EFFECT OF THE LONGITUDINAL DIMENSIONS OF A PLASMA COLUMN ON THE CRITERION FOR EXCITATION OF THE KADOMTSEV-NEDOSPASOV CURRENT-CONVECTIVE INSTABILITY

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The effect of the finite length of a plasma column on the development of Kadomtsev-Nedospasov current-convective instability is studied. The spatial spectrum of the instability harmonics in a bounded plasma is measured. It is shown that when the wavelength of the instability λ is comparable with the longitudinal dimensions of the plasma, the appearance of new regions of stable behavior of the current column should be possible. The experimental results are compared with the theory[1,2]

In spite of the considerable number of investigations of magnetohydrodynamic instabilities in a plasma, there are practically no published data on the behavior of magnetohydrodynamic perturbations with wavelengths comparable with the longitudinal dimensions of the plasma column. Thus, we still do not know the maximum perturbation dimensions that can appear in electrode discharges, and there are no reliable experimental data on the spectra of the spatial harmonics of the instability.

In this paper we attempt to investigate the Kadomtsev-Nedospasov magneto-hydrodynamic instability[1] in an electron-hole plasma of a germanium semiconductor under conditions when the limiting dimensions of the sample L are comparable with the perturbation wavelength λ. An analysis of the results obtained in this paper is best based on the theory of stability of the positive column of a gas discharge in a longitudinal magnetic field[1], suitably modified for the case of a semiconductor plasma[2]. The theory considers the case of a weakly developed instability. The applicability of the theory is limited to a narrow range of electric field intensities $\mathscr{E}$ slightly exceeding the critical field $\mathscr{E}_0^\infty(B)$, starting with which the current-carrying plasma column becomes unstable. The index infinity denotes the relation obtained for the case of an infinitely long plasma column with $L/\lambda \to \infty$.

For convenience in comparing the experimental results with the Glicksman theory[2], shall henceforth find it convenient, besides using the electric and the magnetic fields $\mathscr{E}$ and B, to introduce the dimensionless quantities $\omega_c\tau = \mu_e B$ and $E_0 = \mu_e \mathscr{E} a/D_e$, which are proportional to these fields. Here $\omega_c = eB/m^*c$ is the cyclotron frequency of an electron with reduced mass m^*; τ is the time between two collisions, determined from the expression for the expression for the electron mobility $\mu_e = (e/m^*)\tau = 3900\,(300/T)^{1.66}$; a—radius of the plasma cylinder; $D_e = \mu_e kT/e$ is the electron diffusion coefficient, and T is the lattice temperature. The measurements were made in the region of the intrinsic conductivity of the samples at $\omega_c\tau \gtrsim 1$. The coefficient of surface recombination of the electrons and holes, determined by the Vladimirov method[3], corresponded to a ratio of the plasma density at the boundary of the sample to the plasma density on the axis $\delta = n(a)/n(0) \approx 0$.

We used in the experiments germanium with room-temperature resistivity 45 ohm-cm. The magnetic field was produced in the experiments by a multiturn copper solenoid cooled to liquid-nitrogen temperature, producing a field B that was homogeneous in the sample region within 2–3%. The quantity D could be continuously varied in the experiments from zero to 150 kG. The solenoid operated in the pulsed regime with a field half-period of 3 msec. The bulk of the measurements were made in the region of maximum B, and during the time of the measurements the field did not change by more than 0.5%. By suitable choice of $\mathscr{E}$ and B the main parameters of the instability of the plasma column were determined only under regimes close to critical, corresponding to the instant of onset of the oscillations. The procedure for observing the instability was similar to that described in[4].

In accordance with the considered theory, at small magnetic field intensities, characterized by the inequality $\omega_c\tau \ll 1$, the quantity $X = 2\pi a/\lambda$ depends little on B. When $\omega_c\tau \gtrsim 1$, it decreases rapidly. If, as assumed in the calculations of[1,2], the length of the plasma column is sufficiently large to be able to neglect the end effects, X does not depend on the longitudinal dimensions of the sample. In the region $\omega_c\tau \gtrsim 1$, we have $\lambda = \lambda(B)$, and the magnetic field can serve as a convenient means for a smooth variation of λ.

The main results of the theory proposed in[1,2] are the function $\mathscr{E}_0^\infty(\omega_c\tau)$ calculated under rather broad assumptions, the function $X^\infty(\omega_c\tau)$, and an estimate of the plasma oscillation frequencies near the instability threshold. An experimental verification of the main premises of Glicksman's theory[2] shows good quantitative agreement between the calculated and experimental results, provided the assumptions of the theory of ($L/\lambda \to \infty$ and $\mathscr{E}_0 \sim \mathscr{E}_0^\infty$) are sufficiently rigorously satisfied.

At the same time, there are practically no calculations for the case $L/\lambda \sim 1$. Apparently, one of the most consistent attempts to take into account the finite length of the plasma cylinder was undertaken in a theoretical paper of applied character[5], where, assuming the perturbations on the end to vanish, an explanation was obtained for a previously observed experimental effect[4] of high-frequency stabilization of instabilities of the Kadomtsev-Nedospasov type in a plasma column with $L/\lambda \gtrsim 1$. The obtained agreement between the results of the calculation[5] and the experimental data[4] make it possible to assume in this case that for a qualitative, and frequently also quantitative explanation of the effects connected with the presence of end surfaces of the plasma, one can use the analogy between the vibrational perturbations in the plasma of finite dimensions and, for example, systems of the microwave resonator type. In particular, one should expect that when $L \sim \lambda$ the instability can develop only in definite magnetic-field intensity zones. The positions of the instability zones should correspond to the equation $X(\omega_c\tau) = 2\pi as/L$, where s is an integer or a half-integer.

A qualitative verification of the assumption made above concerning the possible existence of discrete instability-excitation zones with a probable appearance of zones of stable behavior of the current in short ($L/\lambda \sim 1$) samples of a semiconductor is demonstrated by the oscillogram of Fig. 1. The upper trace represents the plasma-density oscillations obtained with the aid of a probe soldered to the surface of the sample, and the lower trace shows the variation of the magnetic field in time, the maximum of $B(t)$ amounting to 100 kG. Under the experimental conditions $a = 1$ mm and $L = 6$ mm, the calculation wavelength of the perturbation in the sample at small magnetic-field intensities can be obtained from the $X^{\infty}(\omega_c\tau)$ plot shown in Fig. 2, taken from[2]. The experimental conditions were chosen such that the condition $B \sim B_0^{\infty}$ was satisfied at the instant when the instability occurred. At this instant, the calculated wavelength in the sample was $\lambda = 3$ mm, and when B is increased, oscillations with harmonics $s = 2$ and $s = 1$ can appear successively in the plasma.

It is seen from the oscillogram that, as expected, when B is varied smoothly, the instability in the plasma occurs in the form of discrete zones. Depending on the intensity of the electric field $\mathscr{E}$ in the sample, it is possible to observe from one to three zones of instability excitation. Although the oscillogram is

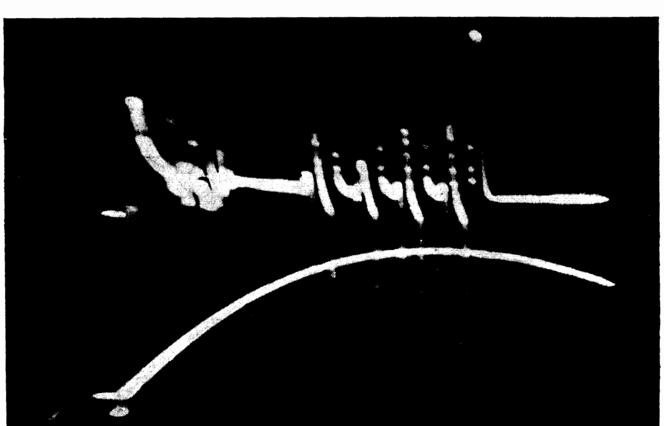

FIG. 1. Oscillogram of the zones of instability excitation in a short plasma column. Upper trace—oscillations in the plasma, lower trace—variation of magnetic field B(t), with time. $B_{max} = 100$ kG, $L = 6$ mm, $a = 1$ mm.

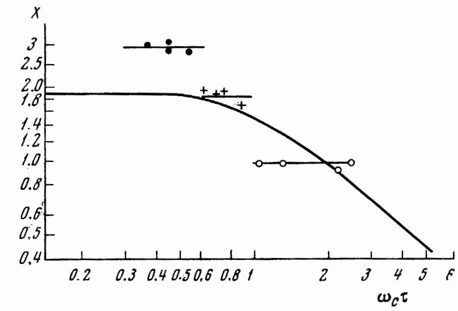

FIG. 2. Plot of $X^{\infty}(\omega_c\tau)$. Solid curve—calculated values from [2], points—experimental data: ●—s = 3, +—s = 2, ○—s = 1 (L = 6 mm, a = 1 mm).

purely illustrative, the demonstrated picture is evidence in favor of an important role of the ends of the plasma in short systems. When $L/\lambda \gg 1$, this effect does not occur.

For a more detailed study of the phenomenon observed in the described experiment, we plotted the threshold electric field E_0, corresponding to the instant of occurrence of instability, against the magnetic field. The plots obtained for two samples with $L = 30$ mm and $L = 6$ mm are shown in Fig. 3. For a comparison, the figure shows also the $E_0^{\infty}(\omega_c\tau)$ plot calculated[2].

A characteristic feature of Fig. 3 is the good agreement between the measured values of E_0 and the calculated E_0^{∞} for $L = 30$ mm, and the large discrepancy between E_0 and E_0^{∞} for $L = 6$ mm in the region $\omega_c\tau < 1$. The reason for such a behavior of the $E_0(\omega_c\tau)$ curves becomes clear if it is recognized, that, as noted above, at zero boundary conditions of the end surfaces of the plasma column the minimum perturbation length cannot exceed 2L for the case of a plane wave or L for a three-dimensional perturbation in our case. Under conditions when the properties of the medium, described by the $X^{\infty}(\omega_c\tau)$ dependence, leads to violation of the equality $L \approx \lambda$, to maintain the oscillations it is naturally necessary to increase B compared with the minimum possible value B_0^{∞}. Further, in accordance with the $X(\omega_c\tau)$ dependence for $a = 1$ mm at $\omega_c\tau \ll 1$, we have $\lambda^{\infty} = 3$ mm = const. This means that in weak magnetic fields, for a long crystal,

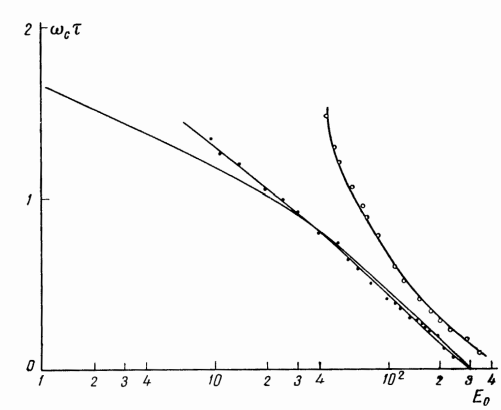

FIG. 3. Dependence of the measured values of E_0 on $\omega_c\tau$. Solid curves—calculated values for an infinitely long plasma column, points—experimental: ●—L = 30 mm, ○—L = 6 mm; a = 1 mm.

the number of the spatial harmonics at the instant of occurrence of the instability should correspond to the equality $s = L/\lambda^{\infty} \approx 10$; for a short crystal under the same conditions we have $s \approx 2$.

When B is increased, $X(\omega_c\tau)$ begins to decrease (λ increases) when $\omega_c\tau > 1$. This makes it necessary for the structure of the perturbation of the plasma column in a short sample to change from $s = 2$ at $\omega_c\tau < 1$ to $s = 1$ at $\omega_c\tau > 1$. Further increase of the magnetic field should either lead to the appearance of a new structure with $s = 0.5$ or, if possible, leads to an experimentally observed sharp deviation of the measured $\mathscr{E}_0(\omega_c\tau)$ dependence from the calculated $\mathscr{E}_0^{\infty}(\omega_c\tau)$ dependence.

Naturally, for a longer crystal, the process of transition from the states with $s = 10$ to $s = 1$ will stretch out the values $\omega_c\tau \sim 10$, corresponding to $X \sim 0.2$. But here, in analogy with the curve for $L = 6$ mm, a strong discrepancy between E_0 and $\mathscr{E}_0^{\infty}$ should appear for the case $L = 30$ mm.

Unfortunately, the range of working values of B in the given series of experiments was limited to the partial deterioration of the insulation of the solenoid, and in the case of a long crystal the obtained deviations of $\mathscr{E}_0$ from $\mathscr{E}_0^{\infty}$ were small. Therefore, the numerical comparison of the values of L with the calculated value of λ^{∞} at the point of maximum deviation of E_0 from E_0^{∞} can be carried out only for the second case. Then $\omega_c\tau \sim 3$ and the calculated value is λ - 6 mm = L, as expected.

For a direct confirmation of the results, we measured directly the wavelength of the perturbation in a sample with $L = 6$ mm for different values of the magnetic field. The value of λ was determined by measuring the phase difference of the oscillations obtained from a pair of probes located along the generatrix of the crystal at a distance 2 mm[6]. The results of the measurements are shown in Fig. 2. It is seen from the obtained data that on moving along the $E_0(\omega_c\tau)$ curve (Fig. 3) with a monotonic increase of the magnetic field intensity in the sample with $L = 6$ mm, the plasma column jumps from the state with $s = 3$ into the state with $s = 2$, and then into the state with $s = 1$. The mean values of X determined from the measured values of λ are in satisfactory agreement with the calculated $X_0^{\infty}(\omega_c\tau)$ curve for $s = 2$ and $s = 1$. At values of B satisfying the inequality $\lambda^{\infty}(\omega_c\tau) > L$, the plasma column turns out to be stable at all times.

Somewhat unexpected in the experiments was the presence of a harmonic of the instability with $s = 3$, with a wavelength $\lambda < \lambda^{\infty} = 3$ mm. The appearance of this harmonic can be attributed to the fact that, in connection with the increasing particle loss to the ends, which is not taken into account in the theory[2], the instability of the plasma in a short sample begins at fields $E_0 \sim 1.5\,E_0^{\infty}$ (Fig. 3). Under these conditions we can expect in the region $\omega_c\tau < 1$ the appearance of instability harmonics of higher order than those predicted for the region $E_0 \sim E_0^{\infty}$[2].

Taking the foregoing into account, we can assume that, at least in the region $\lambda \sim L$, the obtained picture confirms the conclusions based on the results of the measurements shown in Fig. 3, and is in good agreement with the predicted existence of instability zones with discrete values of s. It is seen from the obtained data that for zero boundary conditions the maximum length of the perturbation λ is equal to the length of the sample L, and no oscillations with $\lambda = 2L$ are produced.

In conclusion, notice should be taken also of another important feature of the behavior of the instability in a short plasma column, namely that the harmonic with the maximum oscillation amplitude is always the harmonic with the smallest number allowed by the $X(\omega_c\tau)$ dependence. The contribution of harmonics with large s in the experiments turned out to be very small.

In connection with the fact that under the conditions of the experimental properties of the electron-hole plasma are close to the properties of a gas-discharge plasma ($\omega_c\tau \gtrsim 1$, $n \sim 10^{13}$–10^{14} cm^{-3}), we can expect analogous phenomena to be observed also in an ordinary plasma consisting of electrons and ions. Moreover, inasmuch as in the phenomena under consideration the instability excitation mechanism does not play the principal role, and the preference of the plasma and surfaces is of principal significance in the establishment of oscillations with a given spatial period, we can expect analogous phenomena to occur also for other types of long-wave instabilities. This is confirmed, in particular, by the behavior of the Kruskal-Shafranov instability. A study of this instability[7] revealed only a mode with $s = 1$ in regimes close to critical at $L \sim \lambda$. Similar experiments on gas-discharge plasmas would be of great interest.

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