# THE JOSEPHSON EFFECT IN SMALL TUNNEL CONTACTS

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It is shown that fluctuations result in an anomalous behavior of the Josephson current in tunnel contacts with small cross section. The effect of various types of fluctuations on the direct and alternating Josephson currents is considered. The results agree with available experimental data.

## 1. INTRODUCTION

In superconducting tunnel junctions having a small cross-sectional area of contact, the Josephson coupling energy $\epsilon$[1] may be commensurate with the energy of fluctuations. In this case the phase difference of the ordering parameter in the superconductors provides marked fluctuations, as a result of which[2] a finite voltage appears at the barrier for a given current even in that region of currents where a superconducting current would be observed in the absence of fluctuations. It is important that the tunnel current which flows thereby has the same nature as the ordinary Josephson current.[3] The dependence of current on voltage is nonlinear in the general case and is determined by the fluctuation mechanism. If $\epsilon \sim \Theta$ ($\Theta = kT$, where T is the transition temperature), and we have an ideal external circuit or a superconducting ring closed by the tunnel contact, the basic mechanism disturbing the stabilization of the coherent state of two superconductors with a fixed phase difference consists of thermodynamic fluctuations. When $\epsilon \gg \Theta$ quantum fluctuations of charge can be very important in tunnel contacts of small size. These can be treated by considering the Coulomb interaction of the electrons.

It is clear that the Coulomb interaction of the electrons inside each of the metals is taken into account in the model of Cohen, Falicov, and Phillips,[4] since it simply leads to a renormalization of the expression for the gap in the spectrum of elementary excitations of the superconductors. The interaction of electrons through the barrier is not accounted for in the model. It would seem that accounting for this would not lead to significant changes in the tunnel current because of strong screening of the electron interaction potential. However, in the tunneling process, the electrical neutrality of each metal individually may be virtually destroyed, and a surface charge will accumulate on the plates of the condenser formed by the tunnel contact. For broad contacts the effect of electron interaction through the barrier will be of little significance, since the Coulomb energy of the condenser for a given charge is inversely proportional to the area of the cross section.

Finally, the action of dissipative fluctuations introduced from the external circuit is very important. As Larkin and Ovchinnikov have shown,[5] these fluctuations lead to the appearance of a finite band of frequencies irradiated by the Josephson tunnel structure.

In this paper we shall investigate the effect of dissipative and quantum fluctuations. We neglect retardation effects[6,7] and the effect of the action of the quasi-particle current on the tunnel current of the Cooper pairs.

## 2. DISSIPATIVE FLUCTUATIONS

Consider a system consisting of a Josephson contact supplied from a source with emf E and internal resistance R. The contact is assumed small, so that the current is uniform over its section. Assuming that the main contribution is given by fluctuations introduced from the external circuit, we obtain this equation for the phase difference $\varphi$[6]:

$$C\ddot{\varphi}+\frac{\dot{\varphi}}{R}+\frac{2e}{\hbar}I_0\sin\varphi=\frac{2e}{\hbar}\frac{E+V(t)}{R}, \tag{1}$$

where C is the capacitance of the tunnel contact, $I_0$ is the Josephson current amplitude[3] ($\epsilon = \hbar I_0/2e$), V(t) is the stochastic emf arising as a result of thermal fluctuations in the external circuit. It is assumed that V(t) has the characteristics of white noise (see, for example,[8]).

We consider first the case when the capacitance of the tunnel contact may be neglected. Then the Fokker-Planck equation corresponding to Eq. (1) will have the form

$$\frac{\partial W}{\partial t}=D\frac{\partial^2 W}{\partial\varphi^2}+\Omega\cos\varphi W+(\Omega\sin\varphi-\Omega_0)\frac{\partial W}{\partial\varphi}. \tag{2}$$

Here $D = \Theta_0 R(2e/\hbar)^2$, $\Theta_0$ is the temperature of the external circuit, $\Omega = 2eI_0R/\hbar$, $\Omega_0 = 2eE/\hbar$, $W = W(\varphi, t|\varphi_0, t_0)$ is the conventional density of the probability of finding a phase difference in the interval $\varphi$, $\varphi + d\varphi$ at time t, if when $t = t_0$ the magnitude of $\varphi = \varphi_0$, i.e., $W(\varphi, t_0|\varphi_0, t_0) = \delta(\varphi - \varphi_0)$.

We introduce the quantities

$$x_n(t)=\int_{-\infty}^{\infty}d\varphi\, e^{in\varphi}W(\varphi,t|\varphi_0,0). \tag{3}$$

From (2) we obtain the following equation for $x_n$:

$$\frac{\partial x_n}{\partial t}=(-Dn^2+in\Omega_0)x_n(t)-\frac{\Omega n}{2}[x_{n+1}(t)-x_{n-1}(t)]. \tag{4}$$

As $t\to\infty$, $\partial x_n/\partial t = 0$, and for $x_n(\infty)$ we obtain an equation in finite differences, which is easily solved:

$$x_n(\infty)=\Theta(n)\frac{I_{n-iz_0}(z)}{I_{-iz_0}(z)}+\Theta(-n)\frac{I_{n+iz_0}(z)}{I_{iz_0}}. \tag{5}$$

Here $I_p(z)$ is a Bessel function with imaginary argument $z_0 = \Omega_0/D$, $z = \Omega/D$.

The dc Josephson current can be found from the relation

$$I = I_0 \lim_{t\to\infty} \int_{-\infty}^{\infty} d\varphi \sin\varphi W(\varphi, t|\varphi_0, 0) = I_0 \frac{x_1(\infty) - x_{-1}(\infty)}{2i}. \tag{6}$$

The relation (6) connects the dc current with the emf of the source. However, experimentally one measures the dependence of current on the contact voltage. The dependence of voltage on the source emf is obtained by averaging Eq. (1):

$$V = \frac{\hbar}{2e}\frac{d}{dt}\langle\varphi\rangle = \frac{\hbar}{2e}\left(\Omega_0 - \Omega\frac{x_1(\infty) - x_{-1}(\infty)}{2i}\right). \tag{7}$$

If we eliminate the parameter $z_0$ from (6) and (7), we obtain the dependence of current on voltage, which is given in Fig. 1 for several values of $z$ ($V_1 = I_0R$). In the limiting cases of large and small $z$ it is possible to obtain simple expressions.

For $z \to \infty$

$$I = \begin{cases} E/R; & 0 \leqslant E \leqslant I_0R, \quad V = 0 \\ \dfrac{V}{R}\left(\sqrt{1+\left(\dfrac{I_0R}{V}\right)^2} - 1\right); & V \neq 0 \end{cases} \tag{8}$$

For $z \ll 1$

$$I = \frac{I_0 z}{2}\frac{2eVD/\hbar}{(2eV/\hbar)^2 + D^2}. \tag{9}$$

We now consider the spectrum of the Josephson radiation $R(\omega)$:

$$R(\omega) = \frac{2}{\pi} I_0^2 \operatorname{Re}\int_0^{\infty} dt\, e^{i\omega t} R(t), \tag{10}$$

where $R(t)$ is the correlation function[8]:

$$R(t) = \lim_{t_1 - t_0\to\infty}\int_{-\infty}^{\infty} d\varphi \int_{-\infty}^{\infty} d\varphi_1 \sin\varphi\sin\varphi_1 W(\varphi, t+t_1|\varphi_1, t_1) W(\varphi_1, t_1|\varphi_0, t_0). \tag{11}$$

For calculating $R(\omega)$ it is convenient to introduce the quantities

$$R_{mn}(\omega) = \lim_{t_1 - t_0\to\infty}\int_0^{\infty} dt\, e^{i\omega t}\int_{-\infty}^{\infty} d\varphi \times \int_{-\infty}^{\infty} d\varphi_1\, e^{i(m\varphi + n\varphi_1)} W(\varphi, t+t_1|\varphi_1, t_1) W(\varphi_1, t_1|\varphi_0, t_0). \tag{12}$$

Considering Eqs. (25) and (28), we obtain for $R_{mn}(\omega)$,

$$R_{mn}(\omega) = \frac{1}{m^2 - i(y + mz_0)}\left[\frac{x_{m+n}(\infty)}{D} - \frac{mz}{2}\left(R_{m+1,n}(\omega) - R_{m-1,n}(\omega)\right)\right]. \tag{13}$$

In the general case the solution of the recurrent relation (13) can be obtained only in the form of a series in powers of $z$. If the Josephson coupling energy is much smaller than the energy of the fluctuation ($z \ll 1$), then we get from (13), to terms of second order in $z$, using the definitions (10) and (11) for $R(\omega)$

$$R(\omega) = \frac{I_0^2}{2\pi D}\left\{\frac{1}{1+(y+z_0)^2} + \frac{1}{1+(y-z_0)^2} + \pi D\delta(\omega)\left(\frac{zz_0}{1+z_0^2}\right)^2 + \frac{z^2}{2}[f(z_0) + f(-z_0)]\right\}. \tag{14}$$

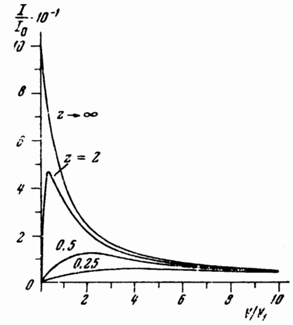


FIG. 1.

Here

$$f(z_0) = \frac{2 - 4z_0^2 - 3z_0y}{(1+z_0^2)(4+z_0^2)[1+(y+z_0)^2]} + \frac{2z_0(y+2z_0)}{(1+z_0^2)^2[1+(y+z_0)^2]} + \frac{1}{[1+(y+z_0)^2][16+(y+2z_0)^2]}\left\{\frac{4 - 4z_0(y+z_0) - (y+2z_0)^2}{1+z_0^2} + 4\frac{2[1-(y+z_0)^2] - (y+z_0)(y+2z_0)}{1+(y+z_0)^2}\right\}. \tag{15}$$

We note that in the general case $R(\omega)$ differs from the expression obtained by Larkin and Ovchinnikov[5] and becomes equal to it only in the two limiting cases $\Omega_0 \gg \Omega$ and $z \ll 1$. This is because the system is nonlinear, and although the input probability process is white noise, the distribution for $W(\varphi, t|\varphi_0, t_0)$ will be normal only in these limiting cases. In Eq. (14) the correction of order $z^2$ to the relation obtained in[5] leads to the appearance of a resonance at $\omega = 2\Omega_0$, to a slight shift of the resonance frequencies relative to $\Omega_0$ and $2\Omega_0$, and to asymmetry of the spectrum near these frequencies. We remark that resonances appear in the vicinity of $n\Omega_0$ if one includes terms of higher order in $z$.

We now return to the general equation (1) and see how these results change when the capacitance of the tunnel contact is taken into account. Since Eq. (1) contains the second derivative with respect to time, then in accordance with Dub's theorem[8] the probability process will not be unidimensional, as when $C = 0$, but two-dimensional. The Fokker-Planck equation for this case has the form

$$\frac{\partial W}{\partial t} = \frac{D}{\tau^2}\frac{\partial^2 W}{\partial q^2} + \frac{\partial}{\partial q}\frac{q}{\tau}W + \frac{1}{\tau}(\Omega\sin\varphi - \Omega_0)\frac{\partial W}{\partial q} - q\frac{\partial W}{\partial\varphi}. \tag{16}$$

Here $q = \dot{\varphi}$, $\tau = RC$, $W \equiv W(\varphi, q, t|\varphi_0, q_0, t_0)$. When $t = t_0$

$$W(\varphi, q, t_0|\varphi_0, q_0, t_0) = \delta(q - q_0)\delta(\varphi - \varphi_0).$$

We introduce the quantities

$$x_n(k, t) = \int_{-\infty}^{\infty} d\varphi \int_{-\infty}^{\infty} dq\, e^{in\varphi} e^{-ikq} W(\varphi, q, t|0, 0, 0). \tag{17}$$

Using Eq. (16), we obtain for $x_n(k, t)$

$$x_n(k, t) = x_n^0(k, t) + \frac{\Omega}{2\tau}\int_0^t dt_1 \times \int_{-\infty}^{\infty} dk_1\, k_1 G_n(k|k_1; t - t_1)[x_{n+1}(k_1, t_1) - x_{n-1}(k_1, t_1)]. \tag{18}$$

where

$$x_n^0(k,t)=\exp\Big\{-\Big[n^2D\tau\Big[\Big(\frac{t}{\tau}-2\,\mathrm{th}\,\frac{t}{2\tau}\Big)+\frac{1}{2}(1-e^{-2t/\tau})\Big(\frac{k}{n\tau}-\mathrm{th}\,\frac{t}{2\tau}\Big)^2\Big]-i\Omega_0[nt-(k+n\tau)(1-e^{-t/\tau})]\Big]\Big\}$$

$$G_n(k|k_1;t)=\delta(ke^{-t/\tau}-k_1-n\tau(1-e^{-t/\tau}))\exp\Big\{-\Big[(Dn^2-i\Omega_0)t+\frac{D}{2\tau}(k+\tau n)^2(1-e^{-2t/\tau})+(i\Omega_0-2nD)(k+\tau n)(1-e^{-t/\tau})\Big]\Big\}$$

The dc Josephson current is connected with $x_n(k, t)$ by a relation like (6)

$$I=I_0\frac{x_1(0,\infty)-x_{-1}(0,\infty)}{2i}\,. \qquad (19)$$

Just as in deriving (7), we can find the connection between the contact voltage and the source emf. However, since we cannot obtain the exact solution of Eq. (18) and must solve it by iterations accurate to terms of first order in z, it is unnecessary to write out the relation between voltage and emf, since they coincide to this accuracy. Solving Eq. (18) with the accuracy stipulated above, and then substituting this solution for $x_1$ into (19), we obtain an expression for the dc current in series form

$$I=I_0\frac{zz_0}{2}e^{1/z_1}\sum_{n=0}^{\infty}\frac{(-1)^n}{n!\,z_1^n}\Big\{\frac{1}{(1+nz_1)^2+z_0^2}-\frac{1}{(1+nz_1+z_1)^2+z_0^2}\Big\}, \qquad (20)$$

where $z_1 = (D\tau)^{-1}$.

As can be seen from this relation, the dc current is reduced in comparison to the current when C = 0 (see Eq. (9)). Otherwise the dependence of the current on the contact voltage is similar to that for C = 0. The power spectrum of the Josephson radiation, unlike the dc current, can be significantly different if $C \neq 0$. This can be clearly seen even in the limit when the nonlinear term in Eq. (1) may be neglected, i.e., when $\Omega \ll \Omega_0$. In this case the probability process becomes normal and the calculation can be carried to its end. The solution to Eq. (16) will be

$$W(\varphi,q,t|\varphi_0,q_0,0)=\frac{1}{2\pi\sigma(t)\xi(t)}\exp\Big\{-\Big[\frac{(q-\hat{q}(t))^2}{2\sigma^2(t)}+\frac{(\varphi-\varphi_0+\psi(t))^2}{2\xi^2(t)}\Big]\Big\} \qquad (21)$$

Here

$$q(t)=q_0e^{-t/\tau}+\Omega_0(1-e^{-t/\tau}),\qquad \xi^2(t)=2D(t-2\tau\,\mathrm{th}\,(t/2\tau)),$$
$$\sigma^2(t)=\frac{D}{\tau}(1-e^{-2t/\tau}),\qquad \psi(t)=\tau(q-q_0)-\Omega_0t-\frac{2\tau}{1+e^{-t/\tau}}(q-\hat{q}(t)).$$

In this approximation there is no dc component of current. The power spectrum is calculated from formulas like (10) and (11). The result for $R(\omega)$ agrees with the expression obtained by Larkin and Ovchinnikov.[5] When $\Omega \gtrsim \Omega_0$ a significant change in the form of the spectrum is possible. Unfortunately, this case does not yield to analysis.

## 3. QUANTUM FLUCTUATIONS OF CHARGE

We now consider the case when a sufficiently small contact is supplied from an ideal generator of current I at $\Theta = 0$. The classical Hamiltonian of such a system, according to Anderson,[9] is

$$H=\frac{Q^2}{2C}-\frac{\hbar}{2e}(I_0\cos\varphi+I\varphi). \qquad (22)$$

Here Q is the charge on the tunnel contact condenser, which is connected with the phase difference by the relation $(C\hbar/2e)\varphi = Q$.

From (22) it is seen that there are metastable states in the system, and the phase difference $\varphi_n$ in these is equal to $\arcsin x + 2\pi n$ ($x = I/I_0$, n is an integer). At finite temperatures the system can make a transition from one metastable state to another under the influence of thermodynamic fluctuations, with a probability $\sim \exp(-\Delta E/\Theta)$, where $\Delta E$ is the height of the barrier separating these states. Transitions of this type, as shown in[2], lead to the appearance of a voltage across the contact. If $\Theta = 0$, then in the classical case the system will be found in some state with a fixed phase difference and, consequently, with V = 0. In the quantum-mechanical case the system can tunnel into a neighboring, quasi-stationary state. The quantum analog of Hamiltonian (22) (see[10]) can be written

$$H=-\frac{2e^2}{C}\frac{\partial^2}{\partial\varphi^2}-\frac{\hbar}{2e}(I_0\cos\varphi+I\varphi). \qquad (23)$$

If the electrostatic energy $e^2/C$ is much smaller than the coupling energy, the system can be in quasi-stationary states in the vicinity of minima in the potential energy $U(\varphi)$ for a rather long time. The wave function $\psi$ for these states can be found in the quasi-classical approximation. Since the wave functions of adjacent states overlap, the system will make stepwise transitions from one state to another with an increase in the number n. For the probability of a transition during an oscillation period to an adjacent quasi-stationary state we have the expression[1]

$$P\sim\exp-2\beta\int_{\varphi_1}^{\varphi_2}d\varphi\Big\{\frac{2}{\varepsilon}[U(\varphi)-U(\varphi_1)]\Big\}^{1/2}, \qquad (24)$$

where $\varphi_1 = \arcsin x$, $\beta = (\epsilon C/4e^2)^{1/2}$, and $\varphi_2$ is the closest turning point to $\varphi_1$.

The difference between the processes leading to the appearance of a voltage drop in the quantum case for $\Theta = 0$ and the analogous (see[2]) classical processes for $\Theta \neq 0$ should be noted. In the classical case the system diminishes its energy by an amount $(\pi\hbar/e)I$ in making a transition to a neighboring metastable state. This energy is given up to the thermostat. In the quantum case tunneling occurs with conservation of energy. After tunneling the system would continue to move quickly in the direction of increasing $\varphi$. The latter, however, does not occur on account of the strong interaction with the electromagnetic field, which also withdraws an energy $(\pi\hbar/e)I$. Of course, such consideration of the processes is impossible to accept as a rigorous one, since there is assumed to be a practically instantaneous transfer of energy $(\pi\hbar/e)I$ to the field. Nevertheless, with this simplification of the problem one gets the correct value of the least possible voltage that appears at the tunnel contact, given $I < I_0$.

Considering these remarks and using (24), it is not difficult to calculate the dependence of current on voltage by the method set forth in[2]. Figure 2 gives the results of a numerical calculation of the dependence of x on $V/V_0$ ($V_0 = (1/20)\alpha(\hbar I_0/2eC)^{1/2}$, $\alpha \sim 1$) for various values of $\beta$. For large $\beta$ the dc Josephson current can run from zero to a certain value without the appearance

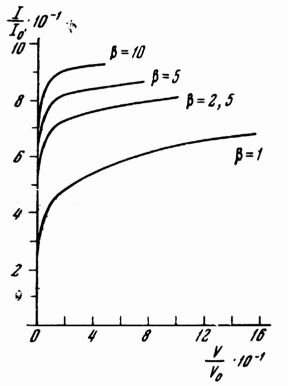


FIG. 2.

of a noticeable voltage drop at the junction. Further increase of current brings about an appreciable voltage drop. We emphasize that this treatment and the results shown in Fig. 2 are more qualitative than quantitative in character.

We now consider processes occurring in the tunnel contact ($\circledast = 0$), the macroscopic state of which was prepared in the following way: at a certain moment of time the contact was connected to a generator of voltage V, and then disconnected at t = 0. In this case the system can be described by the Hamiltonian (23) with I = 0; however, the wave function will be a superposition of eigenfunctions of the operator H due to the fact that at t = 0 we have given a voltage $V = e\langle Q\rangle/C$. The coefficients of the expansion of $\psi$ in the eigenfunctions are determined from the energy minimum of the system with the condition that $\langle Q\rangle = m$, where m is an integer. Solving the variational problem in the usual way, we obtain the equation

$$\left(-\frac{2e^2}{C}\frac{\partial^2}{\partial\varphi^2}+i\mu\frac{\partial}{\partial\varphi}-\varepsilon\cos\varphi\right)\psi(\varphi)=E\psi(\varphi). \qquad (25)$$

Here E and $\mu$ are the Lagrangian multipliers determined from the normalization condition for $\psi(\varphi)$ and the requirement $\langle Q\rangle = m$.

In the approximation $\beta \ll 1$, we find for $\psi(\varphi)$

$$\psi(\varphi)=\left(\frac{\beta}{\pi}\right)^{1/4}e^{im\varphi/2}\sum_k\exp\left[-\frac{\beta(\varphi+2\pi k)^2}{2}\right]. \qquad (26)$$

The average value of the current operator is found in the usual way:

$$I(t)=I_0\int_{-\pi}^{\pi}d\varphi\psi^*(\varphi)\,e^{iHt/\hbar}\sin\varphi e^{-iHt/\hbar}\psi(\varphi). \qquad (27)$$

The result of a calculation of (27) has a satisfactorily simple and clear form only for voltages $\beta e^2/C \ll eV$. This lower limit to the voltage means that the barrier voltage must exceed the quantum-mechanical fluctuations arising during electron tunneling.

There is also an upper voltage limit, $eV < 2\Delta$, due to the fact that when $eV > 2\Delta$ quasi-particle current will be excited.

Neglecting terms of order $\epsilon/CV^2$, we obtain from (27), after elementary transformations,[12]

$$I(t)=\frac{I_0}{\sqrt{\pi\beta}}\sum_k\sin[\Omega_0+(2k+1)\omega_0]t\exp\left[-\frac{1}{\beta}\left(k-\frac{1}{4}\right)^2\right], \qquad (28)$$

where $\omega_0 = 4e^2/C\hbar$.

In spite of the fact that our analysis was made for a disconnected external circuit, the result is applicable also to the case when a generator having sufficiently high resistance is connected when $\circledast = 0$. In this case it is possible to make use of the Hamiltonian (22) with I = 0, if $\beta e/C \ll V$.

## 4. CONCLUSION

Thus, in small superconducting tunnel contacts significant deviations from the ordinary behavior of the Josephson current are possible. Evidently, the effect of the dissipative fluctuations is the most important in an experiment. This is because one usually investigates tunnel contacts with an $\epsilon$ so large that quantum fluctuations may be neglected.

The first experimental evidence of the effect of fluctuations was discovered by Shigi et al.[13] The form of the current-voltage curve they obtained is similar to the curve for $z \sim 2$ in Fig. 1. Quantitative comparison with the results of[13] is impossible because of the lack of detailed information about this experiment. Besides, it is hopeless to look for a good quantitative agreement with the results of[13], since the experiment was performed on a sample with dimensions that exceeded the Josephson penetration depth, whereas our calculation was for the opposite case. We mention one further experiment in which a very significant effect due to dissipative fluctuations was observed, the experiment of Vant-Hull and Mercereau,[14] in which it was shown that even in the absence of a superconducting current through the contact, the ac Josephson current can lead to the appearance of resonance steps associated with the radiation of high-frequency power. The dependence of current on voltage, after elimination of the quasi-particle current and the spikes from the resonance steps, is like the curve for $z \sim 0.5$ in Fig. 1. This dependence oscillates with changing magnetic field just as the superconducting Josephson current does.

It would be of interest to investigate experimentally the spectrum of Josephson radiation for the systems described in Sec. 3. If the apparatus has insufficient resolving power, one will observe a line of Gaussian shape centered at $\omega = \Omega_0$ and having a width

$$\Delta\omega=\frac{4e^2}{C\hbar}\left(\frac{\hbar I_0 C}{2e^3}\right)^{1/4}. \qquad (29)$$

This spectrum will be made up of individual lines separated by $\omega_0$ and run together by the dissipative fluctuations. The measurement of $\omega_0$ is evidently most easily accomplished by measuring the rf radiation of a contact that is supplied from a pulse generator. When the repetition rate of the pulses coincides with $\omega_0$, a sharp spike in the intensity of the radiation will occur.

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