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NEGATIVE LONGITUDINAL MAGNETORESISTANCE ASSOCIATED WITH SCATTERING FROM IONIZED IMPURITY CENTERS

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The behavior of longitudinal magnetoresistance associated with scattering from impurity ions is analyzed. Because of the strong angular dependence of the Coulomb scattering amplitude the magnetoresistance is in this case considerably more negative than for scattering by phonons and depends significantly on the parameter kR (R is the screening length and k is the electron momentum). It is shown that when the finite time between collisions is taken into account the magnetoresistance becomes less negative. $\Delta\rho_{\parallel}/\rho$ is calculated as a function of $\beta = \hbar\Omega/k_0T$ with $\beta > 3$ for several values of kR. The longitudinal magnetoresistance in tellurium single crystals with impurity concentrations $\sim 10^{15}/cm^3$ is investigated experimentally between 1.6° and 20.4°K. The experimental dependences of $\Delta\rho_{\parallel}/\rho$ on β and kR are in good qualitative agreement with the theory.

INTRODUCTION

It has been shown in[1,2] that when current carriers are scattered by acoustic phonons in the absence of degeneracy, the oscillatory behavior of the state density in a magnetic field when the quantum parameter $\beta = \hbar\Omega/k_0T$ ($\beta < 2$) is small leads to a negative longitudinal magnetoresistance of as much as −12%; k_0 is Boltzmann's constant. The possibility that negative longitudinal magnetoresistance might be associated with scattering from ionized impurity centers was first predicted by Argyres and Adams,[3] who were studying magnetoresistance in the quantum limit. In the present work the behavior of longitudinal magnetoresistance associated with the same scattering mechanism is analyzed in the absence of degeneracy. It is shown that the negative magnetoresistance then considerably exceeds (reaching −90%) the effect which accompanies scattering by phonons and that it can remain negative for $\beta \gg 1$.

The described effect is associated with the fact that when scattering from ionized impurities occurs the magnetic field changes the scattering probability not only through an altered behavior of the state density, but principally because of a changed amplitude of scattering with momentum transfer in the direction of the magnetic field.

The existence of negative magnetoresistance can be accounted for as follows. We know that when an electron is scattered by a screened Coulomb center in the absence of a magnetic field and with $kR \gg 1$ (where R is the screening length and k is the mean electron momentum), the small-angle scattering amplitude greatly exceeds the large-angle amplitude. Therefore, although the resistance is determined by the probability of scattering with a change of momentum in the electric field direction, it is caused mainly by scattering at small angles down to $\theta^0_{min} \sim 1/kR$. In a magnetic field $\mathbf{H} \parallel \mathbf{E} \parallel \mathbf{z}$ energy is quantized in the plane perpendicular to the magnetic field, and the z component k_z of the electron momentum changes by discrete amounts in elastic scattering:

$$k_z'^2 = k_z^2 + 2(n - n')/\lambda^2. \quad (1)$$

Here n and n′ are the numerical indices of the Landau bands before and after scattering; $\lambda^2 = c\hbar/eH$ is the magnetic length. Since scattering with no change of k_z ($k_z' = k_z$) does not affect the resistance, the latter is determined by scattering processes with momentum transfer $\Delta k_z \geq \sqrt{2}/\lambda$, i.e., by scattering at angles exceeding $\theta^H_{min} \sim \sqrt{2}/\lambda k_z \sim \sqrt{\beta}$.

We thus find that in a magnetic field $\lambda/\sqrt{2}$ plays the role of a screening length in the sense that the potential of the Coulomb center is cut off at the smaller of the two lengths $\lambda/\sqrt{2}$ and R. When $\lambda^2/2R^2 \ll 1$ ($\theta^H_{min} > \theta^0_{min}$) the magnetic field, by cutting off the most important region of small-angle scattering, drastically reduces the scattering cross section so that we expect $\Delta\rho/\rho < 0$, whereas in the case of $\lambda^2/2R^2 \gg 1$ ($\theta^H_{min} < \theta^0_{min}$) we can expect $\Delta\rho/\rho > 0$.

In a high magnetic field ($\beta \gg 1$), when transitions are possible only within the "zeroth" Landau band (n = n′ = 0), Eq. (1) shows that the magnetoresistance is determined by backward scattering ($k_z' = -k_z$), or more precisely by scattering into the backward sphere, because a large change can occur in the momentum component perpendicular to the magnetic field. Since the large-angle amplitude of Coulomb scattering is small, the probability of scattering is now much reduced by the presence of a magnetic field and we can expect to obtain a pronounced negative value of $\Delta\rho/\rho$.

It is clear that the parameter kR determines the degree of negativity exhibited by $\Delta\rho/\rho$. Since for $kR \ll 1$ scattering is isotropic in the absence of a magnetic field, the application of a field induces no great change in the transport cross section for scattering. Under these conditions $\Delta\rho/\rho$ should behave in about the same way as for scattering from acoustic phonons, in which case the scattering cross section is also isotropic in the absence of a magnetic field. We can therefore expect that $\Delta\rho/\rho$ becomes more negative with increasing values of kR. The calculation of $\Delta\rho/\rho$ in the region of β from

3 to ∞ yielded an extremely large negative value (up to 90%) of $\Delta\rho/\rho$, and for sufficiently large kR the effect is negative even in the quantum limit.

In actuality a circumstance exists that can considerably diminish the negativity of magnetoresistance for $kR \gg 1$. The study of $\Delta\rho/\rho$ was subject to the customary conditions $\Omega\tau \gg 1$ ($\hbar\Omega \gg \hbar/\tau$) and $k_0T\tau/\hbar \gg 1$ (the uncertainty of the electron energy is much smaller than their mean energy), i.e., the finite time elapsing between collisions has been neglected. It is found that for Coulomb scattering in a magnetic field the finite time between collisions is significant even when that time is large ($\tau k_0T/\hbar \gg 1$). The physical situation can be described as follows. As has already been stated, a high magnetic field and energy conservation exclude a contribution to the resistance from processes involving small momentum transfer. However, when the time between electron collisions in a crystal is of the order τ, the uncertainty in its energy is $\Gamma \sim \hbar/\tau$. Therefore the squared momentum change Δk_z^2 is not necessarily zero, but can amount to $\sim 2m/\hbar\tau$. This signifies that small-angle scattering can actually contribute to the resistance. Since the small-angle scattering amplitude is large for $kR \gg 1$, this effect can be substantial even for $k_0T\tau/\hbar \gg 1$ ($k_0T \gg \Gamma$).

We can evaluate the effect in the following simple manner. We compare the transport relaxation times for "backward" (b) and "forward" (f) scattering:

$$\left(\frac{1}{\tau}\right)_b \sim \frac{\Delta k_z}{k_z} W_b = 2W_b,$$

where W_b is the probability of scattering into the backward hemisphere;

$$\left(\frac{1}{\tau}\right)_f \sim \frac{\Delta k_z}{k_z} W_f \sim \sqrt{\frac{m\Gamma}{\hbar^2 k_z^2}}\, W_f,$$

where W_f is the probability of scattering into the forward hemisphere. It is easily proved that for Coulomb scattering with $kR \gg 1$ we have

$$W_b = W_f / 2(2kR)^2.$$

Then the total relaxation time is

$$\frac{1}{\tau} = \frac{1}{\tau_b} + \frac{1}{\tau_f} \sim 2W_b\left[1 + \sqrt{\frac{\Gamma}{k_0T}}(2kR)^2\right],$$

which means that the correction for small-angle scattering can be significant when $kR \gg 1$ despite the smallness of Γ/k_0T. The inclusion of small-angle scattering increases the resistance by making $\Delta\rho/\rho$ less negative. In the present work we have obtained a correction to $\Delta\rho/\rho$ resulting from this effect for $R/l \ll 1$ (where l is the mean free electron path in the field direction), i.e., when the mean free path exceeds the diameter of the scattering center.

CALCULATION OF LONGITUDINAL MAGNETORESISTANCE ACCOMPANYING SCATTERING FROM IMPURITY IONS IN THE ABSENCE OF DEGENERACY

We shall calculate the longitudinal component of electrical conductivity in a magnetic field, taking account of electron quantization, following[4,5]. The longitudinal current density in a magnetic field is

$$j_z = -eg_H \sum_n \int dk_z\, v_z f_n(k_z); \tag{2}$$

where $f_n(k_z)$, which is linear in E and k_z, represents a correction to the equilibrium distribution function; $v_z = \hbar k_z/m^*$; g_H is determined from

$$n_0 = g_H \sum_n \int dk_z F_0(\varepsilon_n), \tag{3}$$

where n_0 is the concentration of conduction electrons and $F_0(\epsilon)$ is the equilibrium electron distribution in a magnetic field:

$$F_0(\varepsilon_n) = \exp\left[\frac{\mu}{k_0T} - \frac{\varepsilon_n(k_z)}{k_0T}\right],$$
$$\varepsilon_n(k_z) = (n + 1/2)\hbar\Omega + \hbar^2 k_z^2/2m^*. \tag{4}$$

We obtain $f_n(k_z)$ by solving the kinetic equation

$$\frac{eE}{\hbar}\frac{\partial F_0(\varepsilon_n)}{\partial k_z} = \sum_{n', k_y', k_z'} W_{nn'}(k_z', k_y', k_z, k_y)[f_{n'}(k_z') - f_n(k_z)], \tag{5}$$

where $W_{nn'}(k_z', k_y', k_z, k_y)$ is the probability per unit time of an electron transition from the n, k_z, k_y state to n′, k_z', k_y'. The known form in the first approximation of perturbation theory for scattering by a screened Coulomb potential is

$$W_{nn'}(k_z, k_z') = \frac{2\pi}{\hbar}\sum_q |C_q|^2 |Q_n^{n'-n}(u)|^2 \delta(\varepsilon_n - \varepsilon_{n'}) \times \delta_{k_y, k_y'-q_y}\delta_{k_z, k_z'-q_z}, \tag{6}$$

where

$$C_q = \frac{4\pi e^2}{(q^2 + R^{-2})\varkappa}\sqrt{\frac{N}{V}},$$

q is the momentum that is transferred in scattering, κ is the dielectric constant, R is the screening length of the potential, $N/V = n_I$ is the concentration of scattering centers,

$$Q_n^{n'-n}(u) = (n!\,n'!)^{-1/2} u^{(n'-n)/2} e^{-u/2} L_n^{n'-n}(u),$$

$L_n^{n'-n}$ is a generalized Laguerre polynomial, and $u = (q_x^2 + q_y^2)\lambda^2/2$. Inserting $W_{nn'}$ of Eq. (6) into (5), we obtain

$$f_n(k_z) = -e\frac{\partial F_0}{\partial\varepsilon}\frac{\hbar k_z}{m^*}\chi_n E. \tag{7}$$

We sum over k_y' and k_z', integrate with respect to q, and introduce the dimensionless variables $y = \epsilon m^*\lambda^2/\hbar^2$ and $\zeta_n = \sqrt{y - n}$ to obtain

$$\sum_{n'}(A_{nn'}^{+}\tilde{\chi}_n + A_{nn'}^{-}\tilde{\chi}_{n'}) = 1, \tag{8}$$

where

$$A_{nn'}^{+} = \frac{1}{\zeta_n\zeta_{n'}}\left\{\int\frac{|Q_n^{n'-n}(u)|^2 du}{[u + (\zeta_{n'} - \zeta_n)^2 + \lambda^2/2R^2]^2} + \int\frac{|Q_n^{n'-n}(u)|^2 du}{[u + (\zeta_{n'} + \zeta_n)^2 + \lambda^2/2R^2]^2}\right\}$$

$$A_{nn'}^{-} = \frac{1}{\zeta_n\zeta_{n'}}\left\{\int\frac{|Q_n^{n'-n}(u)|^2 du}{[u + (\zeta_{n'} + \zeta_n)^2 + \lambda^2/2R^2]^2} - \int\frac{|Q_n^{n-n'}(u)|^2 du}{[u + (\zeta_{n'} - \zeta_n)^2 + \lambda^2/2R^2]^2}\right\}, \tag{9}$$

with the following relation between $\tilde{\chi}_n$ and χ_n:

$$\chi_n = \frac{\hbar^3\varkappa^2}{\pi e^4 n_I \lambda^4 m^* k_z}\tilde{\chi}_n\left(y, \frac{\lambda^2}{2R^2}\right). \tag{10}$$

Thus for Coulomb scattering the kinetic equation (5) is transformed into the system (8) of n algebraic equations. Electrons having energy of the order ϵ are distributed over $n \sim \epsilon/\hbar\Omega$ Landau levels. To obtain the distribution function we must find a set of $\tilde{\chi}_n$ providing a solution of the n-th order system.

It is worth mentioning that in the case of scattering by acoustic phonons, since the interaction matrix element is independent of q the "arrival" term in the kinetic equation vanishes; we may then introduce the relaxation time and arrive at an independent determination of $f_n(k_z)$ corresponding to the energy ϵ_n.

Inserting g_H from (3) and $f_n(k_z)$ from (7) into (2), and making use of (4), we obtain

$$\sigma_{zz} = \frac{n_0 \varkappa^2 \sqrt{2}\,(k_0T)^{3/2}\beta^3(1-e^{-\beta})}{n_I e^2 \pi^{3/2} m^{*1/2}} \times \sum_n \int_0^\infty dy\, \tilde{\chi}_n(y) e^{-\beta y}. \qquad (11)$$

Let us consider the sum over n. For each unit interval of y ($k < y < k+1$) in the integral with respect to y we have an integrand formed by the superposed solutions of a k-th order system:

$$\sum_n \int_0^\infty dy \tilde{\chi}_n(y) e^{-\beta y} = \sum_{k=0}^\infty \int_k^{k+1} dy \sum_{n=0}^{n=k} \tilde{\chi}_n^{(k)} e^{-\beta y}.$$

In the absence of a magnetic field, as we know,[6] we have

$$\sigma_{zz}^0 = \frac{n_0}{n_I} \frac{2^{7/2}(k_0T)^{3/2}\varkappa^2}{\pi^{3/2} m^{*1/2} e^2 \Phi(\eta)};$$

$$\Phi(\eta) = \ln(1+\eta) - \eta/(1+\eta), \quad \eta = (2kR)^2 \quad \text{при } \varepsilon = 3k_0T. \quad (12)$$

Equations (11) and (12) give for the magnetoresistance:

$$\frac{\Delta\rho}{\rho} = \frac{8}{\Phi(\eta) J(\beta,\eta)} - 1. \qquad (13)$$

We have introduced here the notation

$$\beta^3(1-e^{-\beta}) \sum_{k=0}^\infty \int_k^{k+1} dy \sum_{n=0}^{n=k} \tilde{\chi}_n^{(k)} e^{-\beta y} \equiv J\left(\beta, \frac{\lambda^2}{2R^2}\right). \qquad (14)$$

It is seen from (9) that J is a function of the parameter $\lambda^2/2R^2$; but since $\lambda^2/2R^2 = 12/\beta\eta$, we have $J = J(\beta, \eta)$, which we must now calculate in order to obtain $\Delta\rho/\rho$.

The solution has been obtained in the quantum limit by Argyres and Adams.[3] In this case all electrons are in the first Landau band ($n = n' = 0$), and the system (8) becomes a single equation for the determination of $\tilde{\chi}_0$:

$$\tilde{\chi}_0 = \frac{y}{2} \frac{z}{1 + z e^z \operatorname{Ei}(-z)}, \quad z = 4y + \lambda^2/2R^2. \qquad (15)$$

Here χ_0 equals the relaxation time; in dimensional variables we obtain for the backward relaxation time an expression similar to that obtained in[3]:

$$\frac{1}{\tau} = \frac{\pi e^4 n_I}{\varkappa^2 \sqrt{2m^*}} \varepsilon_z^{-3/2} \frac{1 + z e^z \operatorname{Ei}(-z)}{1 + (2k_z R)^{-2}}. \qquad (16)$$

In the quantum limit, when $\beta \to \infty$, we shall have

$$\tilde{\chi}_0 = \frac{y}{2}\left[4y + \frac{\lambda^2}{2R^2}\right] \qquad (17)$$

and for the limiting value of the magnetoresistance we shall have

$$\frac{\Delta\rho}{\rho} = \frac{2}{\Phi(\eta)[1 + 3/2\eta]} - 1. \qquad (18)$$

Equation (18) shows that for $kR \gg 1$ the magnetoresistance as a function of β approaches a negative limit that increases in absolute value as kR increases. For $kR \lesssim 1$, when the scattering is nearly isotropic, $\Delta\rho/\rho$ is positive. Since $\operatorname{Ei}(-z) < 0$ we see from (15) and (14) that in a very high field J is larger than in the quantum limit, i.e.,

$$\frac{\Delta\rho}{\rho}(H) < \frac{\Delta\rho}{\rho}(\infty),$$

and the curve of $\Delta\rho/\rho$ as a function of β approaches the limit from below. This behavior of $\Delta\rho/\rho$ is associated with the fact that the scattering probability is determined by both the scattering amplitude and the state density. The Coulomb scattering amplitude is diminished as the magnetic field increases, while for $\beta \gg 1$ when $\hbar^2 k_z^2/2m^* \sim k_0T \ll \hbar\Omega$ the state density is considerably greater in the absence of a field and increases with β. These two factors lead to an increase of the magnetoresistance for very large values of β and cause a minimum of $\Delta\rho/\rho$ to exist when β is not too large.

We calculated $\Delta\rho/\rho = f(\beta, \eta)$ for $\beta \geq 3$ and several values of η (230, 9.8, 5.0, and 1.42).[1)] Equation (14) shows that the contribution to $J(\beta, \eta)$ from each succeeding band contains the additional factor $e^{-\beta}$; if $\tilde{\chi}_n$ does not increase too rapidly, for $\beta \geq 3$ we can thus limit ourselves to only two Landau bands. Numerical calculations showed that the contribution of the second band to $J(\beta, \eta)$ comprises about 25% for $\beta = 3$ and about 2% for $\beta = 6$. Figure 1 shows the dependence of $\Delta\rho/\rho$ on β with

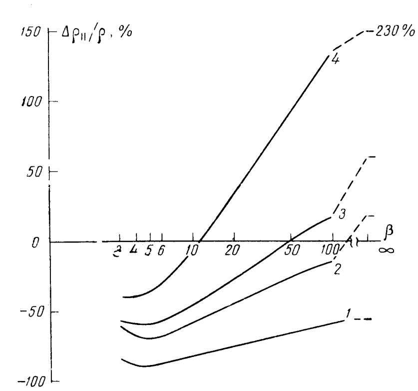

FIG. 1. Dependence of longitudinal magnetoresistance on the parameter β, calculated with two Landau bands for $\beta > 3$. Curves 1,2,3, and 4 were calculated for n = 230, 9.8, 5.0, and 1.42, respectively. The values of $\Delta\rho/\rho$ are given in the quantum limit.

two bands taken into account. All of the foregoing results were obtained in the Born approximation, which can be used for sufficiently fast particles, i.e., when the criterion $e^2/\kappa\hbar v \ll 1$ is satisfied.

EFFECT OF THE FINITE TIME BETWEEN COLLISIONS

As has been indicated in the Introduction, the uncertainty of electron energy that is associated with the finite time between collisions produces the following effect: Scattering with small momentum transfer, which

1)These values of n were selected on the basis of the experimental work that will be discussed below.

is excluded from the resistance in a high magnetic field by the conservation laws, actually does contribute to the resistance. We calculate the correction for this effect in the quantum limit ($\beta \gg 1$, $n = n' = 0$), when we can introduce the transport relaxation time:

$$\frac{1}{\tau_{tr}} = \frac{1}{\chi_0} = -\sum_{k_z} W_{00}(k_z, k_z')\left[1 - \frac{k_z'}{k_z}\right]. \tag{19}$$

Since the uncertainty of electron energy is $\hbar/\tau \sim \Gamma$, energy conservation takes the following form

$$\varepsilon - \varepsilon' = \frac{\hbar}{2m^*}(k_z^2 - k_z'^2) = \frac{\hbar^2}{2m^*}(2k_z q_z + q_z^2) \sim \frac{\Gamma}{2}.$$

The momentum transferred in scattering is

$$q_z^{\pm} \sim -k_z \pm \sqrt{k_z^2 + m^*\Gamma/\hbar^2},$$

so that both backward scattering (q_z^-) and forward scattering (q_z^+) can occur:

$$\frac{1}{\tau_{tr}} = \frac{1}{\tau_b} + \frac{1}{\tau_f}. \tag{20}$$

For backward scattering the correction to the transferred momentum because of the energy spread is small; we may therefore anticipate that the correction to the scattering probability because of this effect is also small, and we shall neglect this effect in connection with backward scattering. Forward scattering results only from the energy spread, and the given effect must be taken into account here. Therefore we use (16) for $1/\tau_b$, and to calculate $1/\tau_f$ we replace $\delta(\epsilon - \epsilon')$ in (6) by

$$\frac{1}{\pi}\frac{\Gamma/2}{(\varepsilon - \varepsilon')^2 + (\Gamma/2)^2}$$

The calculation yields

$$\frac{1}{\tau_f} = \frac{e^4 n_I}{\varkappa^2\sqrt{2m^*}} \times \varepsilon_z^{-3/2}[1 + z_1 e^{z_1}\mathrm{Ei}(-z_1)] \times \frac{R}{2l_z}(2k_z R)\ln\left(\frac{2l_z}{R}\right)^2, \tag{21}$$

where $z_1 = \lambda^2/2R^2$ and l_z is the mean free path in the direction of the field. Equation (21) has been calculated subject to the condition $l_z \gg R$, which means that the mean free path is much larger than the dimension of the scattering center. This condition is required if individual scattering events are to be regarded as independent. We finally obtain

$$\tau_{tr} = \frac{\varkappa^2\sqrt{2m^*}\,\varepsilon_z^{3/2}}{e^4 n_I}\left[\frac{1 + ze^z\mathrm{Ei}(-z)}{1 + (2k_z R)^2} + \frac{1}{\pi}(1 + z_1 e^{z_1}\mathrm{Ei}(-z_1))\frac{R}{2l_z}(2k_z R)\ln\left(\frac{2l_z}{k}\right)^2\right]. \tag{22}$$

To derive the longitudinal magnetoresistance we must calculate σ_{zz} from (3) using τ_{tr} from (22). We obtained a rough estimate of the effect produced by the energy spread for large finite values of β (taking one band into account) by transferring the expression within the square bracket of (22) to a position outside the integral (with $\epsilon_z = 2k_0T$) when calculating σ_{zz}. Then

$$\frac{\Delta\rho}{\rho} = \frac{2}{\Phi(\eta)}\left[\frac{1 + ze^z\mathrm{Ei}(-z)}{1 + 3/2\eta} + \frac{1}{\pi}\frac{R}{2l_z}\sqrt{\frac{2}{3}\eta} \times (1 + z_1 e^{z_1}\mathrm{Ei}(-z_1))\ln\left(\frac{2l_z}{R}\right)^2\right] - 1. \tag{23}$$

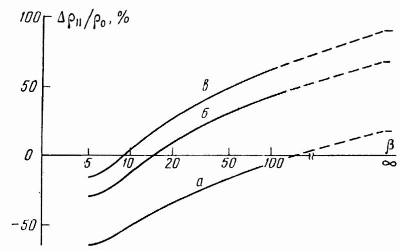

FIG. 2. Magnetoresistance $\Delta\rho_{\parallel}/\rho$ for $\beta > 5$, calculated with account of the finite time between collisions (in the approximation of the first Landau band). Curve b was calculated for $R/2l_z = 0.1$, and curve c for $R/2l_z = 0.2$. These are compared with curve a, which was calculated without account of the finite relaxation time ($R/2l_z = 0$). The values of $\Delta\rho_{\parallel}/\rho$ are given in the quantum limit.

For $\beta \to \infty$ the result is

$$\frac{\Delta\rho}{\rho} = \frac{2}{\Phi(\eta)}\left[\frac{1}{1 + 3/2\eta} + \frac{1}{\pi}\frac{R}{2l_z}\sqrt{\frac{2}{3}\eta}\ln\left(\frac{2l_z}{R}\right)^2\right] - 1. \tag{24}$$

Figure 2 represents the numerical calculation of $\Delta\rho/\rho$ from (23) for $\beta > 5$ and $\eta = 9.8$ and $R/2l_z = 0.1$ or 0.2.

To evaluate $R/2l_z$ we must calculate $l_z = v_z\tau$. We determine l_z from the total lifetime (not the transport lifetime) of all possible scattering processes. For example, an electron-electron interaction, which does not contribute to resistance, reduces the total lifetime of an electron.

AN EXPERIMENTAL INVESTIGATION OF LONGITUDINAL MAGNETORESISTANCE IN TELLURIUM

We have investigated the longitudinal magnetoresistance $\Delta\rho_{\parallel}/\rho$ of tellurium single crystals because the properties of this substance at impurity concentrations $\sim 10^{15}/\mathrm{cm}^3$ and temperatures $T = 1.6-20°$K correspond to the conditions assumed in our calculation. Moreover, it has been reported in[7] that in the indicated temperature and concentration regions tellurium exhibits oscillations of $\Delta\rho_{\parallel}/\rho$ that dip into the region of negative values. A potentiometer was used for dc measurements, and magnetic field signals were recorded with a two-coordinate instrument.

The valence band of tellurium at the investigated concentrations and temperatures is represented by two ellipsoids of rotation along the C_3 axis of the crystal.[8] When the relaxation time is isotropic or possesses cylindrical symmetry, as occurs in tellurium for scattering from ionized impurities,[9] classical theory yields zero longitudinal magnetoresistance for this band. However, tellurium single crystals often exhibit $\Delta\rho_{\parallel}/\rho$ having considerable magnitude even in weak fields and at relatively high temperatures (77°K) when the quantum parameter β is small, apparently because of crystal imperfections.[10] Therefore in order to study the effect of quantization on the longitudinal magnetoresistance (which is our present interest) we had to select tellurium single crystals for which at 77°K we would find $\Delta\rho_{\parallel}/\rho \ll \Delta\rho_{\perp}/\rho$.

Our most interesting results were obtained with a crystal having the concentration $10^{15}/\mathrm{cm}^3$ and

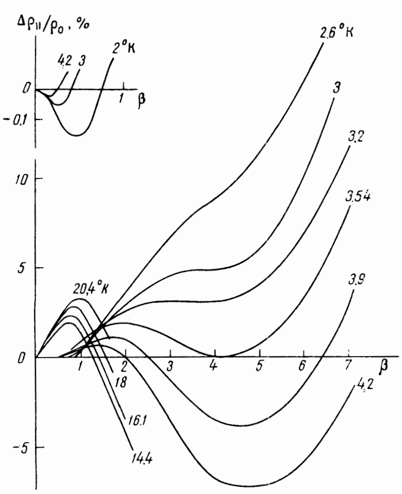

FIG. 3. Experimental dependences of the magnetoresistance $\Delta\rho_{\|}/\rho$ on β at different temperatures. Initial segments of some curves, representing amplified measurements, are shown in the upper left-hand corner.

$(\Delta\rho_{\|}/\rho)/(\Delta\rho_{\perp}/\rho) \approx 0.05$ at 77°K. Figure 3 shows the dependence of $\Delta\rho_{\|}/\rho$ on β for this sample at different temperatures. In the investigated temperature region this sample is nondegenerate; we thus have for the screening length:

$$R=\sqrt{\frac{\varkappa k_0 T}{4\pi e^2 n_0}}, \quad \eta=(2kR)^2=\frac{6m^*(k_0T)^2\varkappa}{\pi n_0\hbar^2 e^2}. \tag{25}$$

We obtained the following values of the reduced chemical potential μ^* and of η for the sample:

T, °K:	1.6	2.6	3.0	3.8	4.2	14.45	20.4
μ^*:	+0.1	−0.8	−1	−1.5	−1.7	−3.5	<−4
η:	1.4	3.8	5	8.00	9.8	116	230

(26)

For the calculation of m* cyclotron masses from[11] were used, and the calculation of the dielectric constant κ was based on data obtained from[12]. The calculated curves shown in Fig. 1 correspond to the temperatures, 1.6, 3, 4.2, and 20.4°K.

It should be noted that at 4°K and lower temperatures in tellurium having the concentration $10^{15}/cm^3$ resistance results mainly from scattering from ionized impurities; scattering by acoustic phonons begins to play a significant part at hydrogen temperatures (14–20°K). It therefore follows from[10] that the ratio between the lattice and ion-dependent components of the mobility is ~60 at 4°K, while at 20°K the ratio is ~0.5. Therefore both scattering mechanisms should be taken into account at hydrogen temperatures.

Figures 1 and 3 show large discrepancies between the calculated and experimental values of $\Delta\rho/\rho$; however, the theory provides a good qualitative description of the behavior of $\Delta\rho/\rho$ for $\beta > 1$. As predicted by the theory, a minimum of the longitudinal magnetoresistance is observed on several experimental curves for $\beta \sim 4$–6. The minimum becomes deeper as kR increases, i.e., as predicted by the theory, the magnitude of the negative effect is associated with the anisotropy of Coulomb scattering.

It has been pointed out in the introduction that a significant negative effect is possible in the region where $\lambda^2/2R^2 = 12/\beta\eta < 1$, i.e., we may expect that for larger values of η (higher T) the transition to the negative region occurs for smaller values of β. Figure 3 shows this effect for temperatures at which only scattering from ionized impurities is important. In the mixed scattering region (at hydrogen temperatures) for $12/\eta\beta < 1$ scattering from ions decreases drastically because of the temperature increase. The inclusion of this scattering mechanism can lead to greatly negative magnetoresistance in this case also. However, since for $\beta > 2$ the magnetoresistance that is associated with scattering by phonons increases linearly with β,[1] the transition to negative magnetoresistance in the presence of both scattering mechanisms occurs for larger values of β than would have been the case for pure scattering from ions at the same value of η. The value of β increases with the temperature, i.e., with the relative role of lattice scattering.

Almost all the experimental curves reveal weak negative magnetoresistance for $\beta \ll 1$. Although we have performed no calculation for $\beta < 1$ and have not considered the asymptotic behavior of $\Delta\rho_{\|}/\rho$ for $\beta \to 0$ in the case of scattering from ionized impurities, we shall attempt to elucidate qualitatively the behavior of $\Delta\rho_{\|}/\rho$ for very small values of β. When $\beta \ll 1/\eta$ the magnetic field is so weak that there is practically no cutting off of small-angle scattering: $\theta^{H}_{min} < \theta^{0}_{min}$ and scattering is determined mainly by η(kR), as in the absence of a field. The effect of quantization is then manifested mainly through the behavior of the state density, as in the case of the isotropic scattering mechanism. It has been shown in[1] that for this case we have

$$\Delta\rho/\rho = \beta \ln C\beta + O(\beta^{3/2}), \tag{27}$$

i.e., when β is very small we have negative $\Delta\rho_{\|}/\rho$, followed by a transition to the positive region or a minimum [if the terms $O(\beta^{3/2})$ begin to play an important role]. We may possibly have observed this effect, upon which a weak dependence on kR (i.e., on temperature) is superimposed in the case of scattering from ions. However, we cannot exclude the possibility that negative magnetoresistance in the initial segment of a curve is associated with the so-called Toyozawa effect.[13]

A comparison of the calculated and experimental curves has thus shown that by taking quantization into effect within the framework of the kinetic equation we are enabled to describe the behavior of longitudinal magnetoresistance qualitatively. What causes the pronounced discrepancies between the experimental and theoretical values of $\Delta\rho_{\|}/\rho$ in the region $\beta > 3$? One source of the discrepancies lies in the fact that the curves in Fig. 1 were calculated without taking account of the finite time between collisions. Through the discussion in the preceding section and Fig. 2 we have become aware that the inclusion of this effect changes $\Delta\rho_{\|}/\rho$ considerably, without essentially altering the dependence on β. It is interesting to estimate the order of magnitude of $R/2l_z$ that is required for this effect under experimental conditions. The mean free path $l_z = v_z\tau$ is determined by the total lifetime of all possible scattering processes. We evaluate τ from the experimental mobility for our sample at 4.2°K. By doing this we exaggerate τ, because the mobility depends on the transport relaxation time, not the total relaxation time. For $n \sim 10^{15}/cm^3$ at 4.2°K in tellurium we have $l_z \sim 5.5 \times 10^{-6}$ cm and $R/2l_z \sim 0.2$. Figure 2 shows that when $R/2l_z$ has this value the aforementioned effect makes an important contribution to the magnetoresistance.

Another source of the discrepancies can be found in the fact that the foregoing calculations were performed subject to the fulfillment of the criterion $\gamma = e^2/\hbar v\kappa \ll 1$, although the following data show that this condition is not satisfied:

T, °K:	1.6	3,0	4,2	20,4
γ:	4	3	2,5	1

(28)

The nonfulfillment has different effects on ρ_H in a high magnetic field and ρ_0 calculated in the Born approximation. The Born expression for the amplitude of scattering by an unscreened Coulomb potential is an exact solution. For backward scattering, induced by interactions at short distances, where screening is insignificant, the solution coincides with the expression for the amplitude of scattering by a screened Coulomb potential. Therefore when ρ_H is calculated in a high field, where it is determined mainly by backward scattering, the nonfulfillment of the Born condition affects the value of ρ_H only slightly.

The resistance in the absence of a field is determined mainly by small-angle forward scattering. This signifies the inclusion of the cross section for scattering by a screened Coulomb potential, for which when $\gamma \ll 1$ is not fulfilled the Born equation yields a value that is too high. As a result the ratio ρ_H/ρ_0 calculated in the Born approximation is too small and the negativity of $\Delta\rho/\rho$ is exaggerated.

We note that at higher temperatures, when the Born condition begins to be better fulfilled and the error resulting from the use of the Born approximation is reduced, the finite time between collisions begins to play an especially large role, since this effect increases with kR.

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