TWO-PHOTON IONIZATION OF THE HYDROGEN ATOM

L. P. RAPOPORT, B. A. ZON, and L. P. MANAKOV

Voronezh State University

Submitted August 5, 1968

Zh. Eksp. Teor. Fiz. 56, 400-401 (January, 1969)

An analytic expression in terms of the Appell function F_1 is obtained for the two-photon ionization cross section of hydrogen atoms.

In our previous work[1-3] analytic expressions were obtained for the amplitudes of two-photon processes in the hydrogen atom for transitions from one bound state to another. In the present note the proposed method is extended to the case of bound state–continuum transitions. As shown below, one can also obtain the amplitude for such transitions in analytic form.

For the sake of definiteness let us consider two-photon ionization of the ns-levels. The cross section of this process for the case of absorption of two photons of the same polarization and frequency ω(Ry) has the following form in the nonrelativistic limit:

$$\frac{1}{I}\frac{d\sigma}{d\Omega}=\frac{4\pi r_0^2}{9\alpha\omega^3 I_0}|M_0^{ns}+e^{i(h_2-h_0)}M_2^{ns}(1-3\cos^2\theta)|^2. \tag{1}$$

Here θ is the angle between the directions of polarization of the incident photons and the momentum $\mathbf{p}$ of the outgoing electron, r_0 is the classical radius of an electron, I is the intensity of the incident radiation in W/cm^2, $I_0 = 7.019\times10^{16}$ W/cm^2. Let us consider then the case which is most important in practice, when ω is less than the ionization potential of the atom. In this connection one can use the dipole approximation, and the reduced matrix elements of M_L have the form

$$M_L^{ns}=\int\int_0^\infty dr\,dr'(rr')^2 g_1(E_{ns}+\omega;r,r')\frac{dR_{ns}(r)}{dr}\times\left[\frac{dR_L(r')}{dr'}+3\delta_{L2}\frac{R_L(r')}{r'}\right], \tag{2}$$

where R_{ns} is the radial part of the wave function for the discrete spectrum, R_L and h_L are the radial part and phase of the L-th partial wave function for the continuous spectrum, having the asymptotic form of a plane wave plus a spherical incoming wave;[4] g_1 is the radial part of the Coulomb Green's function for angular momentum $l = 1$; δ_{L2} is the Kronecker delta.

If the integral representation used in [1-3] is used for g_1, then all of the integrals appearing in Eq. (2) can be evaluated analytically. It is easy to see, by carrying out the integration in (2) first with respect to dr′, then with respect to dr, and finally with respect to the variable appearing in the integral representation, that one can represent M_L in the form

$$M_L^{ns}=N(\nu,k)P_{ns}\left(-\frac{\partial}{\partial x}\right)Q_L(x)|_{x=1/n}, \tag{3}$$

where

$$N(\nu,k)=\frac{32e^{-k\varphi}}{\alpha(1-e^{-2\pi k})^{1/2}(2-\nu)\omega^3\nu k^6}$$

$$Q_0=-\frac{2}{(1+x\nu)^4}\,\mathrm{Im}[(2-k^2+3ik)e^{i\varphi}F(0;1;x)],$$

$$Q_2=\frac{4k\nu[(1+k^2)(4+k^2)]^{1/2}}{(1+x\nu)^4}\left[F(0;3;x)-\left(\frac{1+x\nu}{1-x\nu}\right)^2\frac{2-\nu}{4-\nu}F(2;3;x)\right],$$

$$F(s;t;x)\equiv F_1\left(s+2-\nu;\ t-ik;3+ik;\ s+3-\nu;\ \frac{1+x\nu}{1-x\nu}e^{-i\varphi},\ \frac{1+x\nu}{1-x\nu}e^{i\varphi}\right) \tag{4}$$

denotes the Appell function,[5] $\varphi = 2\tan^{-1}(\nu/k)$, $\nu = n(1-\omega n^2)^{-1/2}$, $k = (ap)^{-1}$, a is the Bohr radius. The differential polynomial P_{ns} is obtained from

$$-a^{3/2}e^{r/an}\frac{d}{d(r/a)}R_{ns}\left(\frac{r}{a}\right)$$

by the substitution $r/a \to -\partial/\partial x$. In particular,

$$P_{1s}=2,\quad P_{2s}=2^{-1/2}\left(1+\frac{1}{4}\frac{\partial}{\partial x}\right).$$

The Appell functions entering into expressions (4) may be evaluated by means of expansion in rapidly converging series.[5] We have compared the cross section for ionization of the 2s-level with earlier calculations by Zernik.[6] Satisfactory agreement with [6] is obtained with the exception of regions of abrupt variation of the cross section (ω = 0.137, 0.163, 0.188, 0.200), where the results differ somewhat.

In conclusion we note that an examination of other types of bound state–continuum transitions may be carried out in similar fashion.

[1] B. A. Zon and L. P. Rapoport, ZhETF Pis. Red. 7, 70 (1968) [JETP Lett. 7, 52 (1968)].

[2] L. P. Rapoport and B. A. Zon, Phys. Lett. 26A, 564 (1968).

[3] B. A. Zon, N. L. Manakov, and L. P. Rapoport, Zh. Eksp. Teor. Fiz. 55, 924 (1968) [Sov. Phys.-JETP 28, 480 (1969)].

[4] H. A. Bethe and E. E. Salpeter, Quantum Mechanics of One- and Two-Electron Atoms, Academic Press, 1957 (Russ. transl., Fizmatgiz, 1960).

[5] A. Erdélyi, ed., Higher Transcendental Functions (Bateman Manuscript Project) (McGraw-Hill, 1953) Vol. 1 (Russ. transl., Izd. Nauka 1965).

[6] W. Zernik, Phys. Rev. 135, A51 (1964).

Translated by H. H. Nickle
50