EFFECT OF SURFACE ON THE MAGNETIC RESISTANCE OF BISMUTH

Yu. A. BOGOD, V. V. EREMENKO, and L. K. CHUBOVA

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

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The transverse magnetic resistance of bismuth single crystals is investigated in magnetic fields up to 20 kOe and at helium and nitrogen temperatures. It is found that in the indicated field strength range and at 4.2°K the resistance in a magnetic field increases more slowly than predicted by the quadratic law. This phenomenon is ascribed to the specular scattering of the carriers by the surface of the semimetal and to the current being forced out towards the surface of the sample. The diffusivity constant q_1 calculated on the basis of the theory of resistance of semimetals is of the order of 10^{-4}. A number of features of the transverse magnetic resistance of Bi, observed in the present investigation, such as the effect of the angle of inclination of the magnetic field to the surface of a single crystal plate on the magnetic resistance, and the linear dependence of magnetic resistance on field strength in cylindrical samples with rough surfaces, are also ascribed to the specular nature of the carrier scattering by the Bi surface.

INTRODUCTION

IN 1951, Borovik and Lazarev[1] observed a shape effect—the influence of the orientation of the magnetic field relative to the surface of thin Bi plates on the magnetoresistance. Since that time, much progress was made in the purification of Bi, so that the resistance singularities connected with the boundaries can appear also in relatively bulky samples. However, in spite of the fact that the magnetoresistance of pure single-crystal Bi has been investigated in sufficient detail[2], no notice was taken of the influence of the surface boundaries on the magnetoresistance. These circumstances, as well as the development of a theory of the resistance of semimetals in a magnetic field with allowance for effects connected with carrier scattering by the surface[3,4], have induced us to return to an investigation of the magnetoresistance of Bi.

EXPERIMENTAL PROCEDURE, SAMPLES

The measurements were made in stationary magnetic fields up to 20 kOe, using a commercial electromagnet (SP-54) in conjunction with a stabilizer and a P-62 dc generator. The sensitivity of the potentiometer circuit was 5×10^{-8} V. The investigated Bi samples were cut by the electric-spark method from a single-crystal ingot 99.9999% pure. The measurements were made on samples of two types: plates whose faces were perpendicular to the principal crystallographic axes, and cylinders.

As is well known, one of the criteria of metal purity is the ratio of its electric resistances at 300 and 4.2°K. Naturally, however, the ratio no longer characterizes the purity of the metal as soon as the carrier mean free path l at helium temperature becomes of the same order of magnitude as the characteristic sample dimension d. Such a phenomenon takes place both in diffuse and specular scattering of the carriers by the sample surface. In the latter case, as shown by Price[5], the Fermi surface of carriers should differ from a sphere. The region of values of d in which the condition $l \sim d$ is satisfied can be obtained by measuring the dependence of the residual resistance of the sample on its thickness. This indeed was done in the present investigation.

The experiments were performed on plates whose thickness was varied by electric polishing. The sample holder was a device consisting of a base with a slot in the form of a "swallow tail" and a slide, to the plane a of which the end face of the sample was secured with Wood's alloy (Fig. 1). The indicated device served

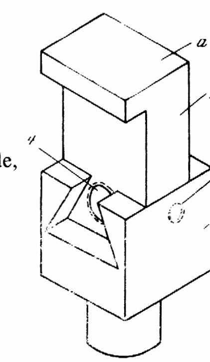

FIG. 1. Diagram of holder: 1 – face, 2 – slide, 3 and 4 – openings for fixing the position of the slide in the base and of the guide.

simultaneously as the current lead. The second current lead and the two potential leads were copper wires soldered with Wood's alloy to the end (bulky contact) and to the plane of the sample. The distance between the potential contacts was usually 4–6 mm. The diameters of the current and potential leads were 0.15 and 0.05 mm, respectively.

The sample thickness was varied by electric polishing without potential contacts between the plane and the mounting stage, making it possible to set the plane of the crystal parallel to the surface of the electrolyte. The electrolyte was a 4 : 1 : 1 mixture of glycerine, concentrated nitric acid, and glacial acetic acid[6]. During the time of the polishing, the slide with the sample were placed in a special guide, also constructed in the form of a "swallow tail."

The construction of the holder and method of polishing made it possible to carry out measurements on samples of different thicknesses without remounting them. For this reason, the defects in the crystal could arise and heal mainly as a result of the temperature drop on cooling to 77 and 4.2°K followed by heating to 300°K. In the present experiment, the thinnest samples ($d \geq 0.3$ mm) still remain sufficiently bulky. Therefore the concentration of the defects resulting from the temperature drop can be regarded to be the same at different thicknesses.

Figure 2 shows the dependence of the ratio $\rho(300°K)/\rho(4.2°K)$ on the thickness of the Bi samples

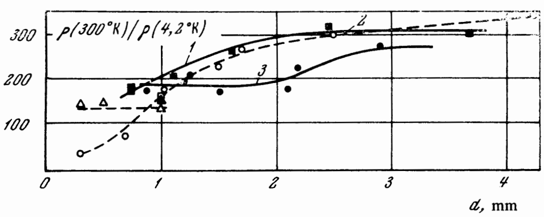

FIG. 2. Dependence of the residual resistance of single - crystal Bi samples on the thickness: ■ – Bi - Ia (curve 1), □ – Bi - Ib, ▲ – Bi - Ic, ● – Bi - IIa (curve 3), △ – Bi - IIb, ○ – Bi - III (Curve 2).

(the initial dimensions of the samples and their orientation are listed in the table; C_1, C_2, and C_3 corre-

Sample	Form	Initial dimensions, *mm	Direction of longitudinal axis	Direction of polishing
Bi-Ia	Plate	$12{,}3(C_3)\times 8{,}0(C_2)\times 3{,}7(C_1)$	C_3	C_1
Bi-Ib		$10(C_3)\times 1{,}5(C_2)\times 1{,}0(C_1)$	C_3	—
Bi-Ic	»	$10(C_3)\times 1{,}4(C_2)\times 1{,}0(C_1)$	C_3	—
Bi-Id	Cylinder	⌀4	C_3	—
Bi-IIa		$9{,}54(C_2)\times 3{,}68(C_3)\times 3{,}03(C_1)$	C_2	C_2
Bi-IIb	»	$15(C_2)\times 6{,}5(C_1)\times 1{,}0(C_3)$	C_2	C_3
Bi-III	»	$13(C_1)\times 4{,}45(C_3)\times 4{,}25(C_2)$	C_1	C_2

*The parentheses contain the indicated directions of the crystallographic axes.

spond to the directions of the bisector, binary, and trigonal crystallographic axis). We see that the resistance of sample Bi-Ia at helium temperature remains practically unchanged when its thickness is decreased to 2 mm, and then, with further decrease of thickness, it begins to increase monotonically. The residual resistance of the samples Bi-IIa has two sections, which hardly vary with changing thickness. They correspond to $d > 2.5$ mm and $d < 2$ mm (see also the dashed line in Fig. 2). The thickness variation of the residual resistance of the crystal Bi-III is qualitatively similar to that of the sample Bi-Ia.

Thus, for the Bi used in the present investigation, the ratio of the resistances at room temperature and at helium temperature characterizes the purity and the physical perfection of the samples only at thicknesses not smaller than 2 mm. The mean free path of the carriers in the samples at 4.2°K is in our case on the order of 10^{-1} cm. At 77°K, the mean free path is close to 10^{-3} cm ($\rho(300°K)/\rho(77°K) \approx 5$).

MEASUREMENT RESULTS

1. Dependence of the Resistance of the Magnetic Field

Figure 3 shows in logarithmic coordinates the dependence of the resistance on the magnetic field for several samples of Bi at 77°K. We see that in fields

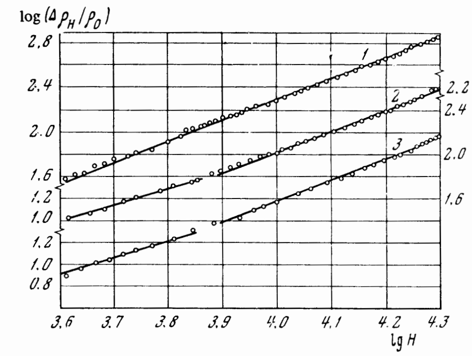

FIG. 3. Dependence of the resistance of single-crystal Bi samples on the magnetic field at 77°K: 1 – Bi-Id, H ∥ C_2, after polishing; 2 – Bi-IIa, H ∥ C_1, corss section 3.68 × 2.1 mm; 3 – Bi-IIb, H ∥ C_1, cross section 655 × 1 mm.

exceeding 6 kOe ($\log H = 3.8$) the resistance is proportional to the square of the field at all orientations of thicknesses, and that the law governing the increase of the resistance does not change on going from sample to sample. The weaker change of $\Delta\rho_H/\rho_0$ in fields up to 6 kOe is due the realization of different sizes of cyclotron orbits in the same direction of **H**, or else to their elongation. The absence of a kink on the curve corresponding to sample Bi-Id is due to the larger mean free path at 77°K, compared with the remaining crystals[7]. The error in the determination of the exponent in the dependence of $\Delta\rho_H/\rho_0$ on H reaches ±0.1.

When T = 4.2°K, the resistance Bi oscillates in the magnetic field[8]. Therefore separation of the monotonic part of the $\Delta\rho_H/\rho_0$ dependence on H, entails certain difficulties. The problem is solved most simply when the magnetic field is parallel to the C_3 axis. Owing to the large values of the cyclotron masses realized in this direction, the amplitude of the oscillations is negligible, so that in logarithmic coordinates the dependence of the resistance of the magnetic field is simply a straight line. The slope of the line does not depend on the transverse dimensions of the sample and on the direction of the electric field, remains constant from sample to sample, and equals 1.65 ± 0.05 (Fig. 4).

When the direction of the magnetic field coincides with the binary axis of the crystal, the following extremal cyclotron masses are realized: $m_1^* = 1.120\ m_0$, $m_2^* = m_3^* = 0.0093\ m_0$ (electrons), $M^* = 0.203 m_0$ (holes)[9] (m_0—mass of free electron). In fields up to 20 kOe, the contribution to the oscillating part of the magneto-resistance is made by practically one harmonic corresponding to $m_2^* = m_3^*$. Consequently, the averaging can be carried out, as shown in Fig. 5. In the coordinates $\log(\Delta\rho_H/\rho_0)$ and log H, the distance between the lines joining the geometric loci of the points of the maxima and minima (see Fig. 5, curve 3a) is divided in equal halves (the distance is measured perpendicular to the abscissa axis). The straight line 3b, which joins these points, is the monotonic part of the dependence of $\Delta\rho_H/\rho_0$ on H in logarithmic coordinates. We see that on going from sample to sample, and also in measurements made on crystals with different orientations and transverse dimensions, all the singularities of the curves are reproduced exactly, and

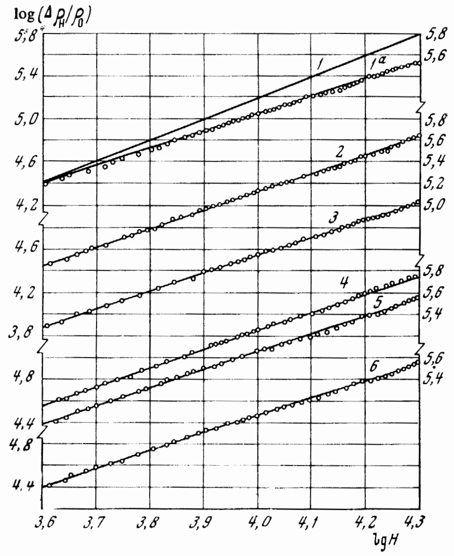

FIG. 4. Dependence of the resistance on the magnetic field at **H** ∥ C_3 (C = 4.2°K). Curves 1a, 2, 3 – Bi-IIa, respective cross sections 3.68 × 2.95, 3.68 × 2.1, and 3.68 × 0.9 mm; curve 4 – Bi-III, cross section 4.45 × 1.7 mm; curves 5, 6, – Bi-IIb, cross sections 6.5 × 1 and 6.5 × 0.3 mm; line 1 is constructed for the case when $\partial \log(\Delta\rho_H/\rho_0)/\partial \log \mathbf{H} = 2$.

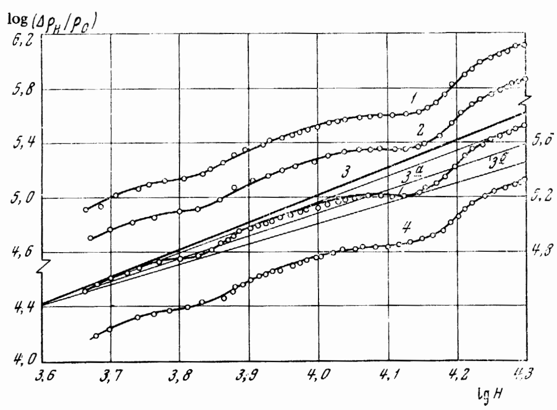

FIG. 5. Dependence of the resistance on the magnetic field at **H** ∥ C_2 (T = 4.2°K): curves 1 and 3a – Bi-Ia, cross sections 8 × 3.7 and 8 × 0.75 mm; curves 2 – Bi-Ib, 4 – Bi-III, cross section 4.45 × 1 mm. The line 3 corresponds to a quadratic dependence of the magnetoresistance..

the exponent of the monotonic part of the magnetoresistance is the same, within the limits of experimental error, for all the curves, and is equal to 1.65 ± 0.05.

When H ∥ C_1, an appreciable contribution to the oscillating part of the magnetoresistance in fields up to 20 kOe is made by two harmonics corresponding to cyclotron masses $0.0081m_0$ and $0.016m_0$[9]; consequently, the method indicated above for separating the monotonic part is not suitable. Neglecting the difference between the orbital and spin splittings, and also the shift of the chemical potential in the magnetic field[10], the dependence of $\Delta\rho_H/\rho_0$ on H can be written in the form

$$\frac{\Delta\rho_H}{\rho_0} = A_x H^{n_x} + H^{1/2}\left[B_x \sin\left(\frac{2\pi c S_1}{ehH} + \delta_{1x}\right) + C_x \sin\left(\frac{2\pi c S_2}{ehH} + \delta_{2x}\right)\right] + \dots \qquad (1)$$

Here S_i are the extremal sections of the Fermi surface, corresponding to the masses indicated above, and δ_{ix} are the phases of the oscillations. In accord with Adams and Holstein[11], it is assumed that the amplitude of the oscillations is proportional to the square root of the magnetic field, if the monotonic part of the resistance exceeds the oscillating part. Further, using the experimental values of $\Delta\rho_H/\rho_0$ and formula (1), we can determine the unknown coefficients, the phases and the exponent n_x, and thus separating the monotonic part of the magnetoresistance. Such a procedure was used for curve 7 of Fig. 6 in conjunction with an

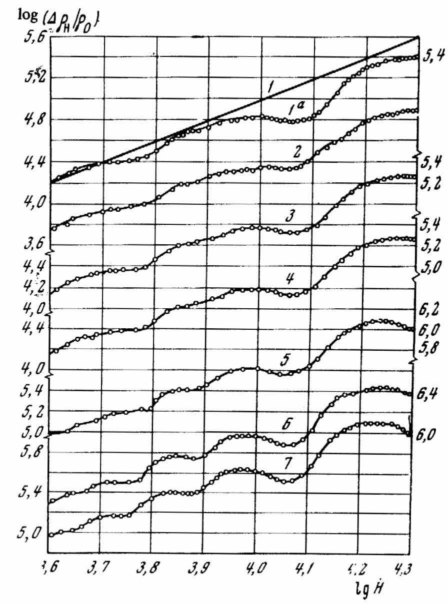

FIG. 6. Dependence of the resistance on the magnetic field at **H** ∥ C_1 (T = 4.2°K).. Curves 1a and 2 – Bi-IIa, corss sections 3.68 × 2.95 and 3.68 × 0.9 mm; curves 3 and 4 – Bi-IIb, cross sections 6.5 × 1 and 6.5 × 0.3 mm; curve 5 – Bi-Ib; curves 6 and 7 – Bi-Ia, cross sections 8 × 3.7 and 8 × 0.75 mm.

electronic computer. (The values of the extremal cross sections and of the cyclotron masses were taken from the papers of Édel'man and Khaĭkin[9] and Brandt et al.[12].) It turned out here that $n_x = 1.6$.

The experimental and calculated curves are shown in Fig. 7. The curves are in satisfactory agreement in fields up to 12 kOe. The discrepancy with further increase of the field is apparently connected with the

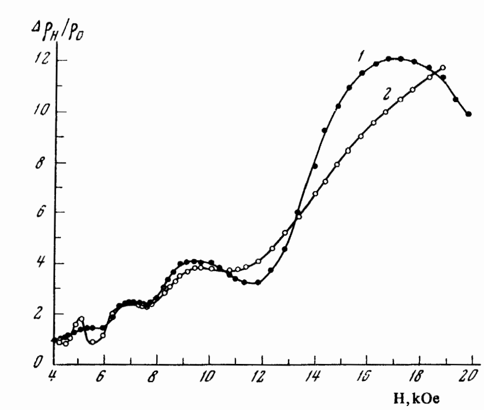

FIG. 7. Experimental (1) and calculated (2) curves of the sample Bi-Ia, **H** ∥ C_1, cross section 8 × 0.75 mm, T = 4.2°K.

shift of the chemical potential in the magnetic field[10]. We note here that an increase of the exponent of H in the oscillating part to $^3/_2$[13] decreases of the exponent n_X in the monotonic part.

As seen from Fig. 6, in the case $H \parallel C_1$, as well as for $H \parallel C_3$ and $H \parallel C_2$, all the singularities of the $(\log \Delta\rho_H/\rho_0) = f(\log H)$ curves are well reproduced on going from sample to sample and in measurements on single crystals with different orientations and with different transverse dimensions. (The possible error in the determination of the direction of the crystallographic axis was $\pm 2°$.)

Resistance measurements was made also on polycrystalline samples of Bi, which were obtained by rapidly cooling, with liquid nitrogen, molten metal drawn through a glass tube (the glass was then removed with fluoric acid, and the surface layer of Bi with nitric acid). It turned out that at 4.2°K the dependence of the resistance of the polycrystal from the magnetic field differed from quadratic. The exponent was 1.6 ± 0.05 for the investigated samples.

2. Influence of Shape of Sample on the Anisotropy of the Magnetoresistance of Bismuth

In the investigation of the resistance of the single crystal Bi-Ia, we observed that the magnetoresistance effect depends on the inclination of the magnetic field to the surface of the single crystal. The rotation diagram of sample Bi-Ia is shown in Fig. 8. We see that

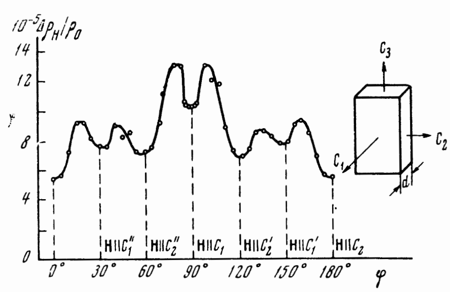

FIG. 8. Rotation diagram of single-crystal Bi-Ia with cross section 8 × 0.75 mm; H = 20 KOe, T = 4.2°K, $\varphi = 0°$ corresponds to $H \parallel C_2$.

for crystallographically equivalent directions of the magnetic field (they are realized when H is rotated through an angle $\varphi = \pm 60°$) the magnitude of the relative change of the resistance at definite values of φ turns out to be different. Thus, for example, when $H \parallel C_1$ and is perpendicular to the plane of the plate ($\varphi = 90°$), the value of $\Delta\rho_H/\rho_0$ is larger than when the angle between the normal to the plane of the plate and the direction of the magnetic field is $\pm 60°$. If $H \parallel C_2$, then the opposite picture is observed. Such a unique lowering of the symmetry of the rotation diagram of Bi to second order was observed for the sample Bi-Ia in the entire investigated range of thicknesses (Fig. 8). Experiments show that such singularities are possessed by single crystals of Bi of the given orientation with rectangular cross section.

Further, the dependence of the anisotropy of the magnetoresistance on the sample thickness was investigated for the single crystal Bi-Ia. The anisotropy of the magnetoresistance is usually characterized by the ratio of the resistances corresponding to different directions of the transverse magnetic field at equal magnitude of the field. As the main directions of the field we chose in this case those at which the magnetic-field vector coincided with the normals to the faces of the plate and simultaneously with the crystallographic axes C_1 and C_2.

Figure 9 shows a plot of

$$\alpha_1(H) = \frac{(\Delta\rho_H/\rho_0)_{\varphi=90°}}{(\Delta\rho_H/\rho_0)_{\varphi=0°}}$$

obtained for different sample thicknesses. We see that when d increases from 3.7 to 0.75 mm, the mean value

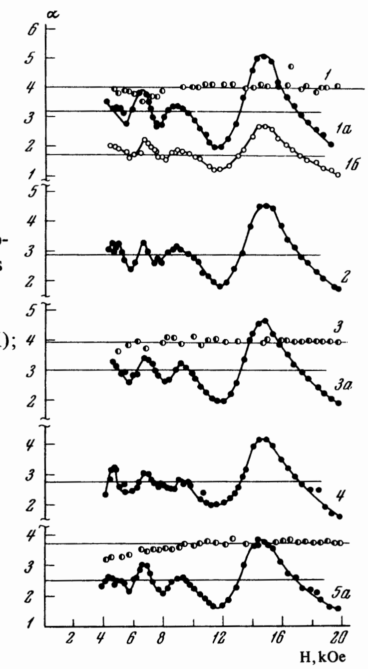

FIG. 9. Anisotropy of the magnetoresistance as a function of the nagnetic field for Bi samples of different thickness: curves 1a, 2, 3a, 4, 5a – α_1 (H) for d equal to 0.75, 1.1, 1.6, 2.5 and 3.7 mm. respectively (T = 4.2°K); curves 1, 3, and 5 – α_1 (H) for d equal to 0.75, 1.6, and 3.7 mm. (T = 77°K); curve 1b – α_2 (H), d = 0.75 mm. (T = 4.2°K).

of the resistance anisotropy as a function of the magnetic field at 4.2°K increases by approximately 30%. A very slight increase of $\alpha_1(H)$ (8%) takes place also at 77°K. A comparison of $\alpha_1(H)$ with

$$\alpha_2(H) = \frac{(\Delta\rho_H/\rho_0)_{\varphi=30°}}{(\Delta\rho_H/\rho_0)_{\varphi=60°}}$$

shows that $\alpha_1 > \alpha_2$ for all values of the magnetic field (Fig. 9). It should be noted that, within the limits of experimental error and of the approximations used in the calculation, the exponent of the function $\Delta\rho_H/\rho_0 = f(H)$ at $\varphi = 0°$ and $\varphi = 60°$ is the same. The same holds also for $\varphi = 30°$ and $\varphi = 90°$.

3. Influence of the Surface Layer on the Magnetoresistance of Bismuth

The electric and galvanomagnetic properties of the cylindrical sample Bi-Id were investigated before and after polishing. In the initial state, the ratio $\rho(300°K)/\rho(4.2°K)$ of the sample was 380, the resistance at 4.2°K increased linearly with the magnetic field for all directions of H, and the amplitude of the Shubnikov–de Haas oscillations was quite negligible. After polishing, $\rho(300°K)/\rho(4.2°K)$ increased to 410, the anisotropy of the magnetoresistance increased appreciably, the amplitude of the oscillations and the exponent of the monotonic part of the dependence of $\Delta\rho_H/\rho_0$ on H also increased. Thus, for example, when $H \parallel C_2$, we have $\Delta\rho_H/\rho_0 \propto H^{1.6\pm0.05}$ at 4.2°K (Fig. 10).

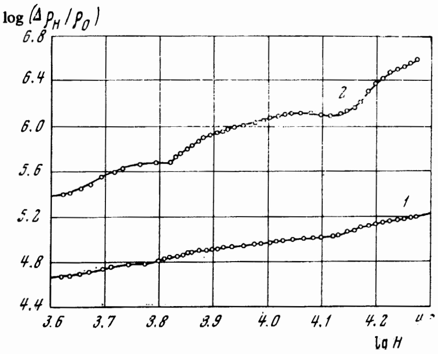

FIG. 10. Magnetoresistance of simple Bi-Id before (curve 1) and after (curve 2) polishing as a function of the magnetic field, T = 4.2°K.

Besides revealing the shape effect (see above), the experiment also shows that the surface influences the galvanomagnetic properties of bismuth

DISCUSSION OF RESULTS

As indicated above, the carrier mean free path in Bi at 4.2°K, determined by measuring the dependence of the residual resistance on the thickness, is of the order of 10^{-1} cm. Consequently, the magnetic field H_0 at which the Larmor radius r is of the same order as the mean free path l is $H_0 \sim pc/el \sim 1$ Oe. (The quasimomentum value $p \sim 10^{-21}$ is taken from[14]).

At 77°K, the field is $H_0 \sim 10^2$ Oe, and thus, the criterion of strong magnetic fields ($r \ll l$ or $H \gg H_0$) is satisfied at $H \geq 10$–100 Oe, if $T = 4.2$°K, and at $H \geq 10^3$–10^4 Oe if $T = 77$°K. The correctness of the foregoing estimates can be verified by determining H_0 by another method. It is known[15] that when $r \sim l$ we have $\Delta\rho_H/\rho_0 \sim 1$. At 4.2°K, the resistance of the bismuth in our experiments was doubled in a magnetic field of ~ 5 Oe. Consequently, $H_0 \approx 5$ Oe and $l \sim pc/eH_0 \sim 10^{-1}$ cm, which is in perfect agreement with estimates based on the results of measurements of the dependence of the residual resistance on the thickness.

1A. We note first that the deviation of the law governing the increase of the resistance of Bi from the quadratic law predicted by the theory[16], observed in the present investigation, is not an anomaly connected with the specific feature of a definite sample, but a general property of the given semimetal, as is evidenced by the good reproducibility of the result in measurements made on different samples.

One of the causes of the indicated phenomenon might be the increase of the number of carriers N in the magnetic field when the Fermi level is shifted, provided the spin and orbital splitting are not equal to each other. To this end it is necessary to satisfy the condition $N \propto H^{0.3}$, which yields $\Delta\rho_H/\rho_0 \propto H^2/N \sim H^{1.7}$ for all the directions of H—a dependence close to that observed experimentally. On the other hand, it is easy to see that if $N \propto H^{0.3}$, the carrier density, the Fermi energy, and the period of the Shubnikov–de Haas oscillations should change in the field range 10^3–10^4 Oe by approximately 100% for all directions of H ($N \propto \epsilon_F^{3/2}$, $\Delta(1/H) \propto {}^1\!/_2\pi m\epsilon_F$). However, as follows from[11,17], in fields up to 20 kOe, the change of the carrier density is insignificant. This is confirmed by Fig. 7, from which it follows that the experimentally observed oscillation minima at $H \parallel C_1$ in fields up to 12 kOe coincide within 10% with those calculated under the assumption that the chemical potential does not shift monotonically and that the spin and orbital splittings are equal.

1B. When a magnetic field $H \sim 10^4$ Oe is directed along the binary or bisector axis of the single crystal, the system of carriers of Bi is in the ultraquantum limit, corresponding to $\epsilon_F \sim \hbar\omega_H$. When $H \parallel C_3$, the ultraquantum limit is still far, but the deviation from the quadratic law at helium temperatures does take place, and is no less clearly pronounced than for other field directions. Consequently, the change of the exponent with decreasing temperature is not connected with the influence of the scattering mechanisms on the magnetoresistance in the case when $\hbar\omega_H \sim \epsilon_F$[11].

1C. Azbel' and Peschanskiĭ[3,4,18] considered theoretically the influence of the boundaries of the sample on the transport phenomena in the presence of a strong magnetic field, and showed that the presence of the surface can lead to galvanomagnetic effects. An important factor in this case is the character of the carrier scattering by the surface of the sample. As follows from electric conductivity experiment[7], and also from measurements of the surface impedance[14,19], the scattering in Bi is apparently specular. This means that the energy of the electrons (holes) and the momentum projections p_x and p_y on the plane tangent to the surface of the sample at the point of incidence of the electron are conserved, and the projection of the momentum on the inward normal to the surface (this direction is chosen to be the z axis) is uniquely determined from the equation

$$\varepsilon(p_z', p_x, p_y) = \varepsilon(p_z, p_x, p_y); \qquad v_z(p_z') > 0, \quad v_z(p_z) < 0. \tag{2}$$

Here $v = \partial\epsilon/\partial p$ is the velocity of the electron (hole). In other words, the probability of the transfer of an electron from one ellipsoid to another—the so-called intervalley transition—is quite small.

In specular reflection, the incident and scattering electrons are strongly correlated. Because of this, in metals with closed Fermi surfaces in a magnetic field, the electron can move on an open trajectory. As is well known[16], carriers with open orbits make a much larger contribution to the electric conductivity than carriers whose orbits are closed in momentum space. Therefore the dependence of the electric conductivity on the magnetic field in a bounded conductor may be different than in an unbounded one. It is clear that in the case of specular scattering the criterion for the bulkiness of the sample is different, and the boundary effects turn out to be appreciable even in samples whose thickness greatly exceeds the electron mean free path ($d \gg l$).

The problem is solved in the simplest manner for samples having the form of a plate. In this case the dependence of the resistance of the magnetic field parallel to the surface of the plate is, accurate to factors of the order of unity[4],

$$\rho_H = \rho_\infty \Big/ \left(1 + \frac{l/d}{q_1 + \gamma}\right), \tag{3}$$

where ρ_∞ is the resistance of an unbounded conductor, q_1 is the diffusivity parameter, and γ is the ratio of the curvature radius r of the carrier trajectory in the magnetic field to the mean free path l. It is easy to see that if

$$q_1, \gamma \ll l / d, \tag{4}$$

then

$$\rho_H \sim \frac{d}{l} \rho_\infty (q_1 + \gamma). \tag{5}$$

Recognizing that $\rho_\infty \sim \rho_0(H/H_0)^2$ and $\gamma = r/l = H_0/H$, we get

$$\frac{\Delta\rho_H}{\rho_0} \sim \frac{\rho_H}{\rho_0} \sim \frac{d}{l}\left[\frac{H}{H_0} + q_1\left(\frac{H}{H_0}\right)^2\right]. \tag{6}$$

On the other hand, when $\gamma \gg l/d$, we get $\Delta\rho_H/\rho_0 \propto (H/H_0)^2$. In other words, in specular reflection, the exponent of the function $\Delta\rho_H/\rho_0 = f(H)$ of a sample of a given thickness, in a given interval of magnetic fields, is determined by the carrier mean free path, and its value may differ from two.

In our case, at 4.2°K the condition (4) denotes that the relation (6) is valid when q_1, $\gamma \ll 10^{-1}$, i.e., when $H \gg 10$ Oe ($H_0 \sim 1$ Oe, the maximum thickness (width) of the samples is about 1 cm), and consequently formula (6) is certainly valid in fields 10^3–10^4 Oe. At 77°K, the mean free path is $l \sim 10^{-3}$ cm, and formula (6) can be used only in magnetic fields much stronger than 10^3 Oe.

Thus, the change of the refractive index of the function $\Delta\rho_H/\rho_0 = f(H)$ when the temperature decreases from 77 to 4.2°K, and the nonquadratic dependence of the resistance on the magnetic field at 4.2°K may be due to the specular mechanism of the carrier scattering by the Di surface. If this is so, then, representing the monotonic part of the magnetoresistance at 4.2°K in the form

$$\Delta\rho_H / \rho_0 = \alpha H + \beta H^2 \tag{7}$$

and comparing this expression with (6), we can easily determine the order of magnitude of the diffusivity coefficient $q_1 = (\beta/\alpha)H_0$. Thus, within the limits of the experimental errors, the exponent of the function $\Delta\rho_H/\rho_0$ is the same for samples of all thicknesses at all directions of the current and of the magnetic field, and for the determination of q_1 we can use any of the available experimental relations. Thus, for example, curve 1a of Fig. 4, represented in the form (7), becomes

$$\Delta\rho_H / \rho_0 = 4.105H + 0.00068H^2 \tag{8}$$

and consequently the diffusivity parameter is $q_1 \sim 10^{-4}$, i.e., the bismuth surface is practically specular.

When the thickness of the sample changes, the exponent of the function $\Delta\rho_H/\rho_0 = f(H)$ in our experiments remains constant. This circumstance, however, can be readily explained by the theory of[4]—in magnetic fields 10^3–10^4 Oe the parameter γ is smaller by several orders of magnitude than l/d in the entire investigated thickness range. The same can be said also with respect to the relation between q_1 and l/d.

It is seen from formula (6) that when $q_1 \ll H_0/H$ the resistance of the plate depends linearly on the magnetic field. When the ratio of the coefficients α and β is the same as in (8), a linear dependence should be observed in fields weaker than 10^2 Oe. However, when $H < 10^2$ Oe, the condition $H_0/H \ll l/d$ is satisfied to a much lesser degree, and therefore apparently no linear variation of the magnetoresistance in samples of Bi with thickness on the order of 1 mm is realized in strong magnetic fields.

2. If the sample thickness d is much smaller than the carrier mean free path l, then, as follows from (3), the resistance in a magnetic field is proportional to d for both purely specular and purely diffuse scattering. Therefore the growth of the resistance anisotropy of the single crystal Bi-Ia with decreasing thickness at 4.2°K (Fig. 9), when $l \sim d$, still does not permit unique conclusions to be drawn concerning the character of the scattering.

The situation is entirely different when $l \ll d$. In this case the magnetoresistance does not depend on the thickness of the sample when $q_1 = 1$. But when $q_1 \ll l/d \gtrsim \gamma$, then $\Delta\rho_H/\rho_0$ is a function of d (see (4)). Consequently, the variation of $\alpha(H)$ of the sample Bi-Ia with thickness at 77°K (see Fig. 9), when $l/d \sim \gamma \sim 10^{-3}$, confirms the assumed specular character of the carrier scattering by the Bi surface.

3. The singularities of the galvanomagnetic properties of the single crystal Bi-Id can be readily understood by making use of the specular mechanism of carrier scattering by the Bi surface. As shown in[4], if the surface of a cylindrical conductor has jogs, each of which is specular (in our case they unavoidably appear during the course of cutting), then a large electric current is concentrated near their corners (static skin effect), and when the distance between the jogs is smaller than or equal to the Larmor radius, the transverse resistance increases linearly with the magnetic field. Polishing removes the frequent jogs, and this, in accordance with the theory of[4], should lead to an increase of the exponent in the field dependence of $\Delta\rho_H/\rho_0$. This phenomenon does indeed take place, as shown in Fig. 10. (The growth of the oscillation amplitude after polishing is apparently connected with the improvement of the single-crystal structure of the surface layer).

Thus, a nonquadratic field dependence of $\Delta\rho_H/\rho_0$ of Bi plates in a magnetic fields parallel to the plates, the variation of the anisotropy of the magnetoresistance of these plates with thickness, and the linear variation of the magnetoresistance with the field in the case of a cylindrical sample with a rough surface can all be described within the framework of the theory of the resistance of semimetals[4], in which account is taken of the presence of the boundary. It should be noted, however, that the theory of Peschanskiĭ and Azbel'[4] does not explain, without additional particular assumptions concerning the structure of the surface layer, the deviation of the law governing the increase of the resistance from quadratic in those cases when the magnetic field is inclined to the surface of the plate. Nonetheless, the presence of the influence of boundaries on the magnetoresistance of Bi, even in the case of bulky samples, can be regarded as experimentally demonstrated. It becomes most clearly manifest in the shape effect.

4. Deviation of the growth of the resistance of Bi from a quadratic law was observed by Mase and Tanuma[20] at 4.2°K in magnetic fields up to several kOe. This question was considered in greater detail by Hattori[21], who carried out his investigations in fields up to 2 kOe at helium temperatures, on single crystals of various thicknesses. For bulky single crystals ($d \sim 3$ mm), according to[21], $\Delta\rho_H/\rho_0 \propto H^2$. The deviation from quadratic dependence at smaller d is connected with the diffusion size effect[22].

The results of Hattori differ from our data in that in our case, at all sample thicknesses, a deviation from a quadratic field dependence was observed, and in a broad range of thicknesses (0.3—8 mm), for samples of different orientations, the exponent of the dependence of $\Delta\rho_H/H$ was equal to 1.65 ± 0.05. This difference can be explained from the point of view of the specular scattering mechanism. Indeed, in the magnetic fields in which the investigations of[21] were carried out, the condition $\gamma \ll l/d$ is satisfied to a much lesser degree than in our investigation; this, first, may lead to large values of the exponent in the magnetoresistance and, second, may increase the sensitivity of the exponent to the crystal thickness.

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