight diagrams (Fig. 6).

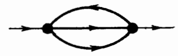

FIG. 4.

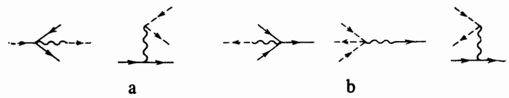

FIG. 5.

In the contribution of these diagrams it is necessary to take into account that each line between points is renormalized by a simple ladder, in order that the diagrams describe the interaction of the spin waves.

The strictly energetic part of the Green function of the second sublattice has an analogous form, except that the solid lines are replaced by dashed lines and vice versa.

The simple interaction is also renormalized; in this case the upper parts and the corresponding diagrams are given in Figs. 7 and 8.

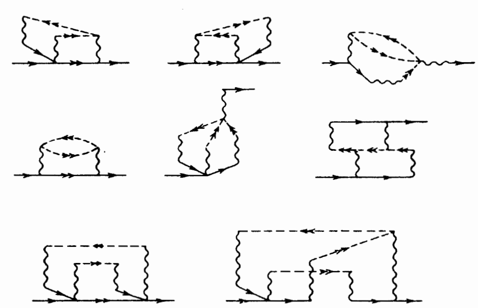

FIG. 6.

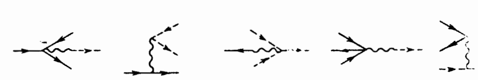

FIG. 7.

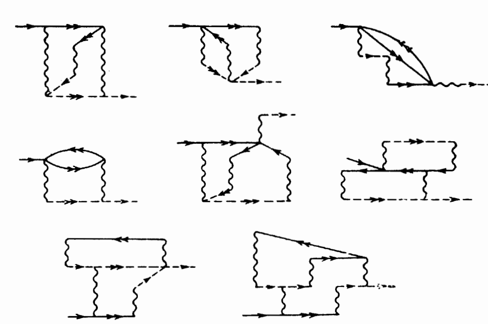

FIG. 8.

After cumbersome calculations and performing the analytical extension $i\omega_n \to \omega + i\delta$ the imaginary part of the strictly energetic part $\Gamma(k, \omega) = \mathrm{Im}\Sigma(k, i\omega_n = \omega + i\delta)$ can be represented in the form

$$\Gamma_{ij}(k, \omega) = \frac{1}{N^2}\sum_{k_1, k_2}\{A_{ij}^{(1)}(k, k_1, k_2)\Delta^{(1)} + A_{ij}^{(2)}(k, k_1, k_2)\Delta^{(2)} + A_{ij}^{(3)}(k, k_1, k_2)\Delta^{(3)}\}, \qquad (3.1)$$

where $i = f, g$ and $j = f, g$ correspond to the indices of the first or second sublattice,

$$\Delta^{(1)} = [n(\varepsilon_{k_1})(1 + n(\varepsilon_{k_2}))(1 + n(\varepsilon_{k+k_1-k_2})) - (1 + n(\varepsilon_{k_1}))n(\varepsilon_{k_2})n(\varepsilon_{k+k_1-k_2})]\cdot\delta(\omega + \varepsilon_{k_1} - \varepsilon_{k_2} - \varepsilon_{k+k_1-k_2}),$$
$$\Delta^{(2)} = [n(-\varepsilon_{k_1})(1 + n(\varepsilon_{k_2}))(1 + n(-\varepsilon_{k+k_1-k_2})) - (1 + n(-\varepsilon_{k_1}))n(\varepsilon_{k_2})n(-\varepsilon_{k+k_1-k_2})]\cdot\delta(\omega - \varepsilon_{k_1} - \varepsilon_{k_2} + \varepsilon_{k+k_1-k_2}),$$
$$\Delta^{(3)} = [n(-\varepsilon_{k_1})(1 + n(-\varepsilon_{k_2}))(1 + n(\varepsilon_{k+k_1-k_2})) - (1 + n(-\varepsilon_{k_1}))n(-\varepsilon_{k_2})n(\varepsilon_{k+k_1-k_2})]\cdot\delta(\omega - \varepsilon_{k_1} + \varepsilon_{k_2} - \varepsilon_{k+k_1-k_2}), \qquad (3.2)$$

and $n(x) = (e^{\beta x} - 1)^{-1}$.

The expressions for the amplitude $A_{ij}^{(1)}(k, k_1, k_2)$

are cumbersome, and hence we give here as an example only $A_{ff}^{(1)}(k, k_1, k_2)$:

$$A_{ff}^{(1)}(k,k_1,k_2) = -\pi S\beta \frac{1}{2\varepsilon_{k_1}}\frac{1}{2\varepsilon_{k_2}}\frac{1}{2\varepsilon_{k+k_1-k_2}} \cdot$$
$$\times\{V_{k_1}V_{k_2}V_{k_1-k_2}(1+\delta)(\varepsilon_{k_2}-\varepsilon_0)(\varepsilon_{k+k_1-k_2}+\varepsilon_0)$$
$$+V_{k_1}V_{k_2}V_{k_1-k_2}(1+\delta)(\varepsilon_{k_1}-\varepsilon_0)(\varepsilon_{k+k_1-k_2}+\varepsilon_0)$$
$$-V_kV_{k_1-k_2}V_{k+k_1-k_2}(1+\delta)(\varepsilon_{k_1}-\varepsilon_0)(\varepsilon_{k_2}-\varepsilon_0)$$
$$-\frac{1}{S}V_{k_1-k_2}^2(1+\delta)^2(\varepsilon_{k_1}-\varepsilon_0)(\varepsilon_{k_2}-\varepsilon_0)(\varepsilon_{k+k_1-k_2}+\varepsilon_0)$$
$$+SV_kV_{k_1}V_{k_2}V_{k+k_1-k_2}(\varepsilon_{k_2}-\varepsilon_0)$$
$$+SV_{k-k_2}V_{k_1-k_2}V_{k_2}V_{k+k_1-k_2}(1+\delta)^2(\varepsilon_{k_1}-\varepsilon_0)$$
$$-SV_{k_1}{}^2V_{k_2}{}^2(\varepsilon_{k+k_1-k_2}+\varepsilon_0)$$
$$-S^2V_{k-k_2}V_{k_1}V_{k_2}{}^2V_{k+k_1-k_2}(1+\delta)\}. \quad (3.3)$$

We now investigate these expressions for small values of the momentum k. Expanding V_k up to squared terms, we have

$$V_k = V_0(1 - {}^1\!/_2\varepsilon^2k^2). \quad (3.4)$$

If $(1+\delta)^2 - 1 \gg \epsilon^2k^2$, then in the approximation (3.4) the energy of the spin waves depends quadratically on the momentum:

$$\varepsilon_k = S|V_0|\sqrt{(1+\delta)^2-1}\left(1+\frac{1}{2}\frac{\varepsilon^2k^2}{(1+\delta)^2-1}\right). \quad (3.5)$$

In the opposite case, when $(1+\delta)^2 - 1 \ll \epsilon^2k^2$, we have

$$\varepsilon_k = S|V_0|\varepsilon k. \quad (3.6)$$

We investigate first the case $(1+\delta)^2 - 1 \ll \epsilon^2k^2$, which is the more interesting experimentally; this is the region of momenta where the effect of anisotropy is insignificant. It is easy to see that in this case

$$\Gamma_{ff}(k,\omega) = -\Gamma_{gg}(k,\omega) = \beta SV_0\Gamma_{fg}(k,\omega) = \beta SV_0\Gamma_{gf}(k,\omega). \quad (3.7)$$

With the aid of (2.15), we find that a pole of the correlation function appears when

$$\omega_k' = \varepsilon_k\left(1 + i\frac{\Gamma_{ff}(k,\varepsilon_k)}{\beta SV_0}\right), \quad (3.8)$$

where the real part of the strictly energetic portion, which gives a slight energy shift, has been neglected. Hence, the relative damping of the spin waves is given directly via Γ_{ij}:

$$\frac{\Gamma_k}{\varepsilon_k} = -\frac{\Gamma_{ff}(k,\varepsilon_k)}{\beta SV_0}. \quad (3.9)$$

Substituting the expansion (3.4) and (3.6) into the expressions for the amplitudes $A_{ij}^{(1)}(k, k_1, k_2)$, we find that

$$\frac{\Gamma_k}{\varepsilon_k} \approx \frac{3\pi}{8}\frac{1}{N^2}\sum_{k_1,k_2}\frac{S^2V_0{}^4\varepsilon^3}{\varepsilon_{k_1}\varepsilon_{k_2}\varepsilon_{k+k_1-k_2}}\{kk_1k_2(1-\cos\vartheta)+k_1{}^2k_2(1-\cos\theta)\}\Delta^{(1)}, \quad (3.10)$$

where ϑ and θ are the angles between the vectors k and k_2, and k_1 and k_2, respectively.

Considering this expression in the two cases, $\epsilon_k \ll T$ and $\epsilon_k \gg T$, we obtain, to within a numerical factor

$$\frac{\Gamma_k}{\varepsilon_k} \sim \frac{1}{S^2}(ak)^3\left(\frac{T}{SV_0}\right)^2 \ln\frac{\min(T, S|V_0|)}{\varepsilon_k}, \qquad \varepsilon_k \ll T,$$
$$\frac{\Gamma_k}{\varepsilon_k} \sim \frac{1}{S^2}(ak)^3\left(\frac{T}{S|V_0|}\right)^2, \qquad \varepsilon_k \gg T, \quad (3.11)$$

where a is the lattice constant.

For small momenta, where the expansion (3.5) is valid, similar calculations for the relative damping give

$$\frac{\Gamma_k}{\varepsilon_k} \sim \frac{1}{S^2}\frac{1}{\sqrt{\delta}}\left(\frac{T}{SV_0}\right)^2, \qquad \varepsilon_k \ll T,$$
$$\frac{\Gamma_k}{\varepsilon_k} \sim \frac{\sqrt{\delta}}{S^2}\exp\left(-\frac{S|V_0|\sqrt{\delta}}{T}\right), \qquad \varepsilon_k \gg T. \quad (3.12)$$

Near the transition point, the simplest process that gives a contribution to the damping is the scattering of spin waves by fluctuations in the momentum S^z.

This is sketched out graphically in Fig. 9.

FIG. 9.

In this diagram the dashed line corresponds to the correlator of the z components of the spin operators. In this temperature region it is more convenient to use, not the ladder approximation, but the general formulas given in[3]. Here we shall symbolize the irreducible parts by $R_{ij}(i\omega_n)$, since in this paper the symbol Σ has been used for the strictly energetic part. Introducing the notation

$$R_{ff} = b(y_f)G_f(1+A_{ff}), \quad R_{gg} = b(y_g)G_g(1+A_{gg}),$$

$$R_{fg} = \beta V_kb(y_f)G_fb(y_g)G_gA_{fg}, \quad R_{gf} = \beta V_kb(y_f)G_fb(y_g)G_gA_{gf}, \quad (3.13)$$

we obtain for the damping of the spin waves

$$\Gamma(k,\omega) = \frac{V_k{}^2b(y_f)b(y_g)}{2\varepsilon_k}\,\mathrm{Im}(A_{ff}+A_{gg}+A_{fg}+A_{gf}). \quad (3.14)$$

The contribution of the simplest diagrams has the form

$$\mathrm{Im}(A_{ff}+A_{gg}+A_{fg}+A_{gf})$$
$$= \pi\frac{1}{N}\sum_q \frac{b'(y_f)}{1-\beta^2V_{k-q}^2(1+\delta)^2(b'(y_f))^2}\frac{1}{2\varepsilon_q}\delta(\omega-\varepsilon_q)$$
$$\times\left\{\left[2V_q{}^2 + V_{k-q}^2(1+\delta)^2\left(\frac{\varepsilon_0+\varepsilon_q}{\varepsilon_0-\varepsilon_q}+\frac{\varepsilon_0-\varepsilon_q}{\varepsilon_0+\varepsilon_q}\right) - 4V_{k-q}V_q\frac{V_0}{V_k}(1+\delta)^2\right]\right.$$
$$\left.+\beta b'(y_f)\left[4V_{k-q}^2V_0(1+\delta)^3 - 2V_{k-q}^3\frac{V_q}{V_k}(1+\delta)^3 - 2V_q{}^2\frac{V_{k-q}}{V_k}(1+\delta)\right]\right\}. \quad (3.15)$$

Considering this expression again for small momenta and substituting ϵ_k in place of ω, we obtain for the relative damping

$$\frac{\Gamma_k}{\varepsilon_k} = v_0\frac{c[(1+\delta)^2-1]}{12a^2\varepsilon^2\pi}\frac{k}{|\tau|^2}, \qquad (1+\delta)^2-1 \gg \varepsilon^2k^2,$$
$$\frac{\Gamma_k}{\varepsilon_k} = v_0\frac{c}{48a^2\varepsilon^2\pi}\frac{k}{|\tau|^2}, \qquad (1+\delta)^2-1 \ll \varepsilon^2k^2, \quad (3.16)$$

where $a = (1/3)S(S+1)$, $c = (1/10)a(6a+1)$, $\tau = (T - T_N)T_N$, and v_0 is the volume of the elementary cell. The second expression is also good for the case of an isotropic antiferromagnet. From this it follows that near the Néel point only long spin waves can exist, i.e., if $k \ll |\tau|^2$. This condition is more stringent than in the case of ferromagnets, where $k^2 \ll |\tau|$, and therefore antiferromagnetic spin waves exist in a narrower interval of momentum for a given τ than in a ferromagnet.

4. FREE ENERGY AND THERMODYNAMIC QUANTITIES

The free energy of the system and the thermodynamic quantities will be investigated at low temperatures, where exponentially small terms of the type $\exp(-\beta\epsilon_0)$ can be neglected. The expression for the free energy in the molecular field approximation contains a new term, compared to the case of an isotropic antiferromagnet, which takes into account single-ion anisotropy:

$$-\beta F^{(0)} = -\beta \sum_{f,g} V(f-g)(1+\delta)\langle S_f^z\rangle_0 \langle S_g^z\rangle_0 - \beta K\Big[\sum_f \langle S_f^z\rangle_0^2 + \sum_g \langle S_g^z\rangle_0^2\Big] + \sum_f \ln\frac{\operatorname{sh}(S+1/2)y_f}{\operatorname{sh}(y_f/2)} + \sum_g \ln\frac{\operatorname{sh}(S+1/2)y_g}{\operatorname{sh}(y_g/2)}. \tag{4.1}$$

In the first approximation, at low temperatures, the anisotropic interaction does not contribute, and

$$\beta F^{(1)} = \sum_{k,\omega_n} \ln(1-\beta^2 V_k^2 b(y_f) b(y_g) G_f(i\omega_n) G_g(i\omega_n)). \tag{4.2}$$

In the second approximation, besides the diagram calculated for the isotropic case, in which, however, the interaction coupling the z components of the spin operators is multiplied by $(1+\delta)$, there appear two new diagrams (Fig. 10). Thus

$$\begin{aligned}-\beta F^{(2)} = {}& \frac{\beta}{N}\sum_{k_1,k_2} V_{k_1}^2 V_{k_2}^2 \frac{1+\delta}{V_0}\langle S_f^z\rangle_0 \langle S_g^z\rangle_0 \frac{1}{2\varepsilon_{k_1}}\operatorname{cth}\frac{\varepsilon_{k_1}}{2T}\frac{1}{2\varepsilon_{k_2}}\operatorname{cth}\frac{\varepsilon_{k_2}}{2T} \\ &+ 2\frac{\beta}{N}\sum_{k_1,k_2} V_{k_1}^2 \langle S_f^z\rangle_0 \frac{1}{2\varepsilon_{k_1}}\operatorname{cth}\frac{\varepsilon_{k_1}}{2T}\left(\frac{1}{2}\operatorname{cth}\frac{\varepsilon_0}{2T} - \frac{\varepsilon_0}{2\varepsilon_{k_2}}\operatorname{cth}\frac{\varepsilon_{k_2}}{2T}\right) \\ &- \frac{\beta}{N}\sum_{k_1,k_2} V_0(1+\delta)\left(\frac{1}{2}\operatorname{cth}\frac{\varepsilon_0}{2T} - \frac{\varepsilon_0}{2\varepsilon_{k_1}}\operatorname{cth}\frac{\varepsilon_{k_1}}{2T}\right)\cdot \left(\frac{1}{2}\operatorname{cth}\frac{\varepsilon_0}{2T} - \frac{\varepsilon_0}{2\varepsilon_{k_2}}\operatorname{cth}\frac{\varepsilon_{k_2}}{2T}\right) \\ &+ 2\frac{\beta}{N}\sum_{k_1,k_2} V_0\alpha \frac{\varepsilon_0}{2\varepsilon_{k_1}}\operatorname{cth}\frac{\varepsilon_{k_1}}{2T}\left(\frac{1}{2}\operatorname{cth}\frac{\varepsilon_0}{2T} - \frac{\varepsilon_0}{2\varepsilon_{k_2}}\operatorname{cth}\frac{\varepsilon_{k_2}}{2T}\right).\end{aligned} \tag{4.3}$$

The magnetization of the sublattice is determined from the free energy by taking the derivative with respect to y_f:

$$\langle S_f^z\rangle = S - \frac{1}{2N}\sum_q \left[\frac{1+\alpha+\delta}{\sqrt{(1+\alpha+\delta)^2-\gamma_q^2}} - 1\right] - \frac{1}{N^2}\sum_{k_1,k_2} S^3 V_0^4 \frac{1}{4}\frac{\gamma_{k_1}^2}{\varepsilon_{k_1}^3 \varepsilon_{k_2}}\{\alpha(1+\alpha+\delta) + [(1+\alpha+\delta)(1+\delta)-1]\gamma_{k_2}^2\}. \tag{4.4}$$

Comparing this expression with that obtained in the case of a isotropic antiferromagnet, we see that the first correction here is less, and therefore the deviation from the saturation moment in the anisotropic case is less; but the second correction increases this deviation.

The susceptibility can be obtained from the free energy or from the correlation function, and in the first approximation we obtain expressions similar to those for the case of the isotropic antiferromagnet:

$$\chi_\parallel = \frac{\mu^2}{N}\sum_q \frac{1}{2T}\left(1 - \operatorname{cth}^2\frac{\varepsilon_q}{2T}\right), \tag{4.5}$$

$$\chi_\perp = \mu^2 \frac{2\langle S^z\rangle}{\varepsilon_0 + S|V_0|(1+\delta)} = \frac{\mu^2}{|V_0|}\frac{\langle S^z\rangle}{S}\frac{1}{1+\delta+\alpha/2}. \tag{4.6}$$

FIG. 10.

In both cases the temperature dependence is exponential at very low temperatures and proportional to T^2 at higher temperatures. Equations (4.5) and (4.6) were obtained earlier by Kubo[7] and by Hewson and ter Haar.[8]

5. CONCLUSION

In this paper we have considered an antiferromagnetic dielectric with magnetic anisotropy of the "easy-axis" type. It was found that the temperature renormalization of the spin-wave energy contains a term proportional to T^2, but this term is smaller than the term proportional to T^4, so that it would be difficult to observe it experimentally. The damping of the spin waves was investigated at low temperatures and near the Néel point. At low temperatures the damping is due to scattering of the spin waves by each other, and for momenta where the dispersion curve is already linear ($\epsilon_k \sim k$), we obtained for the relative damping $\Gamma_k/\epsilon_k \sim k^3T^2$. Near the transition point the damping is associated in the first place with scattering of the spin waves by fluctuations of the magnetization, and the relative damping is proportional to $k/|\tau|^2$, where $\tau = (T - T_N)/T_N$. This result indicates that only long spin waves can exist near the transition point. Since in these calculations we investigated only the simplest processes, it would be interesting to consider how processes of higher order affect the spin wave damping. It would then be necessary to consider also other kinds of interaction, e.g., spin-phonon, dipole-dipole, scattering of spin waves by impurities.

The author thanks I. M, Khalatnikov for the hospitality extended to him during his residence in the Institute for Theoretical Physcis, and I. E. Dzyaloshinskiĭ for valuable discussions.

[1] T. Oguchi, Phys. Rev. 117, 117 (1960).
[2] K. H. Lee and S. H. Liu, Phys. Rev. 159, 390 (1967).
[3] É. M. Pikalev, M. A. Savchenko, and J. Sólyom, Zh. Eksp. Teor. Fiz. 55, 1404 (1968) [Sov. Phys. JETP 28, 734 (1969)].
[4] V. G. Vaks, A. I. Larkin, and S. A. Pikin, Zh. Eksp. Teor. Fiz. 53, 281 (1967) [Sov. Phys. JETP 26, 188 (1968)].
[5] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoi teorii polya v statisticheskoi fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965].
[6] H. B. Callen, Phys. Rev. 130, 890 (1963).
[7] R. Kubo, Phys. Rev. 87, 568 (1952).
[8] A. C. Hewson and D. ter Haar, Physica 30, 890 (1964).

Translated by L. M. Matarrese
263