# *INVESTIGATION OF FOCUSING OF ELECTRON BEAMS IN A METAL BY A LONGITUDINAL MAGNETIC FIELD*

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We have studied the electron focusing effect previously observed by us[1,2] in tin, produced by a longitudinal magnetic field H. Voltage peaks periodic in H appeared at the collector as the result of focusing of groups of electrons with maximum displacement along the field during a single revolution. Maximum values of $\partial S/\partial p_H$ have been measured for different orientations of H, where $p_H$ is the projection of the electron quasimomentum on the direction of H, and S is the area of the intersection of the Fermi surface with the plane $p_H$ = const. The small displacement of the peaks in field strength observed with increasing accelerator voltage v permits us to judge the sign of the charge carriers in the beam. The influence of the temperature and the voltage v on the amplitude of the effect have been investigated. In this connection we present the results of theoretical calculations of the electron mean free path in the metal as a function of v/T.

In the case when the mean free path of the conduction electrons is sufficiently large, it is possible to observe in a metal the propagation of electron beams focused by a longitudinal magnetic field. The arrangement of the experiment, which was proposed by us previously,[1] is shown in Fig. 1. Two wires were welded at points A and B to a single-crystal plate M of high-purity tin of thickness d = 0.4 mm. The diameter of the contacts was estimated from the value of their electrical resistance (see below) and in our experiments did not exceed 1 $\mu$. At the temperature of the experiment the electron mean free path $l \geq d$. If an electric current is passed through the circuit of contact A, then the electrons in the metal are accelerated in the electric field near the contact. If a uniform magnetic field H is applied in the direction AB, then for certain values of the field intensity H an appreciable part of the accelerated electrons can be focused close to the contact B, which produces a change in its potential V. We have previously reported[2] observation in the curve of V versus H of a series of peaks periodically repeated with increasing H, in a background of monotonically rising V. In the present article we report a more detailed study and discussion of this effect.

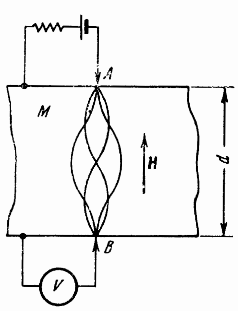



FIG. 1. Experimental arrangement for observation of electron focusing in a metal.

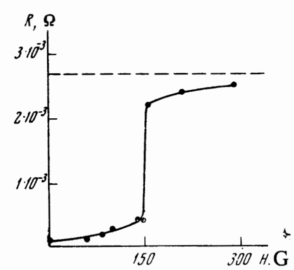
## 1. PREPARATION OF CONTACTS AND DETERMINATION OF THEIR SIZE

In preparing the contacts we tried wires of various materials: Cu, Pt, Au, Sn, Nb, and others, with diameters of 20–60 $\mu$. In order to obtain a reliable electrical contact the wire was welded to the surface of the sample by passing through it a current from a 300-volt battery in series with a resistance of $\sim 10^6$ ohms. In addition, in the focusing experiments the wires were usually cemented by a small drop of diluted BF glue. In preparation of contact A we used wire of the same material as the sample, in order to introduce the smallest amount of contamination in the region of the contact. The material of contact B in general made no difference. In the case of a gold wire and a tin sample, or a platinum wire and indium sample, we could sometimes observe formation in the vicinity of the contact of an alloy with a critical temperature exceeding that of the sample. By transforming the sample from the superconducting state to the normal state, it was possible (in the case of wires of normal metals or Nb) to determine, from the difference in the resistances of the contact circuit in the two cases, the junction resistance, which is localized in the sample itself. In spite of the low resistivity of the material of the samples ($\sim 2 \times 10^{-10}$ ohm-cm), the junction resistance usually had a value of the order of $10^{-2}$ ohm, and it was possible to obtain contacts with even higher resistance values (up to $\sim 1$ ohm). The possibility of obtaining high junction resistance is due to the fact that the resistance value in this case does not depend on the mean free path and is determined in order of magnitude by the relation[1]

$$R = p / e^2r^2N, \tag{1}$$

where p is the Fermi momentum, r is the contact radius, and N is the number of electrons per cm$^3$.

Azbel' and Peshchanskiĭ[3] have pointed out a way to obtain exact solutions of this and similar problems for arbitrary contact shape and electron scattering law, but for estimating the contact diameters we can use Eq. (1), writing it in the form $R = l/\sigma r^2$, and for tin setting $\sigma/l$ equal to $10^{11}$ ohm$^{-1}$cm$^{-2}$.[4] The validity of this formula was checked by determining the contact radius independently, from the value of the critical current $I_c$ necessary for destruction of the superconductivity of the



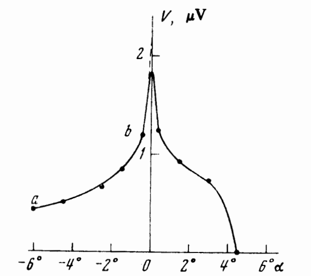




FIG. 2. Contact resistance R as a function of magnetic field strength. The dashed line indicates the value of R for H = 7 kG. T = 1.3°K.

FIG. 3. Voltage in the circuit of contact B as a function of the angle $\alpha$ between the magnetic field direction and the line AB; H = 8 kG.

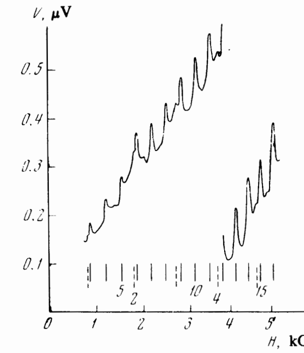



FIG. 4. The function V(H) for contact pair No. 2 (see the table); T = 1.3°, I = 100 mA.

sample in the vicinity of the contact, on the assumption that $r = 0.2\ I_c/H_c$. The radius values obtained by the two means differed by no more than 1.5–2 times. In the case of tin wires the junction resistance of the sample was set equal to the total resistance of the wire-sample system, since the wire resistance far from the contact was negligibly small. The curve showing the destruction of the superconductivity of contact A by a field parallel to the sample surface in one of the focusing experiments is shown in Fig. 2.

## 2. OBSERVATION OF THE FOCUSING EFFECT

Two samples were investigated, Sn-I and Sn-II, of thickness 0.39 and 0.40 mm, respectively. The normal to the surface of Sn-I was directed along the [100] axis with an accuracy of 2°. In the case of sample Sn-II, the normal formed an angle of 65° with the tetragonal axis [001] (which we take as the polar axis in our system of coordinates) and deviated by 10° in azimuth from the (110) plane. The orientation of the line AB with respect to the crystal axes could be varied by changing the location of the contacts on the surface of the sample. Two pairs of contacts were usually installed on a single sample. The orientations of AB which were tested are listed in the table ($\psi$ is the angle of the straight line AB with the (001) plane, $\varphi$ is the azimuthal angle with the (010) plane).

To observe the focusing effect it was necessary to establish the direction of H along the line AB with an accuracy in angle of $r/d \sim 10'$. For this purpose a strong magnetic field (of the order of 10 kG) initially was established at T = 1.3°K approximately in the necessary direction, and through contact A was passed a current of about 100 mA. Under these conditions it was possible to observe between contact B and a peripheral point of the sample, by means of a F116/1 galvanometric amplifier, the potential difference arising from the sharp increase in resistance of the sample in the direction perpendicular to the field. By rotating the sample in the field around two perpendicular axes lying in its plane, it was possible to observe a peak in the signal strength with a width of the order of 10′, corresponding to the direction AB ∥ H (see Fig. 3). The angle $\alpha$ in Fig. 3 corresponds to rotation of the sample around the [001] axis. The rise of the signal on approaching the peak occurred roughly logarithmically (the region ab on the curve). To the right of the peak, at $\alpha = 4.5°$ the signal practically disappears, which is the result of the sharp

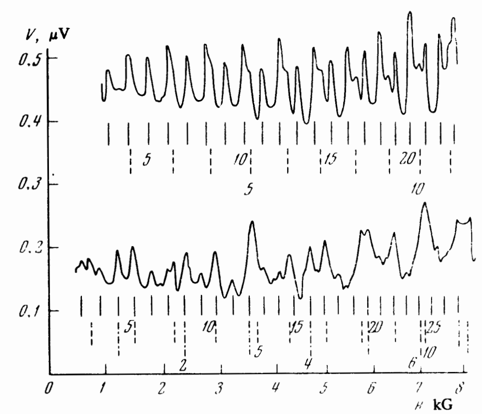



FIG. 5. Recordings of V(H). The upper curve is for contact pair No. 8. The lower curve is for contact pair No. 7. T = 1.3°K, I = 100 mA. The general rise in signal with field has been compensated.

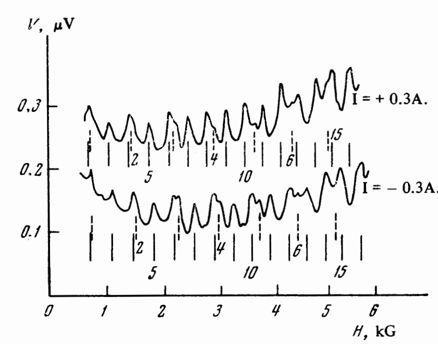



FIG. 6. Curves of V(H) for contact pair No. 8 for different directions of the measuring current I = ± 0.3 A; T = 1.3°K. The general rise in the signal with field has been compensated.

decrease in resistance with the appearance of open trajectories (when H becomes parallel to the (010) plane).

After setting the field direction, we recorded on a two-dimensional plotter the voltage V at contact B as a function of the field intensity H (Fig. 4). More accurately, we plotted directly as abscissa the voltage on a Hall probe, which resulted in some nonlinearity of the H scale in Figs. 4–6. The origin for the ordinate scale is arbitrary, and the second part of the curve in Fig. 4 is displaced downward. Subsequently, in order to compensate the strong increase of signal with field, we fed into the circuit of contact B an additional voltage taken from a tin wire placed in the field beside the sample. A constant current was passed through the wire, and its increase in resistance in the field led to a levelling of the general behavior of the V(H) curve (Figs. 5 and 6).

## 3. EXPERIMENTAL RESULTS

### A. Dependence of the Peak Location on the Direction of the Electron Beam

The peaks in the V(H) curves obtained by us form two or three series equally spaced in field strength, which we have marked by lines under the curves in Figs. 4–6. The origin of these series can be explained as follows.

If some group of electrons which has left point A is focused at point B, then the condition $L = nu$ is obviously fulfilled, where L is the distance AB, and u is the displacement of the electrons along the field in a single revolution. It is well known that for electrons on the Fermi surface which have a given projection of momentum $p_H$ in the direction of the magnetic field, $u = (c/eH)\partial S/\partial p_H$, where S is the area of the intersection of the plane $p_H = \text{const}$ with the Fermi surface. For contacts of finite dimensions the focusing will occur at a certain interval of $p_H$ values, for the electrons of a certain band on the Fermi surface, whose width will be greatest for those portions of the surface where u depends weakly on $p_H$, i.e., in the vicinity of those sections where $\partial^2 S/\partial p_H^2 = 0$. The existence of such regions on the Fermi surface also should lead to appearance of the peaks in our curves.

Generally speaking, still other features in the $u(p_H)$ curve should result in appearance of peculiarities at corresponding points of the V(H) curve, in particular, a break in the $u(p_H)$ curve at a value $p_H'$ at an elliptic limiting point should lead to appearance of periodic steps in the V(H) curve at the points where $nu(p_H') = L$. In this case it would be possible to compute from $\Delta H$ the Gaussian curvature of the Fermi surface at the limiting point $K = (2\pi c/eL\Delta H)^2$. We have so far not been able to observe these steps, which as a rule should be much weaker than the peaks.

For the observed series of peaks we have listed in the table the quantity $p_f = (e/c)L\Delta H$, which has the dimensions of momentum. According to the considerations discussed above, $p_f = (\partial S/\partial p_H)_{extr}$.

The data obtained can be used to check proposed models of the Fermi surface of Sn, although the computational difficulties here are obvious. The simplest model of almost free electrons turned out in this case to be too crude, as the result of the existence of sharp breaks in the Fermi surface. Calculation of $p_f$ according to the model proposed by Weisz[5] would present considerable interest.

Our data must be compared with the results of investigation of the impedance of plane tin specimens in a perpendicular field, performed by Gantmakher.[6] The sinusoidal oscillations of impedance observed in this case are apparently also due to electron groups with extremal values of u, moving in spiral trajectories from one surface of the sample to the other.

For the direction $\psi = 4.5°$, $\varphi = 4.5°$, Gantmakher observed oscillations with $p_f = 42 \times 10^{-20}$ g-cm/sec, close

| No. of contact pair | Sample | $\Psi$, deg | $\varphi$, deg | $p_f \times 10^{20}$ g-cm/sec |
|---|---|---|---|---|
| 1 | Sn-I | 4.5 | 4.5 | 23; 38 |
| 2 | » | 0.5 | 6.5 | 20; 57 |
| 3 | » | 0.5 | 9.5 | 17.5; 56; 70 |
| 4 | » | 2.5 | 18.5 | 16.5; 51; 62 |
| 5 | » | 4.5 | 21.5 | 16; 45; 80 |
| 6 | » | 1.5 | 23 | 16; 80 |
| 7 | Sn-II | 13.5 | 23 | 19; 46; 76 |
| 8 | » | 4 | 43.5 | 22; 47 |

to the second of the values in line 1 of the table. The discrepancy is due, first of all, to inaccuracy in our measurements of $\psi$. For this same direction Gantmakher observed also weaker oscillations, whose periods could not be identified. For $\varphi \sim 15–20°$ and $\psi \sim 0°$ Gantmakher observed the value $p_f = 1.7$ (compare line 4 of the table). The relation of the intensities of the various oscillations as found by us differs from the work of Gantmakher, where the most effective electrons are those entering the skin layer at a small angle to the surface. When contacts are used, on the other hand, these electrons are not effective, since the beam cross section hitting the contacts at an acute angle is small.

### B. The Dependence of Peak Location on Current

The values of $p_f$ listed in the table were obtained with a rather weak current I in the circuit of contact A (not more than 100 mA). For stronger currents a dependence of the peak location was observed on the strength and direction of the current, which is easily visible in Fig. 6. Here values $I > 0$ correspond to motion of electrons from the sample into contact A. For variation of I from +0.3 to −0.3 amperes the lesser of the two periods increases by 4%, and the greater by 3%. The difference in the dependences $\Delta H(I)$ for the different series of peaks leads to a change in the entire shape of the V(H) curve for commutation of the current. A change in the sign of the magnetic field does not affect the location and shape of the peaks.

The phenomenon observed apparently arises as the result of the effect of the magnetic field of the measuring current on the electron motion. Near the contact A this field is not small—of the order of $10^2$ gauss for a contact diameter of $10^{-4}$ cm and $I \sim 0.1$ ampere. The problem of electron motion in this case is considerably complicated. If we consider the circular field of the current as a perturbation which transfers the electrons, precessing in the external field, from one orbit to another, we can reach the qualitative conclusion that the sign of the effect is determined by the direction of precession and that in the case of Fig. 6 we are dealing with hole trajectories occupying a region of momentum space with higher energy values.

### C. Dependence of Peak Amplitude on Temperature

Measurements of the peak locations were made at $T = 1.3°$K. With increase of temperature in the helium region the amplitude of the A peaks increases substantially, evidently as the result of scattering of the focused electrons. We can introduce the mean free path $l$, averaged over the Fermi surface band discussed, assuming that A is proportional to $e^{-s/l}$, where s is the length of the trajectory.

The dependence A(T) was measured for contact pair No. 8 for the series of peaks with $p_f = 22 \times 10^{-20}$ g-cm/sec. By extrapolation it was possible to determine A(0) and from this the quantity

$$\ln\frac{A(0)}{A(T)}=\left(\frac{1}{l(T)}-\frac{1}{l(0)}\right)s,$$

which is proportional to the probability for scattering of electrons by other quasiparticles.

Figure 7 shows the temperature dependence of the quantity $(1/l - 1/l(0))s$ on a logarithmic scale. The straight line corresponds to the equation $1/l - 1/l(0) \propto T^3$. In this way our results agree with the data on the temperature dependence of the mean free path, obtained in study of the radio-frequency size effect in tin,[7] where a cubic law was observed, indicating that removal of an electron from the group being studied occurs as the result of a single interaction with a phonon.

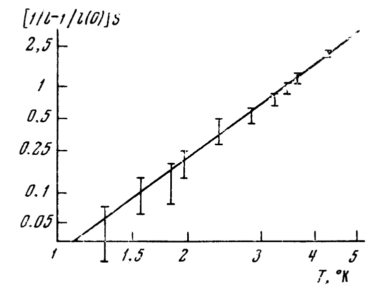


FIG. 7

## D. Dependence of Peak Amplitude on Current

The peak amplitudes increased with increasing current in the circuit of contact A, but the increase was nonlinear, being slower at higher currents. Figure 8 shows ln (A/I) as a function of I for several temperatures for the same series of peaks in which the A(T) dependence was studied.

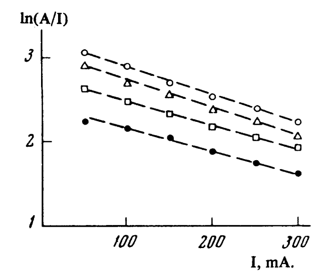


FIG. 8. Plot of ln(A/I) as a function of current I for different temperatures: ○–1.3°K, △–2.12°K, □–2.86°K, ●–3.51°K.

We can point to two mechanisms which can lead to a decrease in the relative amplitude A/I with increasing I. It is evident, in the first place, that the action of the magnetic field of the measuring current should lead not only to a displacement of the peak position, but also to a destruction of the sharpness of focusing and a decrease in A/I. The other mechanism is related to the decrease of the mean free path of the focused electrons, which should occur with increasing voltage on contact A. For a contact resistance $R \sim 10^{-2}$–$10^{-3}$ ohm and $I \sim 0.1$ ampere the electrons emitted by the contact traverse a potential difference $v \sim 10^{-3}$–$10^{-4}$ volt, acquiring an energy ev of the order of several degrees Kelvin. Electrons with such energies can produce other quasiparticles (phonons or electron-hole pairs) with an appreciable probability even at T = 0. For T ≠ 0 the mean free path is a function not only of T but also of v, and decreases with increasing v.

Since, with a change in the relation between v and T, a redistribution of the electrons in energy occurs only immediately in the vicinity of the Fermi surface, the interaction constants do not change in this case and the mean free paths $l_{ep}(v, T)$ and $l_{ee}(v, T)$ for electron-phonon and electron-electron interaction can be determined in terms of the energy distribution functions of Fermi and Bose particles with an accuracy to a factor independent of T and v:

$$1/l_{ep}=C_1(kT)^3 F_1(\gamma),\qquad 1/l_{ee}=C_2(kT)^2F_2(\gamma),$$

where $\gamma = ev/kT$,

$$F_1(\gamma)=\frac{1}{\gamma}\int_{-\infty}^{\infty}dy\{\varphi(y-\gamma)-\varphi(y)\}\int_0^{\infty}x^2dx\{2\nu(x)+1+\varphi(y+x)-\varphi(y-x)\},$$

$$F_2(\gamma)=\frac{1}{\gamma}\int_{-\infty}^{\infty}dz\{\varphi(z-\gamma)-\varphi(z)\}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}dx\,dy\{\varphi(y-z)[1-\varphi(x)-\varphi(y-x)]+\varphi(x)\varphi(y-x)\},$$

where $\varphi(x) = (e^x + 1)^{-1}$, $\nu(x) = (e^x - 1)^{-1}$ (see for example ref. 8).

In the limiting cases, for ev ≫ kT

$$1/l_{ep}={}^1/_{12}C_1(ev)^3,\qquad 1/l_{ee}={}^1/_6C_2(ev)^2,$$

and for kT ≫ ev

$$1/l_{ep}=14.5C_1(kT)^3,\qquad 1/l_{ee}=6.6C_2(kT)^2$$

The values of $F_1$ and $F_2$ for different values of $\gamma$ are as follows:

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $\gamma$: | 0 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 | 20 |
| $F_1(\gamma)$: | 14.5 | 17 | 25.8 | 44.7 | 76.2 | 123 | 191 | 283 | 398 | 534 | 770 |
| $F_2(\gamma)$: | 6.6 | 7.9 | 9.6 | 13.1 | 17.9 | 23.8 | 31.1 | 39.5 | 48.8 | 59.7 | 72.0 |

The numerical coefficients for kT ≫ ev, like the $F_1(\gamma)$ and $F_2(\gamma)$ values listed, are determined with an accuracy of 3–5%.

In the case of Fig. 8 for T = 1.3° (upper curve) and I = 300 mA, the value of $\gamma$ is 5.3. If we estimate the unknown constants from experiments on measurement of the temperature dependence for small values of I, we can calculate the function ln(A/I) as a function of I, which is associated with the scattering of the accelerated electrons. For 1.3°K the change in ln(A/I) with increase of I from 52 to 300 mA as the result of phonon scattering amounts to only about $7 \times 10^{-2}$, and the upper limit for the contribution of electron-electron scattering lies considerably below this value.

Thus, the dependence observed in our experiments of the relative amplitude A/I on current is evidently due mainly to the defocusing action of the magnetic field of the current. In order to observe the decrease in the electron range as the result of their acceleration in our experiment, it is necessary to decrease substantially the contact size, which should lead to a dominance of the second mechanism over the first.

We are sincerely grateful to A. F. Andreev for his advice on theoretical questions and to P. L. Kapitza for his attention to this work.

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