NONLINEAR AMPLIFICATION OF LIGHT PULSES. III. ULTRASHORT PULSE DURATIONS

V. S. LETOKHOV

P. N. Lebedev Physics Institute, USSR Academy of Sciences

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A general description is presented of the evolution of the ultrashort light pulse in a nonlinear-amplifying medium possessing a finite transverse relaxation time T_2 and linear radiation losses. It is shown that in a medium without linear losses the pulse gradually transforms into several stationary "2π-pulses" (the number of which is m + 1, where m is the number of complete oscillations of the quantum system for the initial light pulse), and a "π-pulse". In the presence of linear losses, all "2π-pulses" except the first decay gradually, and the first "2π-pulse" changes into a stationary "π-pulse". It is shown that a stationary "π-pulse" also appears in a medium with an inhomogeneously broadened line, for example, in a gas amplifying medium. It is also shown that self-broadening of the resonance line occurs in the field of the pulse during nonlinear amplification; in this case the transmission bandwidth is greater in the case of linear amplification.

1. INTRODUCTION

THE propagation of a light pulse in a resonantly amplifying medium is a very effective method of obtaining light pulses of very high power. The amplification of a short light pulse from a Q-switched laser yields powers on the order of 10 GW and even higher[1,2]. Amplification of an ultrashort light pulse from a laser with mode-locking[3] yields powers on the order of 10^3 GE[4].

Theoretically, the propagation of a short light pulse in an amplifying medium in the case when the pulse duration τ_p satisfies the condition $T_2 \ll \tau_p \ll T_1$ (T_1 are the times of longitudinal and transverse relaxation of the active medium) has been treated in a number of papers[5-10]. This problem is of interest because nonlinear amplification has made it possible to increase the energy and to reduce the duration of the light pulse[11]. However, it was shown theoretically and experimentally in[1] that when a light pulse from a Q-switch laser propagates in a nonlinear amplifying medium the pulse duration is not shortened. What we get instead is a shift of the maximum of the pulse over the leading front. Under certain conditions[12], such a shift causes the maximum of the pulse to move in the medium with superluminal velocity. The criteria for reducing the pulse duration in nonlinear amplification were found in[13]. A pulse compression regime in nonlinear amplification was realized in[1], where a power of the order of 10 GW was attained in this manner.

In a number of papers[14-17], the propagation of a pulse having a duration on the order of T_2 in an amplifying medium was also considered. It was found that propagation in an amplifying medium gives rise to the so-called stationary "π-pulse," which inverts the level population on passing through the medium. A recent paper[18] deals with the formation of a "2π-pulse" by propagation in an amplifying medium with $T_2 = \infty$. These effects were rather beyond the experimental capabilities, but the latest progress in the generation and amplification of ultrashort light pulses in solid-state lasers[3,4,19] and progress in the development of powerful CO_2 gas lasers[20,21] makes this problem timely.

In this paper we present a general description of the evolution of an ultrashort light pulse in an amplifying medium having a finite transverse relaxation time T_2 and a linear radiation dissipation. The main effect is that the oscillatory response of the two-level system to the strong field causes a gradual breakdown of the powerful pulse into several 2π-pulses. If there are no linear losses at all, then ultimately several stationary 2π-pulses are formed, the number of which equals m + 1, where m is the number of total oscillations in the response of the quantum system to the initial light pulse. The presence of linear dissipation of the radiation changes the evolution of the pulse completely. In this case all the 2π-pulses gradually attenuate, and the first 2π-pulse is transformed into a stationary π-pulse. It is shown that the stationary π-pulse arises also in a medium with inhomogeneously broadened light, for example in an amplifying gas medium.

A major feature of the nonlinear amplification of ultrashort light pulses is the increase of the bandwidth compared with the case of linear amplification. This can be interpreted as a result of self-broadening of the resonance line in the field of the light pulse.

2. FUNDAMENTAL EQUATIONS AND GENERAL ANALYSIS

Just as in[1], we consider the propagation of a light pulse in the form of a plane wave, through an active medium of two-level particles with inverted population. In the approximation in which the changes of the amplitude and phase of the light wave are small over distances on the order of the light wavelength and in times on the order of the period of the light oscillations (the envelope approximation), the intensity E(x, t) of the light-wave field and the polarization p(x, t) of the medium (the dipole moment per unit volume) can be represented in the form

$$E = \mathscr{E}(t, x)\cos[\omega t - kx + \varphi(t, x)], \qquad P = \mathscr{P}(t, x)\cos[\omega t - kx + \psi(t, x)]. \tag{1}$$

If the center of the amplification line of all the particles has the frequency ω_0 (homogeneous broadening of the amplification line), then, as shown in[1], the "slow" variables $\mathscr{E}$, φ, $\mathscr{P}$, ψ and the density N of the inverted population satisfy the equations

$$\frac{\partial \mathscr{E}}{\partial t}+c\frac{\partial \mathscr{E}}{\partial x}+\frac{\gamma}{2}c\mathscr{E}=2\pi\omega\mathscr{P}\sin(\psi-\varphi),$$
$$\mathscr{E}\left(\frac{\partial \varphi}{\partial t}+c\frac{\partial \varphi}{\partial x}\right)=-2\pi\omega\mathscr{P}\cos(\psi-\varphi),$$
$$\frac{\partial \mathscr{P}}{\partial t}+\frac{1}{T_2}\mathscr{P}=\frac{\mu^2}{\hbar}N\mathscr{E}\sin(\psi-\varphi),$$
$$\left[\frac{\partial \psi}{\partial t}+(\omega-\omega_0)\right]\mathscr{P}=\frac{\mu^2}{\hbar}N\mathscr{E}\cos(\psi-\varphi),$$
$$\frac{\partial N}{\partial t}+\frac{1}{T_1}(N-N_0)=-\frac{1}{\hbar}\mathscr{P}\mathscr{E}\sin(\psi-\varphi), \tag{2}$$

where c is the velocity of the pulse in the medium without active particles, γ is the coefficient of linear radiation loss per unit length, T_1 and T_2 are the times of longitudinal and transverse relaxations of the particles (inasmuch as the duration of the ultrashort pulses is smaller than T_1 by many orders of magnitude, the longitudinal relaxation will henceforth be neglected), N_0 is the initial density of the inverted population, and μ is the dipole moment of the transition, connected with the cross section for the radiative transition between the levels σ_0 by the relation $\sigma_0 = 4\pi\omega T_2\mu^2/\hbar c$. In the case of exact resonance ($\omega = \omega_0$), which will be considered in the greatest detail, the number of equations reduces to 3, since in this case $\psi - \varphi = \pi/2$. In terms of the dimensionless variables

$$t=t'/T_2,\quad x=x'\sigma_0N_0,\quad c=c'\sigma_0N_0T_2,\quad \alpha=\gamma/\sigma_0N_0,$$
$$\mathscr{E}=\mathscr{E}'T_2\mu/\hbar,\quad \mathscr{P}=\mathscr{P}'/N_0\mu,\quad N=N'/N_0 \tag{3}$$

these equations become

$$\frac{1}{c}\frac{\partial \mathscr{E}}{\partial t}+\frac{\partial \mathscr{E}}{\partial x}=\frac{1}{2}\mathscr{P}-\frac{\alpha}{2}\mathscr{E},$$
$$\frac{\partial \mathscr{P}}{\partial t}+\mathscr{P}=N\mathscr{E},\quad \frac{\partial N}{\partial t}=-\mathscr{E}\mathscr{P}. \tag{4}$$

We consider first the evolution of an ultrashort pulse qualitatively. To this end we change over in (4) to the variables $\tau = t - x/c$ and $x = x$:

$$\frac{\partial \mathscr{E}}{\partial x}=-\frac{\alpha}{2}\mathscr{E}+\frac{1}{2}\mathscr{P},\quad \frac{\partial \mathscr{P}}{\partial \tau}=-\mathscr{P}+N\mathscr{E},\quad \frac{\partial N}{\partial \tau}=-\mathscr{E}\mathscr{P}. \tag{5}$$

The first equation of (5) (multiplying by 2E and taking the third equation into account) can be reduced to the form

$$\frac{\partial \mathscr{E}^2}{\partial x}=-\alpha\mathscr{E}^2-\frac{\partial N}{\partial \tau}. \tag{6}$$

We integrate (6) with respect to τ from $-\infty$ to τ:

$$\frac{\partial R}{\partial x}=-\alpha R+1-N,\quad R=\int_{-\infty}^{\tau}\mathscr{E}^2(\tau',x)\,d\tau' \tag{7}$$

where the function R characterizes the energy of the pulse and is equal to the total pulse energy at $\tau = \infty$.

In the case of ultrashort pulses of duration $\tau_p \ll T_2$, it is possible to neglect in the second equation of (5) the polarization relaxation. Then the material equation can be integrated and their solution is

$$N=\cos\Phi,\quad \mathscr{P}=\sin\Phi, \tag{8}$$

where

$$\Phi=\int_{-\infty}^{\tau}\mathscr{E}(\tau',x)\,d\tau'. \tag{9}$$

It follows from (8) that N and P can be regarded as the projections of the unit vector, and Φ as the angle of rotation of this vector: $\Phi = \pi$ corresponds to a total transition of the particle to the lower level, and $\Phi = 2\pi$ corresponds to the return to the upper level. In this approximation, Eq. (7) takes the form

$$\partial R/\partial x=-\alpha R+1-\cos\Phi. \tag{10}$$

In the absence of linear loss ($\alpha = 0$), the growth of the pulse energy is determined completely by the angle of rotation Φ of the particles under the influence of this pulse. Figure 1 shows the dependence of the gain $\partial R/\partial x$ on Φ (curve a). If the rotation angle becomes equal to $\Phi = m2\pi$ under the influence of the pulse (m is an integer), then such a pulse propagates without a gain in energy[1)]. If the rotation angle is $\Phi = m2\pi + \delta$, where $\delta > 0$ is an arbitrarily small quantity, then such a pulse will obviously be amplified until the angle of rotation becomes equal to $(m + 1)2\pi$. If $\delta < 0$, then the amplification continues until Φ becomes equal to $m2\pi$. In this

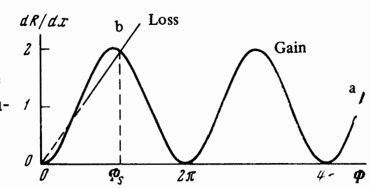

FIG. 1. Dependence of the gain and loss of energy of ultrashort pulse on the rotation angle Φ.

sense, the 2π-pulses are unstable. We can guess from curve a of Fig. 1 that the transmission of a pulse with $\Phi > 2\pi$ proceeds in "batches," each of which corresponds to a rotation angle 2π. As will be shown later, this corresponds to a gradual breakdown of the pulse into 2π-pulses. When $\Phi \ll 2\pi$, a linear amplification of the pulse takes place. The smallness of the gain in this case is due to the condition $\tau_p \ll 1$, which means that the spectrum of the pulse is much broader than the gain line.

In the presence of linear radiation loss ($\alpha > 0$) the evolution of the pulse changes radically. For a qualitative analysis, the function R can be replaced by $\beta\Phi$, where β is a certain dimensionless quantity. For ultrashort pulses ($\tau_p \ll 1$) with $\Phi \gtrsim 1$, we obviously have $\beta \gg 1$. The loss $\alpha R \approx \alpha\beta\Phi$ is plotted in Fig. 1 (curve b). The linear radiation loss gives rise to a stationary value of the phase Φ_s, the determination of which calls for knowledge of the value of β. When the pulse propagates, the coefficient β varies until the pulse assumes a stationary form. As will be shown below, for a pulse of stationary form $\beta = \pi/2\alpha$, and consequently $\Phi_s = \pi$. Thus, in the presence of linear losses we can expect

[1)]In the case of nonlinearly absorbing medium, curve a of Fig. 1 reverses sign, and consequently such pulses propagate in a medium without absorption. This phenomenon, called self-transparency of the medium, is considered in [22], which deals with the case of inhomogeneous broadening, when the inhomogeneous width is much larger than the width of the pulse spectra. In this case the pulses with $\Phi < \pi$ attenuate. It follows from the form of curve a of Fig. 1, that in the case of homogeneous broadening pulses with $\Phi < 2\pi$ attenuate.

formation of one π-pulse regardless of the initial value of the phase Φ_0.

3. AMPLIFYING MEDIUM WITHOUT LINEAR LOSS

To consider the evolution of the pulse, including the relaxation of the polarization, it is necessary to solve the system of equations (4). It is impossible to obtain an analytic solution of this system, and therefore a numerical integration with a computer was used. The results of a numerical solution are quite lucid and make it possible to present a general picture of the evolution of an ultrashort pulse during its propagation.

We considered the propagation of pulses having on the boundary of the medium the form $\mathscr{E}_0(t) = A\cosh^{-2}(t/\tau_0)$ with duration $\tau_0 \ll 1$ and amplitude A. Such a pulse corresponds to a rotation angle $\Phi_0 = 2A\tau_0$. Figure 2 shows the results of the solution for the case $\tau_0 = 0.33$ and A = 3, when $\Phi_0 < 2\pi$. In accordance with the qualitative conclusion of Sec. 2, such a pulse can be transformed into a 2π-pulse. Actually, as it propagates, the pulse is transformed into two pulses: a stationary 2π-pulse and a stationary π-pulse. It is important here that, first, the 2π-pulse is produced even when polarization relaxation exists. In[18] and Sec. 2, the formation of the 2π-pulse is shown for an amplifying medium with $T_2 = \infty$, i.e., without polarization relaxation. Second, the 2π-pulse is followed by the propagation of a π-pulse, which relaxes the level population to the absorbing state. Figure 3 shows the results of the solution for the case $\tau_0 = 0.33$ and A = 15, when $2\pi < \Phi_0 < 4\pi$. Such a pulse is gradually transformed into two 2π-pulses and one π-pulse. This tendency is retained also with further increase of the pulse power. By way of illustration, Fig. 4 shows the results of the solution for the case $\tau_0 = 0.5$ and A = 20, when $6\pi < \Phi_0 < 8\pi$. We see here that the pulse gets broken up into four 2π-pulses and one π-pulse. The results of the numerical solution agree with the foregoing qualitative picture, and can be formulated as follows: if the angle of rotation under the influence of the initial ultrashort pulse $\Phi_0 = 2\pi(m + \delta)$, where m is an integer and $0 < \delta < 1$, then such a pulse propagating in an amplifying medium without linear losses is transformed into m + 1 2π-pulses and one π-pulse.

The form of the stationary 2π-pulse can be obtained analytically. If we neglect the polarization relaxation in the second equation of the system (4), then the propagation of the pulse is described by the equation

$$\frac{1}{c}\frac{\partial\mathscr{E}}{\partial t}+\frac{\partial\mathscr{E}}{\partial x}=\frac{1}{2}\sin\left(\int_{-\infty}^{t}\mathscr{E}(t',x)\,dt'\right). \tag{11}$$

The stationary solution $\mathscr{E}(t - x/v)$ of Eq. (11) satisfies the equation

$$\left(\frac{1}{c}-\frac{1}{v}\right)\frac{d\mathscr{E}}{d\tau}=\frac{1}{2}\sin\left(\int_{-\infty}^{\tau}\mathscr{E}(\tau')\,d\tau'\right) \tag{12}$$

where $\tau = t - x/v$. Eq. (12) has a solution of the form

$$\mathscr{E}(\tau)=\sqrt{\frac{2}{a}}\,\mathrm{ch}^{-1}\frac{\tau}{\sqrt{2a}},\qquad a=\frac{1}{c}-\frac{1}{v}. \tag{13}$$

The indeterminate parameter a can be obtained in the following manner. According to (13), the stationary pulse has exponential fronts. Consequently, the leading front can be represented in the form $\mathscr{E}(\tau) = A\exp(\tau/\tau_0)$, and it should satisfy the initial equation (11) at sufficiently small values of A. From this condition we get the following expression for a:

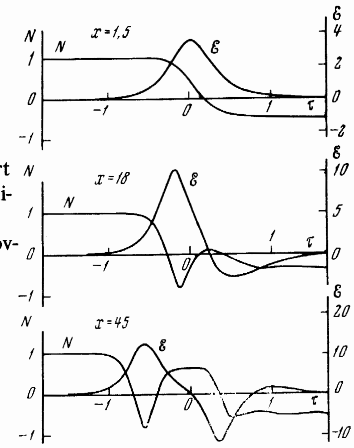

FIG. 2. Evolution of ultrashort light pulse propagating in an amplifying medium without linear loss ($a = 0$). Notation: x – distance covered by the pulse in the medium; $\mathcal{E}$ – field intensity, N – inverted population. The initial angle of rotation under the influence of the pulse is $\Phi_0 = 2$.

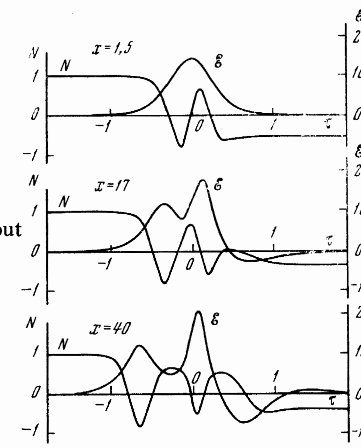

FIG. 3. The same as Fig. 2, but $\Phi_0 = 10$.

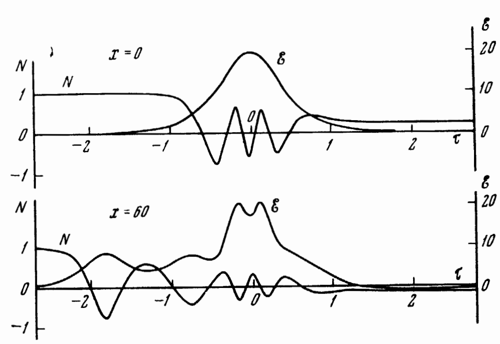

FIG. 4. The same as Fig. 2, but $\Phi_0 = 20$.

$$\frac{1}{c}-\frac{1}{v}=\frac{\tau_0^2}{2} \tag{14}$$

or, changing to dimensional units,

$$\frac{1}{c}-\frac{1}{v}=\frac{1}{2}\left(\frac{\tau_0}{T_2}\right)^2\sigma_0 N_0 T_2. \tag{14a}$$

Then the shape of the pulse (13) becomes

$$\mathscr{E}(\tau)=\frac{2}{\tau_0}\,\mathrm{ch}^{-1}\left(\frac{\tau}{\tau_0}\right). \tag{15}$$

It is easy to verify that such a pulse corresponds to a rotation angle 2π:

$$\Phi = \int_{-\infty}^{\infty} \mathscr{E}(\tau)\, d\tau = 2\pi, \tag{16}$$

i.e., the obtained stationary pulse is a 2π-pulse. The shape of this pulse is similar to the shape of the stationary 2π-pulse in an absorbing medium[22].

It follows from (14) that the propagation velocity v of the 2π-pulse exceeds the velocity of light c. This phenomenon is similar to the propagation of a short pulse with exponential leading front in a nonlinearly amplifying medium with superluminal velocity, considered in[1,12]. Moreover, the expression for the velocity (14a) coincides with the expression of[12]:

$$\frac{1}{c} - \frac{1}{v} = \tau \varkappa_0, \tag{17}$$

where τ is the slope of the exponential front for the intensity, and κ_0 is the initial gain per unit length. Since the width of the spectrum of the ultrashort pulse is larger by a factor T_2/τ_0 than the width of the amplification line, the gain of such a pulse per unit length is $\kappa_0 = \sigma_0 N_0(\tau_0/T_2)$. Substituting this value of κ_0 in (17) and recognizing that $\tau = \tau_0/2$, we obtain (14a). In spite of such a similarity, the shapes of the stationary pulses in these cases are entirely different. The reason is that when $\tau_p \gg T_2$ the stationary pulses must exist in the presence of linear losses γ[12], while the stationary 2π-pulse exists in an amplifying medium without linear loss.

4. AMPLIFYING MEDIUM WITH LINEAR LOSS

In an amplifying medium with linear radiation loss, the picture of the evolution of the ultrashort pulses is entirely different. It was shown already in[14], by numerical integration, that a low-intensity pulse ($\Phi_0 \ll 2\pi$) is gradually transformed into a stationary π-pulse, which propagates when $\alpha \ll 1$ with a velocity that coincides practically with the velocity of light. Is this tendency is retained if $\Phi_0 > 2\pi$? The results of a numerical solution of (4) show that, regardless of the initial power, the pulse will ultimately be transformed into a stationary π-pulse. Figure 5 shows the results of the solution for the case when $\Phi_0 = 3\pi$ and $\alpha = 0.2$. We can see clearly the decrease of the pulse energy and the gradual formation of the stationary π-pulse.

The shape of the stationary π-pulse $E(t - x/v)$ with $v = c$ was obtained analytically in[15-17]. We present a brief derivation based on neglecting the polarization relaxation ($\tau_p \ll 1$), when the propagation of the pulse is described by the equation

$$\frac{1}{c}\frac{\partial \mathscr{E}}{\partial t} + \frac{\partial \mathscr{E}}{\partial x} + \frac{\alpha}{2}\mathscr{E} = \frac{1}{2}\sin\int_{-\infty}^{t} \mathscr{E}(t')\, dt'. \tag{18}$$

A pulse in the form $\mathscr{E}(t - x/c)$ is described by the equation

$$\alpha \mathscr{E} = \sin \int_{-\infty}^{\tau} \mathscr{E}(\tau')\, d\tau', \qquad \tau = t - \frac{x}{c}. \tag{19}$$

Equation (19) has a solution

$$\mathscr{E}(\tau) = \frac{1}{\alpha}\, \mathrm{ch}^{-1}\left(\frac{\tau}{\alpha}\right). \tag{20}$$

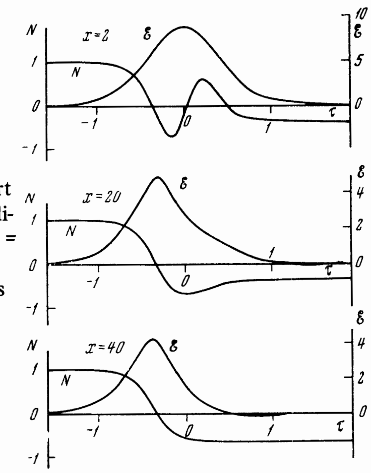

FIG. 5. Evolution of ultrashort light pulse propagating in an amplifying medium with linear losses (α = 0.2). The initial angle of rotation under the influence of the pulse is $\Phi_0 = 3\pi$.

The obtained solution is valid for a pulse duration $\tau_p \ll 1$, and consequently when $\alpha \ll 1$. It is easy to see that a pulse in the form (20) changes the angle of rotation by π, i.e., this is a stationary π-pulse. By direct calculation we can show that for such a pulse the parameter introduced in Sec. 2 is $\beta = R/\Phi = 2/\pi\alpha$.

It is interesting to note that the duration of the stationary π-pulse can be obtained from rather simple qualitative considerations. First, owing to the loss γ, the pulse energy is stationary:

$$\frac{c}{8\pi}\mathscr{E}^2 \tau_p{}' = \hbar\omega_0 \frac{N_0}{\gamma}, \tag{21}$$

Second, the pulse produces population inversion:

$$\mu \mathscr{E} \tau_p / \hbar = \pi, \tag{22}$$

We have used here the dimensional units τ_p and τ_p', which are respectively the pulse durations in terms of the amplitude and intensity. If we recognize that $\tau_p \approx \sqrt{2}\,\tau_p'$, and take into account the connection between μ and σ_0, then we obtain for the pulse duration the expression

$$\tau_p{}' \approx \frac{T_2}{2}\frac{\gamma}{\sigma_0 N_0}\frac{\pi^2}{2}. \tag{23}$$

The approximate expression (23) differs from the exact one[15]

$$\tau_p{}' = \frac{T_2}{2}\left(\frac{\sigma_0 N_0}{\gamma} - 1\right)^{-1} \ln\frac{3 + 2\sqrt{2}}{3 - 2\sqrt{2}} \tag{24}$$

by only 10%.

The main conclusion from relations (20) and (23) is that the duration of the π-pulse can be much shorter than T_2 if $\alpha \ll 1$. This raises the question of how such a pulse is produced in a medium if its spectrum is much broader than the spectral gain line. In a resonant electromagnetic field, the spectral line is broadened by the saturation effect by an amount[23]

$$\Delta\omega_{\mathscr{E}} = \mu \mathscr{E} / \hbar. \tag{25}$$

Then the inversion condition (22) can be represented in the form

$$\Delta\omega_{\mathscr{E}} \approx \Delta\omega_p, \tag{26}$$

where $\Delta\omega_p \approx \pi/\tau_p$ is the width of the pulse spectrum. Thus, the field intensity of the π-pulse is maintained such as to broaden the amplification line to a value equal to the width of the pulse spectrum. In other words, self-broadening of the spectral line takes place when a powerful ultrashort pulse propagates.

From this point of view, we can expect the formation of a π-pulse in an amplifying gas medium with inhomogeneous level broadening, if the line broadening in the field exceeds the Doppler width $\Delta\omega_{\mathrm{Dop}}$. In this case, owing to the broadening by the field, the line becomes homogeneously broadened with a width on the order of $\Delta\omega_{\mathscr{E}}$, and the relations obtained for the case of homogeneous broadening should hold. This follows also from a more rigorous analysis.

Let the centers of the particle amplification line be distributed about the frequency ω_0 with a distribution function $w_\Omega = w(\omega_0 + \Omega)$. Then the line inverted population at the frequency $\omega_0 + \Omega$ is

$$n_\Omega = N w_\Omega, \qquad \int_{-\infty}^{\infty} w_\Omega d\Omega = 1 \tag{27}$$

where N is the total density of the inverted population. The system (2) is replaced in this case by

$$\begin{aligned}
&\frac{\partial\mathscr{E}}{\partial t} + c\frac{\partial\mathscr{E}}{\partial x} + \frac{\gamma}{2}c\mathscr{E} = 2\pi\omega\mathscr{P}\sin(\psi - \varphi),\\
&\mathscr{E}\left(\frac{\partial\varphi}{\partial t} + c\frac{\partial\varphi}{\partial x}\right) = -2\pi\omega\mathscr{P}\cos(\psi - \varphi),\\
&\frac{\partial\mathscr{P}_\Omega}{\partial t} + \frac{1}{T_2}\mathscr{P}_\Omega = \frac{\mu^2}{\hbar}n_\Omega\mathscr{E}\cos(\psi_\Omega - \varphi),\\
&\left[\frac{\partial\psi_\Omega}{\partial t} + (\omega - \omega_0 - \Omega)\right]\mathscr{P}_\Omega = \frac{\mu^2}{\hbar}n_\Omega\mathscr{E}\cos(\psi_\Omega - \varphi),\\
&\frac{\partial n_\Omega}{\partial t} + \frac{1}{T_1}(n_\Omega - n_{\Omega 0}) = -\frac{1}{\hbar}\mathscr{P}_\Omega\mathscr{E}\sin(\psi_\Omega - \varphi),
\end{aligned} \tag{28}$$

where $\mathscr{P}_\Omega$ and ψ_Ω are the amplitude and phase of the polarization of the particles with line center at the frequency $\omega_0 + \Omega$, and $\mathscr{P}$ and ψ are the amplitude and phase of the total polarization of all the particles:

$$\mathscr{P}\cos(\omega t + \varphi) = \int_{-\infty}^{\infty}\mathscr{P}_\Omega\cos(\omega t + \psi_\Omega)d\Omega. \tag{29}$$

In the case of exact resonance ($\omega = \omega_0$) and a symmetrical line, we have $\psi - \varphi = \pi/2$, and the system (28) simplifies greatly:

$$\begin{aligned}
&\frac{\partial\mathscr{E}}{\partial t} + c\frac{\partial\mathscr{E}}{\partial x} + \frac{\gamma}{2}c\mathscr{E} = 2\pi\omega\mathscr{P},\\
&\frac{\partial\mathscr{P}_\Omega}{\partial t} + \frac{1}{T_2}\mathscr{P}_\Omega = \frac{\mu^2}{\hbar}n_\Omega\mathscr{E}\cos\psi_\Omega,\\
&\left(\frac{\partial\psi_\Omega}{\partial t} - \Omega\right)\mathscr{P}_\Omega = \frac{\mu^2}{\hbar}n_\Omega\mathscr{E}\sin\psi_\Omega,\\
&\frac{\partial n_\Omega}{\partial t} = -\frac{1}{\hbar}\mathscr{P}_\Omega\mathscr{E}\cos\psi_\Omega,
\end{aligned} \tag{30}$$

where ψ_Ω now denotes the quantity $\psi_\Omega - \psi$, and we have omitted the term corresponding to the longitudinal population relaxation ($\tau_p \ll T_1$).

The stationary solution of the system (30), having the form $\mathscr{E}(t - x/c)$, satisfies the following equations:

$$\begin{aligned}
&\frac{d\mathscr{E}}{d\tau} + \frac{1}{T_2}\mathscr{E} = \frac{\sigma_0}{\gamma T_2}\left(N\mathscr{E} - \frac{\hbar}{\mu^2}\int_{-\infty}^{\infty}\Omega\sin\psi_\Omega\mathscr{P}_\Omega d\Omega\right)\\
&\frac{dN}{d\tau} = -\frac{\gamma}{4\pi\omega c\hbar}\mathscr{E}^2,
\end{aligned} \tag{31}$$

which follow from (30) after substituting $\mathscr{E}(t - x/c)$ and integrating over the frequencies with allowance for the following relations:

$$\mathscr{P} = \int_{-\infty}^{\infty}\mathscr{P}_\Omega\cos\psi_\Omega d\Omega, \qquad \int_{-\infty}^{\infty}\mathscr{P}_\Omega\sin\psi_\Omega d\Omega = 0. \tag{32}$$

Equations (31) differ from the corresponding equations for the case of homogeneous broadening[15] in the integral term of the first equation. If the pulse duration satisfies the condition

$$\tau_p\Delta\omega_{\mathrm{Dop}} \ll 1, \tag{33}$$

then the estimate for this additional term is

$$\int_{-\infty}^{\infty}\Omega\sin\psi_\Omega\mathscr{P}_\Omega d\Omega \approx \frac{\mu^2}{\hbar}\mathscr{E}N(\tau_p\Delta\omega_{\mathrm{Dop}})^2 \tag{34}$$

Consequently, in the approximation (33) we can neglect this term. Then Eqs. (31) describe a stationary π-pulse in the medium with homogeneous level broadening. To satisfy condition (33), the linear loss γ should satisfy the condition

$$\gamma / \sigma_0 N_0 \ll \Delta\omega_{\mathrm{hom}}/\Delta\omega_{\mathrm{Dop}}, \tag{35}$$

where $\Delta\omega_{\mathrm{hom}} = 2T_2$ is the homogeneous width of the line, which is perfectly attainable in amplifying gas media.

If an inequality weaker than (35) holds, then a π-pulse is also produced. To be sure, in this case only particles within the limits of the line width are inverted, and the particles on the line wings are not inverted. This case was investigated by numerically solving the system (31). Figure 6 shows the results of the solution for the case

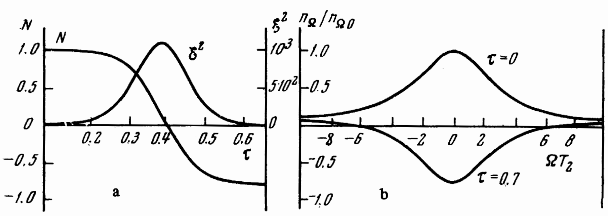

FIG. 6 Stationary π-pulse in an amplifying mediun with homogeneous line broadening ($\Delta\omega_{\mathrm{Dop}}T_2/2 = 0.3$) and with linear radiation loss ($a = 2.0$): a – pulse shape and change of the total inverted population; b – spectral distribution of inverted population before and after passage of the pulse.

$\alpha = 0.2$ and $\Delta\omega_{\mathrm{Dop}}T_2/2 = 0.3$, when the condition (35) is slightly violated. It follows from Fig. 6b that actually the particles on the line wings are not inverted.

5. CONCLUSIONS

The results obtained in the present paper greatly supplement the picture of the phenomena arising in the propagation of a powerful light pulse in a nonlinearly amplifying medium, given in[1,2,5-18]. In addition, the nonlinear amplification regime of ultrashort pulses is of practical interest, since it makes it possible to shape light pulses with a duration much shorter than the limit attained in the case of linear amplification. In practice this effect can be used, for example, for a pulse passing through an amplifying gas medium.

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