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PROPAGATION OF FOURTH SOUND IN SUPERFLUID He^3–He^4 *SOLUTIONS AT TEMPERATURES DOWN TO 0.5°K.*

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Measurements have been made of the speed of fourth sound in superfluid He^3–He^4 solutions with He^3 content up to 50% and in the temperature range from 0.5°K to the λ points of the solutions. The data obtained were used to determine the density of the normal component of the solutions, which in turn made it possible to clarify certain features of the energy spectrum of the solutions, Bose as well as those with impurity branches.

IN superfluid He^4 and He^3–He^4 solutions, propagation of sound oscillations called fourth sound is possible for full retardation of the normal component. This has been shown theoretically[1-3] and experimentally.[4,5] The expression for the speed of fourth sound u_4 is obtained from the system of hydrodynamic equations in which the assumption of Landau and Pomeranchuk[6], as to the participation of He^3 only in the normal motion, is used for solutions. In this case,[3-5]

$$u_4^2 = \frac{\rho_s}{\rho} u_1^2 \left(1 + \frac{c}{\rho} \frac{\partial \rho;}{\partial c} \right)^2 \left[1 + \frac{\rho_s}{\rho_n} \left(\frac{c}{\rho} \frac{\partial \rho}{\partial c} \right)^2 \right]^{-1} + \frac{\rho_n}{\rho} u_2^2 \left[1 + \frac{\rho_s}{\rho_n} \left(\frac{c}{\rho} \frac{\partial \rho}{\partial c} \right)^2 \right], \tag{1}$$

where u_1 and u_2 are the speeds of first and second sound in the solution, $\rho = \rho_s + \rho_n$ is the total density of the solution, and c is the weight concentration of He^3.

The speed of fourth sound in superfluid solutions has been measured previously[5] at temperatures above 1.5°K and for He^3 concentrations less than 20%. Inasmuch as there were independent measurements of all quantities entering on the right side of Eq. (1) in this region, the values obtained for u_4 could be used for a test of the validity of the formula (1). As was shown in[5], Eq. (1) is satisfied within the limits of experimental error in the studied range of solutions.

In the experiments described below, the measurements of the speed of fourth sound were extended to the region of lower temperatures (down to ~0.5°K) and much higher concentrations (up to ~50% He^3).

DESCRIPTION OF APPARATUS AND TECHNIQUE FOR MAKING MEASUREMENTS

1. **Low-temperature part.** The experiments were conducted in a cryostat in which the low temperature was obtained by pumping off vapor over liquid He^3. The He^3 was condensed in the upper part of the copper block 3 (Fig. 1), in which about 150 holes of diameter 1.5 mm were drilled. Twelve normal liters of He^3 were used, and about half of the liquid was located in the holes. On the one hand, this increased the area of thermal contact between the copper and He^3, and on the other hand, it decreased the temperature gradient along the vertical height of the He^3 column. Such a construction of the low-temperature chamber guaranteed a greater cold production in the initial lowering of the temperature from 1.5 to 0.5°K (because of the greater evaporation surface). According to estimates, half of all the He^3 was expended in this process. A smaller cold production is required subsequently, needed only to compensate for the heat intake to the chamber. To maintain the low temperature for a period of about three hours, that part of the He^3 which remained in the holes was sufficient.

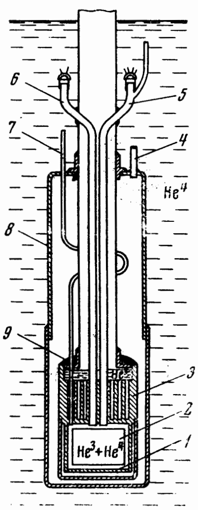

FIG. 1. Low-temperature cell: 1–cap, 2–sound cell, 3– housing of block, 4–tube for outpumping vacuum jacket, 5,6–tube for introducing conductors and letting in mixture, 7–tube leading to McLeod gauge, 8–vacuum jacket, 9–carbon.

The lower part of the block 3, together with the lid 1, formed a volume in which the sound cell 2 was placed and in which the solution under study was condensed. The solution was admitted through a thin-walled German silver tube 5, which served as the conductor for supplying the radiator. The conductor from the detector was located in another such tube 6. Both tubes and all the leads from the sound cell were brought out first into the external helium bath and then to room temperature.

The entire low-temperature chamber was separated from the outer He^4 bath by the vacuum jacket 8, in which gaseous He^4 was admitted at room temperature through the tube 4 up to a pressure of 7–10 torr. When the entire system was cooled to helium temperatures, this heat-exchange gas was adsorbed by a small quantity of carbon 9 specially placed in the jacket. A high vacuum was achieved here, so that the heat intake because of the thermal conduction of the gaseous helium was negligibly small.

Measurement of the temperature was carried out from the vapor pressure of the He^3 (T_{62} scale). A McLeod gauge was used, which was connected with the volume of He^3 by a German silver tube 7. The temperature was determined with account of the thermomolecular correction.[7] Moreover, a carbon resistance thermometer was placed in the sound cell itself; this served as a control of the temperature stability.

The pumping out of the He^3 was carried out by means of an adsorption pump, which was placed in another dewar and which was cooled by liquid helium to a temperature of 4.2°K. Palladium-coated silica gel in the amount of 140 g was used as the adsorbent. In comparison with the carbon, this had a larger adsorption capability (per unit of dry material) and cooled more rapidly.[8]

The pump was connected with the cooled volume by a supply line which at room temperature had a diameter of 30 mm and at low temperature a diameter of 20 mm; the total length of the line was about 70 cm.

The rate of pumping was regulated by two valves with passing cross sections of 20 and 4 mm, respectively; these were connected in parallel and were at room temperature. Such a system made it relatively simple to maintain the necessary temperature. With the small valve completely open, a temperature of ~0.7°K was achieved, and complete opening of the larger valve produced ~0.45°K. The time of achieving the desired temperature was about 30—60 min, and the accuracy of maintaining the temperature was ±0.003°K.

The experiments were carried out in solutions containing 8.5, 15.6, 21.7, 33.2 and 46.4% He^3 (weight concentrations are given), which were prepared by mixing of pure gaseous He^3 and He^4 in a special apparatus. The error in the determination of the concentration did not exceed 0.1% He^3.

2. **Measuring system.** The speed of fourth sound was determined by means of the method described previously.[5] Capacitive pickups served as radiator and detector of the sound; here the movable electrode was a film of aluminized polyester 14 microns thick. The sound was propagated along a system of thin, branched channels, formed by finely powdered rouge and filled with helium. In this way, practically complete stoppage of the normal component was achieved, which is necessary for the realization of fourth sound. (According to estimates, the size of the channels was less than 0.5 micron, which is much less than the penetration depth of the viscous wave.) A sound cell was used with an internal diameter of 18 mm, and the separation between radiator and detector was 12 mm.

Measurement of the speed of sound was carried out by the pulse method: video pulses of length 2 μsec, amplitude ~400 V, and repetition frequency ~200/sec were applied to the radiator. The frequency of oscillations propagated in the liquid depends on the properties of the film, and the maximum contribution was made by frequencies around 30 kHz. The time of passage of the pulse was determined by means of a short-time-interval recorder I2-9A.

3. **Account of multiple sound scattering in the filter.** In the propagation of sound in a system of branched channels, the acoustic path is not identical with the distance between the receiver and the source because of multiple scattering of sound by the powder particles. Therefore, it is necessary, in calculating the sound speed, to introduce a correction for this effect. The magnitude of this correction and the effect on it of different factors were investigated in the research of Shapiro and Rudnick,[4] where a number of empirical relations were obtained. Usually, the correction is determined by reconciling the measured values of the speed for a fixed temperature to the theoretical curve for the speed of fourth sound. In experiments with solutions,[5] the correction was found by means of calibration measurements in He^4.

A somewhat different method of determination of the correction for multiple scattering is possible, because of the fact that the experimental data are obtained in a region of sufficiently low temperatures (~0.5°K), where the speed of fourth sound for He^4 differs from the speed of first sound by no more than $\sim 10^{-3}\%$. Therefore the correction was so chosen that the value of u_4 at 0.5°K would be identical with the data of Whitney and Chase[9] for the speed of first sound in He^4, i.e., so as to use the weaker requirement $u_4 \rightarrow u_1$ as $T \rightarrow 0$ for He^4. The correction for the filter used was turned out to equal 1.15. All the measurements of u_4 in solutions were made on this filter.

4. **Experimental procedure and discussion of errors.** The solution under study was condensed in the volume with the sound cell (Fig. 1), after which the measurements were carried out for a constant amount of solution. The volume of the vapor phase amounted to only a small part of the total volume of the solution; therefore, the change of concentration in the liquid connected with this was negligible. The speed of sound was measured for constant temperature, and to obtain equilibrium conditions, an isothermal run of 30—40 min was used. The speed of fourth sound was determined from the time of passage by the pulse of a given path with account of corrections for multiple scattering of sound on the particles of the filter. The principal error in the time determination depended on the accuracy of the determination of the beginning of the previous pulse. The error in the determination of the speed of fourth sound in the solutions associated with this increased with increase of the concentration and temperature, and amounted to 0.3—0.5%. An exception was the solution containing 46.4% He^3, for which we succeeded in carrying out only very rough measurements. The error in the measurement of the temperature was equal to 0.003°K.

2. RESULTS

1. **Speed of fourth sound.** The temperature dependence of the speed of fourth sound in He^4 and in He^3—He^4 solutions is shown in Fig. 2. As is shown in the drawing, all the curves have the same character—the speed decreases monotonically as the temperature increases. Inasmuch as it was not possible, because of strong absorption, to carry out measurements very close to the λ point, the curves in this temperature region were extrapolated to zero values at the λ points, in accordance with theory (dashed portions of the curves). The ex-

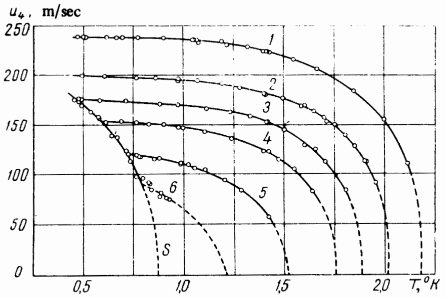

FIG. 2. Temperature dependence of the speed of fourth sound in He^4 (curve 1) and in He^3-He^4 solutions with He^3 weight content: 8.5% (curve 2), 15.6% (curve 3), 21.7% (curve 4), 33.2% (curve 5), 46.4% (curve 6), S—line of stratification of solutions.

perimental values of the speed of fourth sound are given in the table.

At low temperatures, the $u_4(T)$ dependences for concentrated solutions have kinks connected with the stratification of the solutions (curve S). For measurements in this region, such a quantity of the solution was condensed in the sound cell that the boundary between the two phases that were formed passed above the receiver and radiator, and the fourth sound in this case was recorded in the lower superfluid phase. The temperatures of the stratification, determined from measurements of u_4, agree well (with an error of 0.003—0.005°K) with data obtained previously.[10]

For solutions, in contrast with He^4, the speed of fourth sound does not approach the velocity of first sound as $T \rightarrow 0$. This is connected with two circumstances. First, impurity excitations make a significant contribution to the density of the normal component of the solution, so that the density $\rho_n \neq 0$ for $T = 0$. Second, significant oscillations of the concentration in the wave of fourth sound in solutions lead to the result that expression (1) for u_4 depends explicitly on the concentration.

In research on concentrated solutions, the concentration was monitored during the experiment, and this has shown that the solution in the channels of the filter has the same concentration as in the remaining liquid. This is confirmed by the following circumstances:

a) The measured values of the vapor pressure of the solution in the sound cell are always the same as the equilibrium values of the saturated vapors for the given concentration. Inasmuch as about half the liquid is located in the pores of the filter, a change in the content of He^3 in the channels by even 0.3—0.5% should lead to such a change in its concentration in the remaining solution as would easily have been established by the vapor pressure.

b) In that region of concentrations and temperatures where there are independent measurements of u_1, u_2, ρ_n and ρ, the experimental values of u_4 agree with the theoretical values within the limits of experimental error.

c) The stratification curve of the solutions, measured by the speed of fourth sound, is in excellent agreement with the data in the literature.

d) If measurements are made during heating and during cooling of the solution (with corresponding isothermal measurements), hysteresis phenomena are entirely absent.

Speed of Fourth Sound in Superfluid He^3—He^4 Solutions

T, °K	u_4, m/sec	T, °K	u_1, m/sec	T, °K	u_4, m/sec
He^4		8.5 wt. % He^3		15.6 wt. % He^3	
0.469	238.3	0.488	199.1	0.474	176.1
0.494	238.2	0.607	198.6	0.485	175.3
0.693	238.2	0.760	197.1	0.582	174.1
0.770	237.6	0.861	196.6	0.710	173.8
0.900	236.9	0.974	194.9	0.880	170.5
1.059	235.3	1.072	194.0	0.971	169.4
1.073	233.1	1.190	191.1	1.113	165.7
1.190	232.6	1.280	186.5	1.230	162.2
1.216	229.8	1.405	180.3	1.332	157.8
1.332	227.9	1.410	180.0	1.413	151.9
1.400	223.0	1.495	176.1	1.416	151.9
1.412	224.2	1.585	167.6	1.502	144.0
1.425	223.8	1.600	168.5	1.600	128.5
1.538	217.7	1.650	163.0	1.654	120.7
1.550	214.7	1.700	156.4	1.700	111.7
1.667	204.5	1.712	151.8	1.800	83.0
1.848	182.2	1.750	149.1		
1.997	153.6	1.800	139.6	46.4 wt. % He^3	
2.109	109.7	1.850	128.0	0.823	91.5
		1.900	112.7	0.825	90.8
21.7 wt. % He^3		1.905	111.0	0.834	84.8
0.576	156.4	1.950	90.1	0.877	78.1
0.608	152.8			0.883	81.5
0.657	153.6	33.2 wt. % He^3		0.919	76.3
0.762	151.9	0.719	119.1	0.924	75.1
0.825	150.3	0.761	119.1		
0.902	148.8	0.771	118.1		
0.975	147.0	0.850	115.2		
0.993	146.4	0.874	116.2		
1.103	142.5	0.979	110.6		
1.239	135.5	0.981	111.3		
1.401	123.3	1.021	108.8		
1.419	122.8	1.052	105.5		
1.471	114.3	1.106	103.5		
1.547	103.1	1.200	94.0		
1.637	81.3	1.285	83.3		
		1.420	56.2		

2. **Density of the normal component.** Measurements of the speed of fourth sound made it possible, with the aid of Eq. (1), to compute the value of ρ_n/ρ in that region of temperatures and concentrations where such data have been lacking. The values of u_1, u_2 and ρ entering into (1) were taken from[11-14].

The error in the calculated values of ρ_n/ρ is due to errors in the determination of u_1, u_2 and, in significant measure, to the error in u_4. According to the estimates for He^4, such an error amounts to ~50% at 0.7°K and ~20% at 1.0°K. For solutions, the errors are much smaller: for very low temperature, for a given solution, where the error is maximal, it amounts to ~7% for the 8.5% solution and ~4% for the 2.7% solution; the error decreases with increase in temperature.

The values of ρ_n/ρ calculated in this manner are shown in Fig. 3. For comparison, the values of ρ_n/ρ for He^4 calculated on the basis of the exact form of the energy spectrum of He II[19] are also given; the numerous experimental data of different authors (not shown in the drawing, in order not to obscure things) are distributed close to the line with a scatter of ±8%. For weak solutions, the results of[15-16] are also given; in these the density of the normal component was measured by means of a stack of disks. The results of the present research are in satisfactory agreement with these data. Thus the speed of fourth sound represents one more method of determination of the density of the normal

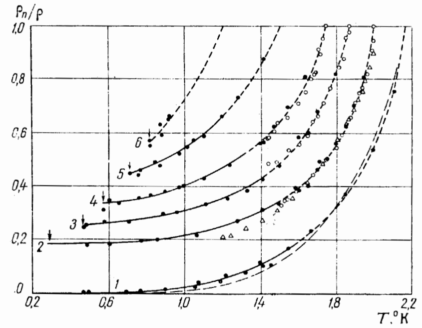

FIG. 3. Density of the normal component, determined from data on u_4. Curve 1—pure He^4, curve 2—8.5% He^3m, 3—15.6% He^3, 4—2.7% He^3, 5—33.2% He^3, 6—46.4% He^3. The solid lines correspond to values obeying Eq. (10) with values of Δ, b and ρ_{ni}, given in Figs. 5–6. The dashed curve is drawn through the experimental points, the shaded curve is computed according to [19]; ●—present research. ○—[15]; △—[16].

component.

As is seen from Fig. 3, the density of the normal component of all the solutions studied falls off with decrease in temperature, approaching a finite value. For concentrated solutions, the ρ_n/ρ data are given up to the stratification temperatures, which are indicated by arrows in the drawing. No singularities associated with the Fermi degeneracy of the solution are noticed on the curves, although the measurements were carried out below the degeneration temperature for a majority of the solutions investigated.

Another question which needs special discussion is associated with the fact that in very narrow channels, the density of the superfluid component and the temperature of the λ transition decrease in comparison with the macroscopic liquid.[17,18] Attention was turned to this in a previous work,[5] where there were independent data on ρ_n/ρ in the temperature range 1.6°K—T_λ. After substitution in Eq. (1) of the value of ρ_n/ρ for a macroscopic volume of liquid, excellent agreement was obtained of the calculated values of u_4 and the experimental measurements, which testified to the fact that the effect shown above is absent in the channels used. The presence of the effect would have led to a significant decrease in the experimental values of u_4 in comparison with the theoretical, especially near the λ point, i.e., precisely in the investigated region of temperatures, which in fact does not take place.

3. ESTIMATE OF THE PARAMETERS OF THE ENERGY SPECTRUM OF THE SOLUTIONS

1. Fundamental assumptions and treatment of data. The values obtained for the density of the normal component can be used for the clarification of some information on the spectrum of elementary excitations of solutions.

Inasmuch as such solutions consist of atoms that obey two different statistics, two branches arise in the energy spectrum of solutions—the Bose statistics, which are obeyed by He^4 and the Fermi or impurity statistics. The simplest description which takes into account the interaction of particles is that any weakly excited state of the system as a whole is considered as a set of noninteracting elementary excitations, i.e., as an ideal gas of quasiparticles. Inasmuch as any sort of model representations describing the elementary excitations in He^3—He^4 solutions of arbitrary concentrations are lacking, we shall start out from the following assumptions in further treatment of the data. We shall assume that the Bose branch of the energy spectrum is similar to the He^4 spectrum, i.e., that there is also a roton minimum, but its parameters depend on the concentration of He^3.

For solutions, as estimates show, the contributions of rotons and phonons are comparable in the range 0.2—0.3°K, and therefore in the studied temperature region (T > 0.5°K) the contribution of the phonons can be neglected. In this case, the density of the normal component, which is governed by the Bose branch of the spectrum, is equal to

$$\rho_{nr} = \frac{b}{\sqrt{T}} e^{-\Delta T}, \tag{2}$$

where

$$b = \frac{2}{3} \frac{(m_4^*)^{1/2} p_0^4}{(2\pi)^{3/2} \hbar^3}, \tag{3}$$

and m_4^*, p_0 and Δ are spectrum parameters that depend on the concentration. One must also take another fact into account. At certain temperatures, there are sufficiently many rotons that they begin to interact with one another, and the quasiparticle description becomes incorrect. For He^4 this takes place in the temperature range where the energy gap Δ depends on temperature, i.e., for T > 1.7°K. It is natural to suppose that this phenomenon will take place in solutions also, and it is connected with the closeness of the λ point, i.e., it depends on the ratio T/T_λ. In this connection, we shall carry out treatment of the data only for temperatures below some value T_{1c}, which is defined by the condition

$$\frac{T_{1c}}{T_{\lambda c}} = \frac{T_1(He^4)}{T_\lambda(He^4)} = \frac{1.7}{2.17} \approx 0.8. \tag{4}$$

The possible form of the impurity (Fermi) branch of the spectrum was first discussed by Landau and Pomeranchuk,[6] who pointed out two possible types:

$$\varepsilon = \varepsilon_0 + p^2 / 2m_3^* \tag{5}$$

and

$$\varepsilon = \varepsilon_0 + (p - p_1)^2 / 2m_3^*. \tag{6}$$

Here m_3^* is the effective mass of the impurity excitations, p_1 is some constant.

Somewhat later, Pomeranchuk[21] calculated the contribution of the impurity excitations to the density of the normal component, limiting himself to the classical region only. It was shown that for the energy spectrum (5) the density of the normal component is determined by the impurities,

$$\rho_{ni} = m_3^* N_3, \tag{7}$$

where N_3 is the number of He^3 atoms per cm^3, and for the spectrum (6).

$$\rho_{ni} = \frac{p_1^2}{3T} N_3. \tag{8}$$

(The latter expression is valid for $p_1^2 \gg 2m_3^* T$.)

On the basis of the analysis of the experimental values of ρ_n for weak solutions obtained previously,[22,23] a choice was made between the two possible types of spectrum in favor of the spectrum (5), inasmuch as no increase in ρ_{ni} was observed in the lowering of the temperature. This circumstance was also confirmed by the results of the present research (Fig. 3), where the monotonic decrease of ρ_n for decrease in temperature takes place also in the region of lower temperatures and higher concentrations.

Very recently, additional considerations have appeared in support of the idea that the quadratic spectrum of the type (5) can be used with excellent approximation. Slyusarev and Strzhemechnyĭ,[24] using the energy gap of weak solutions of He^3 in He^4, made it clear that the impurity spectrum becomes essentially nonquadratic in the momentum region $p \gtrsim 2.5$ Å^{-1}. At temperatures of the Fermi degeneracy (and such temperatures were achieved in the experiments described) only those excitations are important that have momenta of the order of the Fermi momentum. The limiting Fermi momentum is determined by the expression $p_F = (3\pi^2\rho c/m_3^*)^{1/3}$ and for solutions with a content of $\sim 40\%$ He^3, $p_F \approx 0.6$ Å^{-1}, which permits the use of the quadratic dispersion law. We note in passing that the value of the effective mass of the impurities obtained in this research was $m_3^* = 2.4m_3$, which is very close to the experimental values of m_3^* for weak solutions.

In spite of the favorable estimates, it is nevertheless not possible to consider the discussion that has been given as a rigorous proof of the quadratic character of the impurity spectrum, inasmuch as the theoretical consideration was limited to small concentrations of He^3 and was made within the framework of perturbation theory, although the interaction between He^3 and He^4 is not very small, as is noted in[24]. However, one can use the quadratic spectrum of type (5) as a first approximation.

For such a spectrum, as has already been noted in[15], there is one circumstance which greatly simplifies the treatment of the experimental data. That is, both in the classical and in the quantum regions, for solutions of arbitrary concentration,

$$\rho_{ni} = m_3^* N_3 = \rho c m_3^* / m_3. \tag{9}$$

Thus, we write the total density of the normal component of the solutions in the form

$$\rho_n = \rho_{ni} + \frac{b}{\sqrt{T}} e^{-\Delta/T}. \tag{10}$$

Further treatment of the experimental data was carried out by means of a high speed computer.[1] The three parameters ρ_{ni}, b, and Δ entering in Eq. (10) were determined in such a way that the mean square deviation of the experimental values of ρ_n from the curve described by Eq. (10) with the given parameters was minimal. This program was carried out in the following fashion. By giving successively the values of Δ in the interval 2–11°K in steps of 0.2°K, those values of the two remaining parameters were found which would give the minimum mean square deviation $\delta = [\overline{(\delta\rho_n/\rho)^2}]^{1/2}$ for the given Δ. The dependence of δ on Δ found in this way is plotted in Fig. 4 in the case of solutions with He^3 content of 8.5% and 21.7%. The minimum on this curve is the absolute minimum of the mean square deviation of ρ_n/ρ, while the value of Δ corresponding to this minimum best describes the temperature dependence of ρ_n for the given solution (the continuous curve on Fig. 3 describes Eq. (10) with the given values of the parameters).

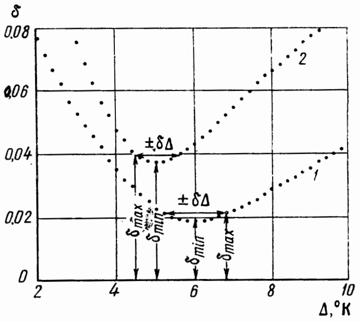

FIG. 4. Dependence of the mean square deviation of the density of the normal component δ on the parameter Δ. 1—solution with 8.5% He^3, 2—21.7% He^3.

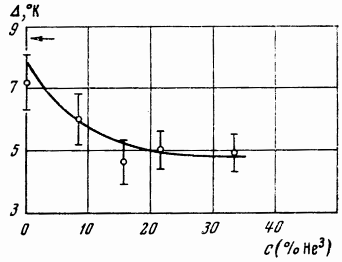

FIG. 5. Dependence of the value of the roton minimum on the He^3 concentration; δ_{max}—maximum experimental error.

2. **Parameters of the bose branch of the spectrum.** The deformation of the phonon part of the spectrum, brought about by the addition of He^3, reduces to the dependence of the speed of first sound on the concentration of He^3. As is shown in[12], the speed of first sound for not very low temperatures decreases linearly with concentration; this was explained (with accuracy of 0.5–1.0%) on the basis of the linear dependence of the density of the He^3–He^4 solutions on the concentration of He^3.

The concentration dependence of the energy gap Δ of the roton part of the spectrum, obtained by the method described above, is given in Fig. 5. The arrow on this graph corresponds to the roton minimum in the He^4 spectrum, which was obtained in experiments on neutron scattering. A satisfactory agreement with the data for He^4 and in the presence of a dependence of the roton minimum on the He^3 concentration follow from this graph. Upon increase in the concentration, Δ at first decreases, and then is practically independent of the concentration.

Such a dependence was explained qualitatively, by using the consideration developed in[24]. (The calculation given below was completed by V. A. Slyusarev and M. A. Strzhemechnyĭ.) The change in the energy spectrum ω_k of pure He^4 as the result of the interaction of the rotons with impurity excitations is given by the expression

$$\tilde{\omega}_k^2 = \omega_k^2 + 2\omega_k\Pi, \quad \Pi = c\frac{\varepsilon_p\Gamma_0^2}{\tilde{\omega}_k^2 - \varepsilon_p^2}, \tag{11}$$

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[1] The computer calculations were carried out by E.M. Livshitz and V.A. Plenkova, to whom the authors express their indebtedness.

where

$$\varepsilon_p = p_0^2 / 2m_3^*,$$

$$\Gamma_0 = \frac{m_4 - m_3}{m_3} \cdot \frac{1}{\sqrt{S_{p_0}}} \left\{ \beta_{p_0}\varepsilon_0 S_{p_0} + (1 - S_{p_0}) \frac{\varepsilon_p m_3^*}{2m_4} \right\},$$

S_{p_0} are the Fourier components of the He^4 density correlation function, Γ_0 the matrix element corresponding to a transition between the states "impurity" and "impurity-bosons," p_0 is the momentum corresponding to the roton minimum. The value of β_k is connected with the kinetic energy—density correlator $\langle n_k^+ \epsilon_k \rangle$ by the relation $\epsilon_0 S_k \beta_k = \langle n_k^+ \epsilon_k \rangle - \epsilon_0 S_k$. Setting $k = p_0$, we have, at the roton minimum,

$$\Delta^2 = \Delta_4^2 + \frac{\varepsilon_p^2 - \Delta_4^2}{2} \left\{ 1 - \left[1 + c \frac{2\Delta_4 \varepsilon_p \Gamma_0^2}{(\Delta_4^2 - \varepsilon_p^2)^2} \right]^{1/2} \right\} \qquad (12)$$

(Δ_4 is the value of the roton minimum in He^4).

The dependence $\Delta = \Delta(c)$ cannot be computed numerically, since it is not possible, at the point p_0, to compute the value of the kinetic energy—density correlation function on the basis of theoretical considerations only, or to estimate this value from other experiments. However, it is easy to obtain the result that $\partial\Delta/\partial c < 0$ from Eq. (12), and for sufficiently high concentration, $\partial\Delta/\partial c \to 0$, i.e., for low concentrations Δ is a decaying function of c; as the concentration increases, the roton minimum becomes independent of the concentration, while $\partial^2\Delta/\partial c^2 > 0$. The experimentally determined dependence $\Delta = \Delta(c)$ is of just this sort (Fig. 5).

From the data on ρ_n/ρ, it is not possible to determine the concentration dependence of the parameter b with satisfactory accuracy, inasmuch as the effect of the pre-exponential factor is, as usual, small in comparison with the size of the exponent. For this reason, the error in the determination of b depends weakly on the magnitude of Δ. Estimates show that even 50% error in b changes Δ by no more than 10%.

3. Effective mass of the impurity excitations. Information on the impurity (Fermi) branch of the spectrum of solutions gives the value of ρ_{ni} entering into Eq. (10), which was assumed to be independent of temperature in the model chosen. Figure 6 shows the concentration dependence of ρ_{ni}, determined from the experimental values of u_4; here the values of ρ_{ni} obtained by other authors[15,16,22,25] are shown.

For solutions with He^3 concentrations $c \leq 10\%$, our data agree with the results of other authors within the limits of accuracy. In this region of concentrations, ρ_{ni} is an almost linear function of c. For $c > 10\%$ there is a significant departure of ρ_{ni} from the initial linear dependence and for $c > 25\%$, a linear dependence is again observed, but with a different, smaller slope. In this same region a significant divergence is observed between the values of ρ_{ni} determined from the speed of fourth sound and measured by the method of oscillating disks.

This divergence can be explained by the following circumstance. All experiments on the determination of ρ_n of superfluid solutions, excluding[28], were carried out at relatively high temperatures—the lower limit was 1.3—1.4°K. At such temperatures, the contribution of the rotons ρ_{nr} to the total density of the normal component is still important. Because of the relatively high temperatures, it was not possible to observe the fact that ρ_{nr} for pure He^4 and for solutions at the same temperature have different values. Therefore the contribution of the impurities is determined as $\rho_{ni} = \rho_n - \rho_{nr}(He^4)$. Actually, because of the decrease of Δ with increase in concentration (Fig. 5) the value of ρ_{nr} of the solution is higher for a given temperature than $\rho_{nr}(He^4)$. If this is not taken into consideration, the computed values of ρ_{ni} will be excessive.

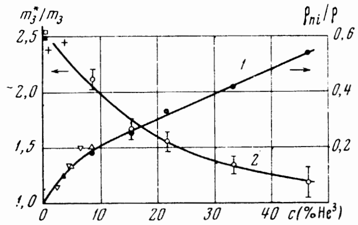

FIG. 6. Dependence of the impurity part of the density of the normal component (ρ_n / ρ) (curve 2) and the effective masses of the impurity excitations (curve 1) on the He^3 concentration from data of various researches: ○,●—present research, △—[16], ▽—[22], ▲—[25], □—[26], +—[27], ■—[29].

We also emphasize the fact that the values of ρ_{ni} are practically identical with the lowest measured values of the total density of the normal component of the solutions; therefore they do not depend on the model chosen or on the method of treatment of the experimental results. The model of an ideal gas of quasiparticles for the impurity excitations allows us to compute their effective mass, using Eq. (9). The nonlinearity of ρ_{ni} in the concentration leads to a dependence of the effective mass m_3^* on the concentration (Fig. 6). The values of m_3^* found by other methods[26-28] are also plotted in this drawing.

The concentration dependence of the effective mass is satisfactorily extrapolated for very small concentrations to the value $m_3^* = (2.4—2.5)m$, which is determined from the speed of second sound.[26,28] For an increase in concentration, m_3^* decreases and approaches the mass of the "bare" He^3 atom, which is characteristic for an ideal gas of real particles.

Figure 6 also shows the data on the effective mass m_3^* in the solution, obtained on the basis of measurement of the heat capacity.[27] These measurements were carried out at very low temperatures and the observed growth of m_3^* is in excellent agreement with the theory of Bardeen, Baym, and Pines,[29] which takes into account the Fermi liquid interaction in degenerate solutions of He^3 in He^4. But such interaction is important only at very low temperatures and therefore the low temperature data on the effective mass cannot agree with the high temperature.

One of the reasons leading to a decrease in the effective mass in concentrated solutions can be the decrease in the density of the solutions with increase in He^3 concentration. In this connection, we turn our attention to the change in the speed of second sound in 0.35-percent solution of He^3 in He^4, which was carried out at various pressures.[26] It turned out that m_3^*, computed from these data, increase with increase in the density of the liquid. For an increase in the concentration of the solution, its density decreases, and the decrease of m_3^*,

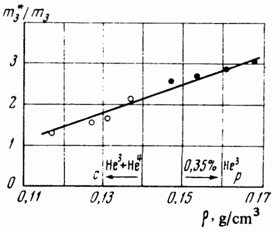

FIG. 7. Dependence of the effective mass of impurity excitations of He^3 in solution on the density of the liquid. ○—present research; the arrow indicates increase in the concentration of solutions; ●—data [26] for a solution with 0.35% He^3; the arrow indicates increase in pressure. (at lower right, ρ, g/cm³)

found in the present work, would appear to be a continuation of the dependence $m_3^* = m_3^*(\rho)$ found in[26], but in the opposite direction. Graphically, such a comparison is shown in Fig. 7, which reinforces the assumption introduced, although the dependence on the density can be a second reason. The primary reason for the dependence $m_3^*(c)$ can be, for example, the change in density, inasmuch as the distance between them changes in this case.

In conclusion, it should be emphasized that the assumptions and treatment of the data carried out in the present work are obviously not unique possibilities, but a theory of the He^3—He^4 solutions is not available at the present time that is valid over a wide range of temperatures and concentrations. The consideration that has been set forth is an attempt to estimate the parameter of the energy spectrum of superfluid solutions in the simplest fashion.

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[1] J. R. Pellam, Phys. Rev. **73**, 608 (1948).

[2] K. R. Atkins, Phys. Rev. **113**, 962 (1952).

[3] D. G. Sanikadze and D. M. Chernikova, Zh. Eksp. Teor. Fiz. **46**, 1123 (1964) [Sov. Phys.-JETP **19**, 760 (1964)].

[4] K. A. Shapiro and I. Rudnick, Phys. Rev. **137**, 1383 (1965).

[5] B. N. Esel'son, N. E. Dyumin, E. Ya. Rudavskiĭ and I. A. Serbin, Zh. Eksp. Teor. Fiz. **51**, 1064 (1966) [Sov. Phys.-JETP **24**, 711 (1967)].

[6] L. D. Landau and I. Ya. Pomeranchuk, Dokl. Akad. Nauk SSSR **59**, 669 (1948).

[7] A. Freddi and I. Modena, Lab. naz. Frascati (rep. A), No. 7, 12, 1967.

[8] B. N. Esel'son, V. P. Babiĭchuk, L. S. Dikina and I. A. Sorbin, Works (Trudy) Physicotechnical Institute of Low Temperatures Academy of Sciences, Ukrainian SSR (in press).

[9] W. M. Whitney and C. E. Chase, Phys. Rev. **158**, 200 (1967).

[10] T. R. Roberts and S. G. Sydoriak, Phys. Rev. **118**, 901 (1960).

[11] T. R. Roberts and S. G. Sydoriak, Phys. Fluids **3**, 895 (1960).

[12] B. N. Esel'son, N. E. Dyumin, E. Ya. Rudavskiĭ and I. A. Serbin, Zh. Eksp. Teor. Fiz. **51**, 1665 (1966) [Sov. Phys.-JETP **24**, 1126 (1967)].

[13] H. A. Fairbank, Nuovo cim. **9**, 325 (1958).

[14] E. C. Kerr, Proc. X Intern. Conf. on Low Temp. Phys. and Chemistry, Madison (1957), p. 158.

[15] B. N. Grigor'ev, B. N. Esel'son, V. P. Mel'khanov and V. I. Sobolev, Zh. Eksp. Teor. Fiz. **51**, 1059 (1966) [Sov. Phys.-JETP **24**, 707 (1967)].

[16] J. G. Dash and R. D. Taylor, Phys. Rev. **107**, 1228 (1957).

[17] I. Rudnick, E. Guyon, K. A. Shapiro and S. A. Scott, Phys. Rev. Lett. **19**, 488 (1967).

[18] D. F. Brewer, C. W. Leppelmeier, C. C. Lim, D. O. Edwards and I. Landau, Phys. Rev. Lett. **19**, 491 (1967).

[19] P. I. Bendt, R. D. Cowan and L. L. Yarnell, Phys. Rev. **113**, 1386 (1952).

[20] I. M. Khalatnikov, Vvedenie v teoriyu sverkhtekuchesti (Introduction to the Theory of Superfluidity) (Nauka Press, 1965).

[21] Ya. Pomeranchuk, Zh. Eksp. Teor. Fiz. **19**, 42 (1949).

[22] N. G. Bereznyak and B. I. Esel'son, Zh. Eksp. Teor. Fiz. **31**, 902 (1956) [Sov. Phys.-JETP **4**, 766 (1957)].

[23] I. M. Khalatnikov, Dokl. Akad. Nauk SSSR **111**, 332 (1956) [Sov. Phys.-Doklady **1**, 655 (1957)].

[24] V. A. Slyusarev and M. A. Strzhemechnyĭ, Proceedings of the 15th All-Union Conference on Low Temperature Physics, Tbilisi, 1968.

[25] J. Pellam, Phys. Rev. **99**, 1327 (1955).

[26] D. Sandiford and H. A. Fairbank, Phys. Rev. **162**, 192 (1967).

[27] A. C. Anderson, J. C. Wheatley, W. R. Roach and R. F. Sarwinski, Phys. Rev. Lett. **16**, 263 (1966).

[28] C. G. Niels-Hakkenberg, L. Meermans and H. C. Cramers, Proc. VIII Intern. Conf. on Low Temperature Physics, London, 1963, p. 45.

[29] J. Bardeen, G. Baym and D. Pines, Phys. Rev. **156**, 207 (1967).

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