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EXPERIMENTAL INVESTIGATION OF INTENSITY FLUCTUATIONS OF A SINGLE-FREQUENCY GAS LASER OPERATING AT 3.39 μ

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The results of experimental investigation of the natural intensity fluctuations of a single-frequency gas laser operating at the wavelength of 3.39 μ are given. The main features of the intensity fluctuation spectrum are determined; in particular it is found that as the generation power increases, the fluctuation modulation index decreases while the spectral width increases, which is typical of self-oscillating systems. The experimental results are compared with the predictions of the phenomenological theory. The agreement between experiment and theory leads to the conclusion that the spontaneous emission of the active medium is the cause of the fluctuations under consideration. Some suggestions are offered for ways of producing laser signals with minimal level of natural intensity fluctuations. The obtained results are compared with those obtained in the investigation of fluctuations from other optical transitions.

1. INTRODUCTION

THE present work contains experimental results of investigating the spectrum of natural intensity fluctuations of a single-frequency gas laser emission at the wavelength of 3.39 μ.[1)] The literature reports on measurements of intensity fluctuations of He-Ne gas lasers, both natural and forced[1-4], carried out at the wavelengths of 0.63 and 1.15 μ. Such an investigation was not performed previously (judging from the literature) at the wavelength of 3.39 μ. This seems to be due to the specific features of lasers and receivers operating at this wavelength. We note that, apart from its intrinsic value, an investigation of fluctuations at this wavelength is of interest because of the sharp distinction of the corresponding transition, in terms of its amplifying and nonlinear properties, from other transitions of the He-Ne mixture. Our investigation was carried out at various laser powers, at the center of the transition line and away from it, at various discharge currents, and with tubes with different pressures of the gas discharge mixture.

The experimental results are compared with the theory of natural fluctuations developed in[5][2)]. According to the analysis presented in that paper, the main cause of fluctuations is spontaneous emission of the active medium. The most adequate quantity in characterizing laser intensity fluctuations is $\overline{M_F^2}$, or the spectral density of the laser intensity random modulation index at the observation frequency F (see[4]). According to[5] the expression for $\overline{M_F^2}$ has the form[3)]:

$$\overline{M^2_F} = \frac{\overline{M_0^2}}{1 + (2\pi F/p)^2} = \frac{8(\Delta\nu)^2 h\nu\tilde{\beta}\alpha\varkappa_2^0}{[1 + (2\pi F/p)^2](p/2\pi)^2 P\tilde{\varkappa}_2}, \quad (1)$$

where $\Delta\nu$ is the resonator bandwidth, $h\nu$ is photon energy, P is total generation power, p is the strength coefficient of the limit cycle, and α, $\tilde{\beta}$, and $\kappa_2^0/\tilde{\kappa}_2$ are coefficients characterizing the initial inversion, depletion of the upper atomic energy level, and the ratio of the imaginary dielectric susceptibility components of the

[1)] Intensity fluctuations due to various technical causes (acoustic and mechanical vibration, various instabilities of the gas discharge, etc.) are not considered here. Special measures were adopted to eliminate these instabilities.

[2)] Formulas cited from this paper are henceforth referenced by the letter A.

[3)] Since intensity fluctuations are the subject of discussion, the cited expression (1) differs by a factor of 4 from formula (38A) giving the spectral density of the amplitude random modulation index. Furthermore, compared with (38A), (1) lacks the term in figured brackets. According to estimates this term can be assumed equal to unity because the resonator band is less than the width of the transition line.

active medium in the absence and presence of oscillations.

We used the following approximation for $\tilde{\kappa}_2$

$$\tilde{\varkappa}_2 = \frac{\varkappa_2^0}{\sqrt{1+\gamma(\nu)P}} \qquad \varkappa_2^0 = -A\exp-\left(\frac{\nu-\nu_m}{0.6\Delta\nu_m}\right)^2 = -Ae^{-x^2}, \quad (2)$$

where $(\nu - \nu_m)/0.6\,\Delta\nu_m = x$ is the relative mismatch between the generation frequency and the mean frequency ν_m of the transition line, $\Delta\nu_m$ is the width of this line, and γ is a coefficient characterizing saturation in the medium, whose value is a function of $|\nu - \nu_m|$.[4)] Such an approximation was used in the literature (see[6]); our measurements proved its validity in our case and the experimental data yielded numerical values for A and γ.

We note that the expression (2) is exact only if the natural width of the emission line of a single atom (taking collisions into account) is much less than the Doppler line width $\Delta\nu_D$ of the transition. However, it can be shown that even when $\Delta\nu_N \leq \Delta\nu_D$ with mismatch values $x < 1$, (2) can be used with a corresponding value of $\Delta\nu_m$ ($>\Delta\nu_D$) which then becomes a function of the ratio $\Delta\nu_N/\Delta\nu_D$.

The value of κ_2^0 can be determined by measuring the gain G of the generator tube for the weak-signal case from the relation

$$\varkappa_2^0 = \frac{\lambda \ln G}{8\pi^2 l} \quad (3)$$

where l is the discharge length and λ is the wavelength. The measurements of G for various values of $\nu - \nu_m$ yield the value of $\Delta\nu_m$.

The experiments described below were performed pursuing two directions: on the one hand, research was carried on to investigate intensity fluctuations, and on the other, the dynamic parameters of (1) were measured and analyzed in order to provide a future basis for the comparison of experimental data with theoretical predictions.

1. APPARATUS AND METHODOLOGY OF FLUCTUATION MEASUREMENT

The measurement of intensity fluctuations was performed with a laser having discharge tubes equipped with Brewster windows. The tubes were filled with a Ne-He mixture in a 1:5.6 ratio under the pressure of 1.2 or 0.6 Torr. The discharge length was 16 cm and the distance between the laser mirrors was 32 cm. The required frequency stability was provided by a rigid construction of the resonator using two rigid flanges connected with four invar rods. Special measures were taken to eliminate air flow in the resonator. The laser was mounted on a massive steel plate.

The resonator was frequency tuned by supplying voltage to a piezoceramic cylinder supporting one of the resonator mirrors. The presence of a single-frequency regime was indicated by a scanning pattern, well-known in the literature, and displayed on the oscilloscope screen in response to an AC laser signal supplied to the oscilloscope by the photodiode. In the experiment the length of the resonator varied periodically (with a frequency of 200 Hz) by the amount of $\sim\lambda/2$ due to the AC voltage supplied to the piezocylinder. The same voltage was used to sweep the oscilloscope.

Since the gain of the active medium (discharge of the He-Ne mixture) is much larger (by two orders) at the wavelength of 3.39 μ than at other wavelengths, the specifications that must be met by the reflection coefficients of the mirrors and the Q-factor of the resonator are much less rigid than is the case with other wavelengths. In our experiments we used various mirrors, such as silvered surfaces with a reflection coefficient of ~95% and dielectric coatings (ZnS and LiF) with a varying number of layers having a reflection coefficient of 60–80%. Both flat mirrors and a combination of a flat and spherical mirror with a radius of curvature of 60 cm were used in the resonator.

The laser emission from one of the mirrors was focused by a lens on the photoreceiver. The DC component of the photoreceiver signal was measured by a V2-3 DC voltmeter. The alternating component was amplified within the band from 10 KHz to 12 MHz and fed to the SCh-8 osciloscopic spectrum analyzer. The noise spectra displayed on the analyzer screen were photographed and processed, taking into account the frequency characteristic of the entire receiver channel. The equipment was calibrated with respect to the thermal noise of a known resistance connected across the amplifier input. The quantity $\overline{M_F^2}$ was determined as a ratio of spectral density of the noise component to the squared DC component of the laser signal at the photoreceiver. The radiation receiver was an indium antimonide photodiode mounted in a dewar with liquid nitrogen. The dip in the diode frequency characteristic due only to the RC parameters occurred at the frequencies of 1–1.5 MHz. A photodiode correction circuit, shown in Fig. 1, was used to measure fluctuations in a broad frequency range up to 10–12 MHz. The circuit provided for the supply of a small (0.2–0.3 V) negative bias to the photodiode to compensate for the background illumination[5)] and to widen the linear region (in terms of power) of the photodiode.

We know that the photodiode frequency characteristic is determined by carrier inertia within the semiconductor and the p-n junction as well as by the RC parameters. To determine the resultant frequency character-

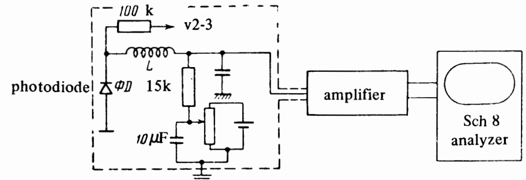

FIG. 1

[4)]The dependence of γ on frequency in our approximation for $\tilde{\kappa}_2$ is due to the fact that in the course of generation at and near the center the atoms interact with waves propagating in both directions, whereas in the case of a mismatch the atoms interact mainly with a wave traveling in one direction.

[5)]The illumination is due to the temperature difference between the ambient medium and the photoreceiver.

istic of the photodiode and the entire measurement channel as a whole we used the noise spectrum of spontaneous emission at the output of the high-gain (~ 100) 3.39 μ amplifier tube. Preliminary investigation of this spectrum by the method described in[7] showed that the spectrum can be considered uniform within the bandwidth of 30 MHz (with an error not exceeding 10%). This noise signal was delivered to the photodiode; the photodiode signal was amplified, fed to the spectrum analyzer, and photographed. Since the spectrum of the supplied optical signal was known, processing of the photographs yielded the frequency characteristics of the photodiode and the entire receiver channel as a whole. We found that the dip in the frequency characteristic is mainly determined by the RC parameters which, in turn, depend on the thermal illumination and can vary (up to 20%) with changing ambient temperature. They also depend on the nitrogen charging time. Reliable measurements require a stabilization time of about two hours. Power calibration of the photodiode was performed with a low optical power meter. The measurement gave the magnitude of the quantum yield of 0.44.

2. METHOD OF DETERMINING DYNAMIC PARAMETERS

In order to compare the values of $\overline{M_F^2}$ found from the experiment with the theoretical predictions we must determine a number of quantities entering in (1). The values of some quantities were taken from the literature, while experiments and special measurements were performed to determine the others.

To determine the total power P generated by the laser we used the obvious relation $P/P_T = \Delta\nu/\Delta\nu_T$ where $\Delta\nu$ is the bandwidth related to all the losses in the resonator, while $\Delta\nu_T$ is that portion of the bandwidth which is due to the transmission losses of the measured power P_T. As we know, the bandwidth $\Delta\nu_T$ is related to the mirror transparency T by:

$$\Delta\nu_T = \frac{c}{2\pi L}\ln\frac{1}{(1-T)^{1/2}} \cong \frac{cT}{4\pi L}$$

where L is the length of the resonator.

One of the methods of determining the resonator bandwidth $\Delta\nu$ is a direct measurement of it according to a method described in[8], for example, to measure Q at 0.63 μ. However with our substantially large values of $\Delta\nu$ it is difficult, when exciting the resonator, to avoid the appearance of close transverse modes ($\Delta\nu$ is comparable to the distance between the longitudinal and the neighboring transverse modes) that can cause a significant error in the determination of $\Delta\nu$.

To determine $\Delta\nu$ we can also use (19A), (37A), and (3) and thus obtain the following expression:

$$\Delta\nu = \frac{c\ln G\exp(-x_p^2)}{2\pi l}, \qquad (4)$$

where $x_p = (\nu_p - \nu_m)/0.6\,\nu_m$ is the relative mismatch that causes the termination of generation, and l is the length of the active medium (length of discharge). The relation (4) means that the generation threshold occurs when gain compensates for the losses in the resonator. It follows from (4) that to determine $\Delta\nu$ we must also measure $\Delta\nu_m$. The results of corresponding measurements of $\Delta\nu_m$ are given below.

The bandwidth of the resonator was determined by both methods described above. The results are in agreement within the limits of measurement error (10—15%).

The quantity $p/2\pi$, or the strength coefficient of the limit cycle, can be determined by various methods. One consists of determining p from the fluctuation spectrum since the observation frequency F at which $\overline{M_F^2}$ drops to half the value of M_0^2 equals $p/2\pi$. The value of p can be computed from (24A) and (2):

$$p = \pi\Delta\nu\frac{\gamma(\nu)P}{1+\gamma(\nu)P} \qquad (5)$$

In addition to the resonator bandwidth, this expression contains the coefficient of nonlinearity γ which should be either computed or measured.

To determine γ we plotted the frequency dependence of laser power by tuning the resonator without changing Q or the generation region (Fig. 2). Using the accepted approximation for $\tilde{\kappa}_2(2)$ and also (18A), (19A), and (37A) we obtain

$$\gamma = \frac{1}{P}\left[\left(\frac{e^{-x^2}}{e^{-x_p^2}} - 1\right)\right]. \qquad (6)$$

We note that the same relations yield for A

$$A = \frac{L}{l}\frac{\Delta\nu}{4\pi\nu_0}\exp(-x_p^2). \qquad (7)$$

From (6) we see that the values of γ and consequently of p are very sensitive to $\Delta\nu_m$. (A 10% change in $\Delta\nu_m$ produces approximately a 25% change in γ). The values of $\gamma(0)$ and A computed for various generation "regions" coincide within the limits of experimental error. This is a further confirmation of the correct selection of the approximation of $\tilde{\kappa}_2$.

To determine the value of $\Delta\nu_m$ we performed experiments according to the method described in[9]. These experiments showed that $\Delta\nu_m$ differs for tubes with different pressure; tubes with larger pressure yielded larger $\Delta\nu_m$. This could have been expected since $\Delta\nu_N$ grows with increasing pressure due to the increased number of collisions. According to measurements a rising discharge current also results in some increase of $\Delta\nu_m$. Insofar as the tubes were filled with a natural mixture of Ne^{20} and Ne^{22}, their gain curves were asymmetric because of the superposition of two gain curves. Consequently the accuracy of measurement of $\Delta\nu_m$ was low in the described experiments. The results of measuring $\Delta\nu_m$ are given in the table for various currents, tubes, and computed values of γ. According to (6) the values of γ decrease with increasing $\Delta\nu_m$.

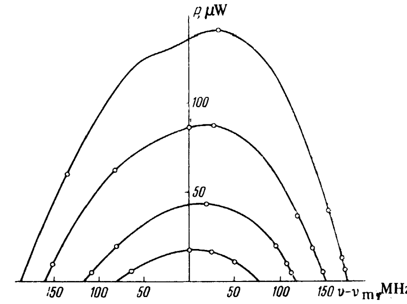

FIG. 2

The value of p can also be measured by the modulation method (this method was used by Bolwijn[10] to determine the dynamic parameters). Let the discharge current I in the tube be modulated by a low frequency (~ 1 kHz) with a degree of modulation m_1. This causes modulation of gain G and dielectric susceptibility $\widetilde{\kappa}_2$, and hence the modulation of laser emission to a degree m_e. Using (22A), (24A), and (2) we can after simple transformation obtain an expression of the limit cycle strength

$$p = \frac{2\pi m_I I \Delta\nu dG/dI}{m_1 \ln G}.$$

The values of p obtained by the three above methods for a tube under the pressure of 0.6 Torr in generation at the transition line center for various powers are given in Fig. 3 (curve 1). The results obtained by the modulation method are indicated by triangles, computed results by crosses, and data obtained from the drop of fluctuation spectrum are marked by circles. According to the diagram all three methods yield values of p that coincide within the limits of experimental error. We note that the data represented by points correspond to the discharge current of 5 mA; data represented by squares (curve 2) correspond to fringe generation (mismatch). These results are discussed below.

The parameter α was not measured directly; it was computed instead according to the method described by Bershteĭn[11] and the experimental results of White and Gordon[12] for a wavelength of 3.39 μ. The computations were performed for three currents (5, 20, and 50 Ma) yielding values for α of 1.14, 1.28, and 1.35 respectively. Thus in our case we can set $\alpha \cong 1.3$.

Since we operated in a single-frequency regime at low power (up to 150 μW) the depletion of the upper level was negligible. Therefore we set the value of 0.9 for $\widetilde{\beta}$.

Tube pressure, Torr	Discharge current, mA	$\Delta\nu_m$, MHz	γ, mW^{-1}
1.2	5	550	15
	20	800	3
0.6	5	360	20
	20	500	10

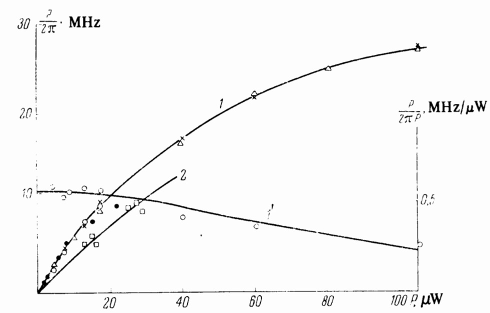

FIG. 3

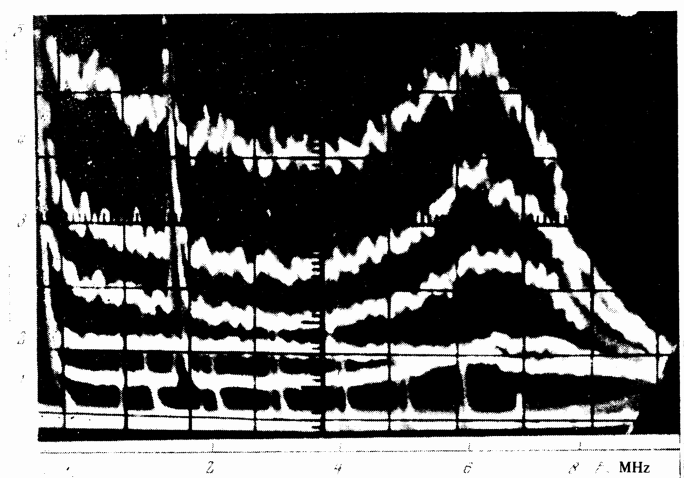

FIG. 4

3. RESULTS OF INTENSITY FLUCTUATION MEASUREMENT

The spectral density of the laser intensity fluctuation was measured at the observation frequencies F from 10 kHz to 12 MHz and powers from 0.5 to 100 μW. The power was varied both by changing the beam aperture and by using mirrors of different transparency. The mixtures were used at pressures of 1.2 and 0.6 Torr for discharge currents of 5–25 mA. The fluctuation spectra were recorded both in generation at the center of the transition line and in generation away from the center. The conditions of the measurements are marked by points in Fig. 2. The experimental results were obtained by processing the photographs of spectra displayed on the screen of the SCh-8 oscilloscopic analyzer. As an illustration, Fig. 4 shows a photograph of noise spectra; curve 1 represents the instrument background noise in the absence of generation, curve 2 depicts the noise of the 1.5 K resistor connected across the input of the amplifier channel for the purpose of calibration, and curves 3, 4, and 5 represent noise due to the photodiode illumination by laser emission at various generation powers[6]. The amplitude characteristic of the spectrum analyzer was taken into account in analyzing the data based on these photographs.

We note that we could neglect the shot noise of the photodiode in our experiments. This was verified by illuminating the photoreceiver with white light of such an intensity as to render the DC component of the photovoltage equal to that in the laser signal.

The main results of the investigation are presented in the form of graphs shown in Figs. 3, 5, and 6. Crosses represent results of computation based on (1) and (5), and the remaining symbols (points, circles, triangles, etc.) refer to experimental data. The following relationships were studied.

1. Spectral density $\overline{M_F^2}$ as a function of observation frequency F. This dependence is shown in Fig. 5 for a tube at a pressure of 1.2 Torr and a discharge current of 20 mA. Curves 1–6 were plotted for powers of 2.6, 4.6, 9, 17, 22.5, and 60 μW respectively. The curves

[6] A burst of external electrical interference appears near 2 MHz. Timing marks are also visible every 1 MHz.

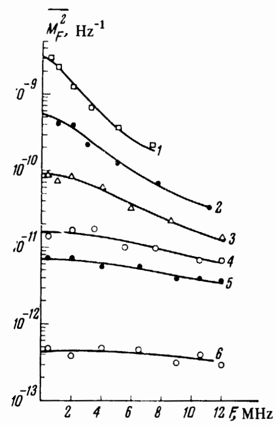

FIG. 5

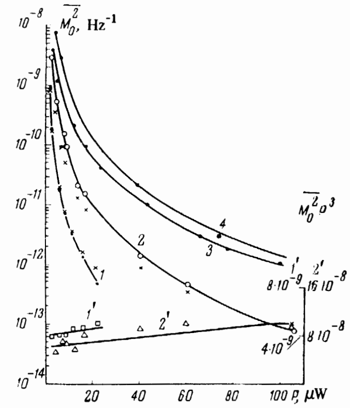

FIG. 6

contain a level section (plateau) and a drop toward high frequencies[7]. The fluctuation level in the plateau region decreases with increasing power and the spectral width characterized by p increases. These relations typical of a self-oscillating system are in full agreement with (1). We note that we were not able to measure p from the fluctuation spectrum when the generation power exceeded 20 μW since the equipment allowed for spectrum measurements only up to 12 MHz and the spectral dip in the indicated power range lies above 12 MHz.

2. Figures 3 and 6 show $\overline{M_0^2}$ and $p/2\pi$ as functions of generation power in operation at the center of the transition curve. Curves 1 and 2 in Fig. 6 were plotted at a tube pressure of 0.6 Torr for discharge currents of 5 and 20 ma respectively. Curve 3 was plotted at a tube pressure of 1.2 Torr and discharge current of 20 ma. An analysis of these diagrams shows that the values of p/P and $\overline{M_0^2}P^3$ (curve 1′ Fig. 3 and curves 1′ and 2′, Fig. 6) remain constant below 20 μW, i.e., the strength of the limit cycle increases in proportion to power while the fluctuation level drops in an inverse proportion to P^3. As power increases, p/P drops and $\overline{M_0^2}P^3$ grows. According to (5) the drop of p/P is due to the fact that generation power increases at the expense of decreasing resonator losses, i.e., decreasing $\Delta\nu$, on the one hand, and to the fact that increasing P is accompanied by an increase in the term $1 + \gamma P$, on the other. It is apparent from (1) that according to (5) the significant rise in $\overline{M_0^2}P^3$ occurs when power is comparable with $1/\gamma$.

The computation results shown in Figs. 3 and 6 (crosses) do not deviate from the experimental data by more than a factor of 2. Considering the low accuracy by the measurement of a number of parameters we can regard this deviation as within the limits of experimental error.

3. The dependences of $p/2\pi$ and $\overline{M_0^2}$ plotted by shifting the frequency within the bandwidth are shown in Fig. 3 (squares) and in Fig. 6, curve 4 (tube with 1.2 Torr pressure, current 20 mA). According to the diagrams the values of $p/2\pi$ are somewhat lower and those of $\overline{M_0^2}$ are somewhat higher in the detuned operation than when generation proceeds at the center of the transition line, for the same generation power. The larger the detuning from the center of the transition line the greater the drop of p and consequently the rise in fluctuation. This is mainly due to a reduction of γ during detuning.

4. The comparison of experimental data obtained in the measurement of the fluctuation spectrum in tubes with different pressures and different discharge currents shows the same nature of the dependence of the spectral noise density on the observation frequency, generation power, and detuning within the bandwidth. The numerical values of p and $\overline{M_0^2}$ are different however. Figure 6 shows the experimental results for tubes with various pressures and various discharge currents (curves 1 and 2 were plotted at 0.6 torr and discharge currents of 5 and 20 mA respectively; curve 3 was plotted at 1.2 Torr and 20 mA). According to (1) and (5) this difference is due to two causes. The first cause is due to the change of the tube gain that results from a change in the operating regime of the tube (pressure or current); consequently to maintain constant generation power one should vary the losses, i.e., the resonator bandwidth. The second cause consists in the fact that changes in current and pressure vary the value of $\Delta\nu_m$, and consequently the value of γ (see the table). According to the experiments a four-fold increase of current with constant generation power leaves p almost unchanged (Fig. 3, points at 5 ma and circles at 20 mA), because an increase in $\Delta\nu$ is compensated for by a reduction in γ, while $\overline{M_0^2}$ increases by an order of magnitude (Fig. 6, curves 1 and 2). A doubling of pressure also results in a negligible change in p while $\overline{M_0^2}$ increases by an order of magnitude (Fig. 6, curves 1 and 2). We may conclude from the above that to achieve laser operation with the lowest possible natural intensity fluctuation level for a given generation power it is desirable to use tubes with the lowest possible pressures and low discharge currents.

In the measurements of fluctuations we noted certain deviations of the experimental conditions from the ideal conditions postulated by the theory. In particular, the large losses in the resonator caused a deviation from the standing-wave regime; the presence of a diaphragm at one of the mirrors of the resonator produced a marked divergence of the beam in the tube, etc. Furthermore, the parameters in (1), such as $\tilde{\beta}$ or α, were estimated fairly approximately and the measurements of

[7] We note that the fluctuation spectrum rises in the low frequency region, beginning with 2 – 5 kHz, due to various technical reasons.

such parameters as $\Delta\nu$, p, and γ were performed with an accuracy not exceeding 10–20%. Nevertheless in spite of these limitations the performed comparison of experimental results with theory shows that we achieved not only a qualitative but also a satisfactory quantitative agreement. This permits us to draw the conclusion that the spontaneous emission of the active medium is the cause of the natural intensity fluctuations.

The analysis of (1) for $\overline{M_F^2}$, taking into account (5) for $p/2\pi$, shows that $\overline{M_F^2}$ depends significantly on γ. The magnitude of γ is in turn approximately proportional to λ^3, according to[13], where λ is the operating wavelength. This results in a significant difference between the fluctuation spectra parameters (p and $\overline{M_0^2}$) for various wavelengths at the same generation power. The corresponding comparison for lasers with the wavelengths of 3.39 and 0.63 μ is given in[14].

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