CONCERNING THE THEORY OF THE ANOMALOUS HALL EFFECT IN PARAMAGNETIC METALS

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The anomalous Hall effect in paramagnetic transition metals is examined. Formulas are derived for the anomalous Hall field arising from the proper spin-orbit electron interaction during electron scattering by phonons in a magnetic field. The field is different for ferromagnetic metals above the Curie point and paramagnetic metals with a temperature independent or weakly dependent susceptibility (metals with Pauli paramagnetism). In the first case the anomalous field is proportional to the magnetic susceptibility which varies with the temperature in accordance with the Curie-Weiss law. In the second case it is approximately proportional to the square of the absolute temperature. Both theoretical conclusions are in agreement with the available experimental data. It is pointed out that the smooth variation of the anomalous Hall field constant R_a on passing through the ferromagnetism-antiferromagnetism transformation may serve as a criterion of the conservation of exchange splitting of the conduction bands in the paramagnetic state at temperatures near the Curie point.

In ferromagnets the Hall field E_h is, as is well known, sufficiently well described by a formula of the form

$$E_h = E_n + E_a = R_0 B + R_a \cdot 4\pi I, \qquad (1)$$

where B is the magnetic induction, $I \leq I_s$ is the magnetization, I_s is the saturation magnetization at a given temperature, R_0 is the constant for the normal Hall field and R_a the constant for the anomalous Hall field. The electric current density j is taken to be unity. The second term of the right-hand side of (1) is referred to as the anomalous Hall field. In ferromagnetic metals at temperatures above the Debye temperature T_D this term is usually larger than the first one by an order of magnitude. In the theory of galvanomagnetic phenomena the anomalous Hall field is explained as the result of the spin-orbit interaction of the electrons, the carriers of the magnetic moment. Consequently the anomalous Hall field can exist not only in ferromagnets. However, separation of the anomalous part from the observed field E_h in para- and antiferromagnetic substances presents certain difficulties, since at temperatures $T > T_D$, when one can expect the appearance of an appreciable anomalous Hall effect, the magnetization in practically attainable fields is still proportional to the field. In many paramagnetic transition metals the constant R_0 which characterizes the normal Hall field can depend on the temperature more strongly than the magnetic susceptibility[1)].

As has been shown by Kikoin[1] and by Kevane and Legvold,[2] the separation of the Hall field into the normal and anomalous part can be carried out in ferromagnetic metals above the Curie point where the magnetic susceptibility follows the Curie-Weiss law. In this case, assuming that the parameter R_0 has a considerably weaker temperature dependence than κR_a, one separates the temperature dependent part of E_h and considers it to be equal to the anomalous part of the Hall field.

The temperature dependence of the Hall field in nonferromagnetic metals has until recently practically not been studied and no theory of the anomalous part of this field existed. The author of this article, Kostina, Kozlova, and Sokolova studied the temperature dependence of the Hall field in chromium and vanadium. Volkov, Kozlova, Prudnikov, and Kozis have recently measured the Hall fields of zirconium and rhenium in a broad range of temperatures.[3]

In this article we consider the problem of the anomalous Hall field in paramagnetic transition metals theoretically and derive for this field an approximate formula which is in qualitative agreement with the obtained experimental data.

In calculating the anomalous Hall field for transition metals in the paramagnetic state we make the same assumptions as in calculations of the state of ferromagnetic ordering (see[4]). That is, we assume that the principal contribution to the anomalous Hall field of pure transition metals at a temperature $T > T_D$ is due to the intrinsic spin-orbit interaction of the conduction-band electrons in scattering by phonons and that the main part of this contribution is represented by terms which do not depend or depend weakly on the relaxation time. As in[4], we shall assume that the lifetime of an electron with given spin exceeds considerably its lifetime in a state with given quasimomentum vector k and band number n; because

[1)]We note that the proportionality between the anomalous Hall field and the average value of magnetization I observed in polycrystalline ferromagnets for $I < I_s$ is explained in the theory by means of purely geometrical considerations. However, analogous considerations are by far not applicable to all cases of paramagnets where the anomalous Hall field is by no means necessarily proportional to the magnetization or to the magnetic susceptibility.

of this one can separate in the system two weakly interacting subsystems of electrons with opposing spins. The formulas for the intensity E_a of the anomalous Hall field and the density j of the anomalous current in the case of the paramagnetic state under consideration are then derived from the corresponding formulas for the ferromagnetic transition metals. The latter, as shown by the author[4,5] and by Gurevich and Yassievich[6] are of the following form:

$$E_a = E_{ay} = -\rho j_{ay}, \tag{2}$$

where ρ is the electrical resistivity and j_{ay} is a parameter which is referred to as the anomalous Hall current density:

$$j_{ay} = ie^2E_x \sum_l \frac{\partial \rho_l}{\partial \varepsilon_l} v_x{}^l J_y{}^n + j_y{}^{(q)} \approx CE_x \sum_n M_n \int_{S_n} \left(q_l \frac{v_x k_x}{|\operatorname{grad} \varepsilon_l|} \right)_{\varepsilon_l = \varepsilon_F} dS_n, \tag{3}$$

$$C = \bar{\nu} e^2 / (2\pi)^3 \Delta^2 < 0. \tag{3'}$$

The summation here is over states with quasimomentum k of n conduction bands, the bands with spin "up" and "down" being enumerated separately. The integration is over the Fermi surfaces of the bands. The x axis is chosen along the current density vector $\mathbf{j}(j = 1)$ and the z axis—along the magnetization vector $\mathbf{I} \perp \mathbf{j}$. The notation in formulas (2) and (3) is the same as in[4], in particular M_n is the average z component of the magnetic moment of the electrons of the band n; at temperatures T sufficiently far from the point of the magnetic transformation $M_n \approx M_{n0} = \pm \mu_B$ (μ_B is the Bohr magneton); for T close to Θ it is sensible to assume that $M_n \approx M_{n0} I_S / I_0$ where I_S is the spontaneous magnetization, $I_0 = (I_S)_{T \to 0}$; q_l is a positive dimensionless parameter of the order of unity which depends weakly on k and n; $\bar{\nu}$ is a negative parameter whose magnitude depends on the average distribution of electric charge density around the ions of the crystal lattice; Δ^{-2} is the value of $(\epsilon_{nk} - \epsilon_{n'k})^{-2}$ averaged over the conduction bands and over the quasimomentum where $\epsilon_{nk} = \epsilon_l$ is the electron energy; iJ_y^n is a correction due to the spin-orbit interaction linear in the spin, to the diagonal matrix element of the coordinate y calculated in the Bloch function representation:

$$J_y{}^n = \frac{1}{m} \sum_{n'} \frac{\mathcal{H}^{so}_{nn'} p^{(y)}_{n'n} - p^{(y)}_{nn'} \mathcal{H}^{so}_{n'n}}{(\varepsilon_{nk} - \varepsilon_{n'k})^2}, \tag{4}$$

where $\hat{\mathcal{H}}^{so}$ is the periodic part of the operator of the intrinsic spin-orbit interaction, and $\hat{p}$ is the electron momentum operator. It follows from the work of Irkhin and Shavrov[7] for crystal lattices with a center of inversion that

$$J_y{}^n \approx -i \frac{\nu_n'}{\Delta^2} [\mathbf{k}\mathbf{M}_n]_y, \tag{5}*$$

where ν_n' averaged over the conduction bands is equal to the above-mentioned parameter $\bar{\nu}$. Expression (5) is obtained from formulas (31) and (34) of[7] in which, however, the average values of M_n of the magnetic moments of the electrons of different conduction bands are not replaced by the value of $\bar{M}_n$ averaged over all bands, as was done in[7]. The product of constant factors entering in the indicated formulas of[7] has been included in the parameter ν_n'.

In deriving the second equation of (3) the value of J_y^n entering into the sum over l and into the sums entering in $j_y^{(q)}$ is substituted from (5). In order to calculate J_y^n in accordance with (4) one would require more accurate data on the energy spectra and Fermi surfaces of ferromagnetic metals than those available at present. The use of data on the dispersion curve of these metals along several axes in reciprocal space known to date and interpolation would hardly result in higher accuracy in estimating J_y^n than the use of Eq. (5)[2]. Here, as in[4] it will be quite sufficient to estimate the orders of magnitude and signs of the anomalous fields and magnitudes on which these fields depend.

Below we shall consider the anomalous Hall field only in the two simplest cases of paramagnetism of transition metals, namely: 1) in the case of paramagnetism of ferromagnetic metals above the Curie point, and 2) in the case of paramagnetism of metals with a magnetic susceptibility which does not depend or depends weakly on the temperature (Pauli paramagnetism).

In the first case the effective temperatures corresponding to the energy of the exchange splitting of bands with spins "up" and "down" exceed considerably the temperatures at the Curie points. In fact, according to available estimates the magnitude of this splitting for nickel is of the order of 0.4—0.7 eV and for iron it is 1.4—1.8 eV.[10] In this case, in the paramagnetic as well as in the ferromagnetic state there should exist as a result of the remaining splitting of conduction bands quasilocalized magnetic moments of the band electrons, although the spontaneous magnetization above the Curie point is zero. The author of the present article has become aware of the fact that this was first pointed out by Slater.[11]

In the case when the exchange splitting of conduction bands is preserved in the paramagnetic state, Eqs. (2) and (3) for the anomalous Hall field derived for ferromagnetic transition metals remain valid, however now the values of M_n are

$$M_n \approx M_{n0} \varkappa H / I_0. \tag{6}$$

where $M_{n0} = \pm \mu_B$, $\varkappa$ is the magnetic susceptibility and

*$[\mathbf{k}\mathbf{M}_n]_y = [\mathbf{k} \times \mathbf{M}_n]_y$

[2]Leribaux[8] attempted recently to calculate J_y^n in accordance with (4) and the anomalous constant R_a for iron using dispersion curves for the principal axes obtained by Wood. Unfortunately, in calculating R_a Leribaux retained in the initial formula for the anomalous Hall current precisely only those terms which drop out entirely in the correct calculation (this was shown in a previous paper of Gurevich and Yassievich[6] which was apparently unknown to Leribaux). After carrying out the necessary calculations we have convinced ourselves that a correction of Leribaux's results does not change the sign or order of magnitude of R_a compared with the value cited in his article. We have also convinced ourselves that an estimate of R_a according to (3) and (5) of this article leads to the same sign and order of magnitude of R_a for iron as in the indicated calculation employing Eq. (4).

I_0 is the saturation magnetization close to absolute zero. Substituting (6) in (3) and taking into account (1) and (2), we obtain

$$E_h = (R_0 + R_a 4\pi\varkappa)H, \tag{7}$$

where

$$R_a = -\frac{j_{ay}\rho^2}{4\pi I E_x} \approx -C\frac{\rho^2}{4\pi I_0}\sum_n M_{n0}\int_{S_n}\left(q_l\frac{v_x k_x}{|\operatorname{grad}\varepsilon_l|}\right)_{\varepsilon=\varepsilon_F} dS_n \tag{8}$$

coincides with the expression for the anomalous Hall field constant in the ferromagnetic state. Thus, R_a changes smoothly in passing through the Curie point. This conclusion is in agreement with the results of experimental investigations for nickel and ferromagnetic alloys.[12] The smoothness of the variation of R_a in passing through the Curie point can serve as proof of the maintenance of exchange splitting of the conduction bands in the transition to the paramagnetic state.

Let us now consider the case when Pauli paramagnetism occurs in transition metals. In the ideal case there is no such exchange splitting of the conduction bands. When the external field H is switched on the Fermi level shifts by $\Delta\epsilon_F = \pm\mu_B H$ and this is accompanied by a change in the filling of the bands. Retaining the initial assumption that in this instance one can also divide the conduction electrons into two weakly interacting systems with spins "up" and "down," we can represent the density j_{ay} of the anomalous Hall current in the form of a sum $j_{ay\uparrow} + j_{ay\downarrow}$ and use formula (3) for calculating it. Then, substituting in (3) $M_{n\uparrow\downarrow} = \pm\mu_B$ and taking into account (2), we obtain

$$E_{ay} = a\rho E_x H, \tag{9}$$

$$a = -2C\mu_B^2\sum_n\frac{\partial}{\partial\varepsilon_F}\left[\int_{S_n}\left(q_l\frac{v_x k_x}{|\operatorname{grad}\varepsilon_l|}\right)_{\varepsilon_l=\varepsilon_F} dS_n\right]. \tag{9'}$$

Hence after replacing E_x by ρj we obtain for the Hall field the expression

$$E_h = R^* H,$$

where the effective Hall constant is

$$R^* = R_0 + a\rho^2. \tag{10}$$

In the range of temperatures $T > T_D$, $\rho \propto T$, and

$$R^* = R_0 + bT^2, \qquad b = a\rho^2/T^2. \tag{11}$$

It is seen from formulas (9) and (3) that a depends on the temperature approximately as the density of states on the Fermi surface, i.e., rather weakly. Thus, under the condition when R_0 also depends weakly on the temperature, the T dependence of R* is basically determined by the value of ρ^2 (as in the case of ferromagnetic transition metals!), i.e., for $T > T_D$ the magnitude of R* will be an approximately linear function of T^2. This conclusion agrees with the experimental results of Volkov, Kozlova, Prudnikov, and Kozis[3] who have recently found that the temperature dependence of the constants R* of zirconium and rhenium is well described by formulas similar to (10) and (11) with constant coefficients R_0, a, and b[3)].

In the special case in which the Fermi surfaces consist of spherical portions or portions close to spherical, the parameter a is proportional to the magnetic susceptibility. In fact, in the indicated special case the sum over n in (9′) is proportional to the density of states $N(\epsilon)$ on the Fermi surface. Then

$$E_{ay} \propto \rho^2 N(\varepsilon_F) \propto \rho^2\varkappa. \tag{12}$$

Since in the special case under consideration the curvature on the Fermi surfaces conserves its sign (positive on some surfaces and negative on others), according to (9′) the magnitude of a and the anomalous part of the Hall field E_{ay} turn out to be always positive[4)].

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[3)] It should be noted that according to data cited in [3] on increasing the absolute temperature from 90 to 300°K in zirconium R* increases by about a factor of 5. Under the condition that R_0 depends weakly on the temperature, the latter could be explained by a very large value of the parameter ν, i.e. by a large electron density near the nucleus. However, it is more probable that the observed temperature dependence of R* of zirconium should be ascribed to the temperature dependence of R_0. In this connection it would be essential to have data on the temperature dependence of R* of zirconium, rhenium, and other transition metals with Pauli paramagnetism in the region of sufficiently strong magnetic field (for $\omega_c\sigma \gg 1$).

[4)] In the special case under consideration both the electrons and the holes make a positive contribution to the anomalous Hall field. Here we have the same circumstances as in the case of ferromagnetic iron where a positive contribution to the anomalous Hall field was made by both types of majority current carriers – electrons with positive and holes with negative components M_n (see[4]).