THEORY OF FLUCTUATIONS IN A NONEQUILIBRIUM ELECTRON GAS

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A method is developed for calculating the fluctuation correlation functions in a nonequilibrium electron gas in which electrons are scattered by phonons, impurities (in crystals), or molecules (in gases). Extraneous fluctuation fluxes in an element of phase space are introduced into the kinetic equation. The correlation function is found for these fluxes and employed for determining the correlation functions of the occupation numbers and the current density. These quantities are expressed in terms of the Green's function of the kinetic equation, which, in the case of a nondegenerate electron gas in a strong stationary homogeneous electric field, is calculated for almost elastic collisions, taking or not taking into account weak spatial dispersion.

1. INTRODUCTION

In nonequilibrium thermodynamic systems, the fluctuation spectral density is not connected by a universal relation with the corresponding kinetic coefficient, i.e., the fluctuation-dissipation theorem does not hold. Therefore, the calculation (and measurement) of the fluctuation correlation function for such systems represents a completely independent problem.

We shall set forth briefly the basic results obtained in fluctuation theory for a nonequilibrium electron gas. The fluctuation spectral density in a plasma at high frequencies, when the collisions can be neglected, has been calculated in a number of papers.[1-4] Bunkin[5] considered fluctuations in a nonequilibrium plasma with a finite mean time of flight of the electrons. In the calculation of the correlation function of the velocities, it was assumed that the correlation exists during the relaxation time of the momentum $\tau_1(\epsilon)$ and vanishes after a collision. Actually, such an approach is valid only for frequencies that are greater than the reciprocal of the energy relaxation time, or for spatially homogeneous fluctuations in the plasma, in which the time of scattering of the momentum τ_1 does not depend on the energy of the electrons ϵ_p. Besides the fact that essentially τ_1 usually depends on ϵ_p, here the rather fundamental effect, which is characteristic precisely for nonequilibrium systems, is lost.

The fact is that the energy fluctuation of the electrons is damped after the energy scattering time τ_0, which, for small nonelasticity of the collisions of the electrons with molecules or with lattice vibrations, is much greater than τ_1. The energy of the electron gas fluctuates even in a state of thermodynamic equilibrium. However, only under nonequilibrium conditions (in the presence of a strong field), and when $\tau_1(\epsilon) \neq \text{const}$ are energy fluctuations observed in electric (in particular, current) fluctuations. (We note that under these same conditions, a nonlinear dependence of the current on the field exists.) A significant dispersion of the spectral density of the current fluctuations arises in the frequency region $\omega \sim \tau_0^{-1}$.[6,7] Owing to the mutual effect of the current fluctuations and the energy of the electrons, the fluctuation-dissipation theorem is invalid in a nonequilibrium electron gas even when the energy distribution of the electrons has the form of the equilibrium function with a certain electron temperature.[8,9]

The theory of spatially homogeneous fluctuations of current in a nondegenerate electron gas of a semiconductor in the presence of a strong electric field has been developed in the researches of Gurevich[6] and Gurevich and Katilyus.[10] It was assumed that the correlation function of the occupation numbers satisfies the same kinetic equation as the mean value of the occupation number (the distribution function). The problem was reduced to the calculation of the instantaneous correlation function of the occupation numbers (mean square value). Such a method could be called the method of moments.

In the present work, a different (Langevin) approach is used, based on the introduction of extraneous random fluxes in the kinetic equation for the fluctuations of the occupation numbers. In the final analysis, i.e., from the viewpoint of the results obtained, both approaches are equivalent. However, the Langevin procedure developed here has some advantages, which allow us to calculate the correlation function of the fluctuations (with ease) also for those cases (interelectronic collisions, degenerate gas) for which this problem has not been solved by the method of moments. Furthermore, in the method of moments, in consideration of the spatially homogeneous fluctuations, one takes into account the conservation of the total number of particles N and computes the mean square of the fluctuation of the occupation numbers, keeping components of order N^{-1},[10] which is generally a difficult task. The method of extraneous random fluxes has such a great clarity that it is of no small value in the search for approximate ways of solution of the equations (a detailed comparison of both approaches is given in [11], see also [12], Sec. 24). The Langevin approach is ordinarily used in the theory of fluctuations in electrodynamics ([13,14], Ch. XIII).

The kinetic equation for fluctuations of the occupation numbers with extraneous random fluxes was introduced earlier by a number of authors.[15-17] In the work of Kadomtsev,[15] a scheme for solution of the problem of fluctuations in a nonequilibrium gas with

pair collisions was given in the Langevin approach. The correlation function for random fluxes of particles in a given element of phase space has been found.

Fluctuations about the thermodynamic equilibrium state of the gas was considered in [16, 17].

In a study of fluctuations in a nonequilibrium weakly ionized plasma, Angeleĭko and I. Akhiezer[18] introduced extraneous fluctuations of the occupation numbers, which play the role of sources of the observed fluctuations. Use of extraneous fluctuations of occupation numbers in place of extraneous fluctuation fluxes is less suitable, particularly because, for an estimate of the correlation functions of extraneous sources, it is generally necessary to solve a certain integral equation. Moreover, the correlations of the symmetric parts of extraneous fluctuations of occupation numbers were discarded in [18], which is invalid for spatially inhomogeneous fluctuations in a gas of hot electrons, even for τ_1 = const.

A series of researches by Lax should be noted; these were devoted both to the general theory of fluctuations in nonequilibrium systems and to their application to problems of quantum electronics (see [11, 19]). In particular, Lax showed, under definite assumptions, that the method of moments and the Langevin method are equivalent.[19]

A theory of fluctuations in an electron gas is developed in the present work. In this gas, the transport phenomena are described by the classical kinetic equation. The correlation function of extraneous particle fluxes is computed in a unified fashion, both for the case in which the electrons are scattered by a "thermostat" (by phonons, impurities, molecules), and in the presence of pair collisions. The fluctuation spectral densities of the occupation numbers are expressed in terms of these correlation functions, as are the extraneous currents that enter into the equations for the fluctuation electromagnetic fields. For a nondegenerate gas, these quantities are computed explicitly, with account of spatial dispersion. Although the current fluctuation spectral density is not proportional to the real part of the differential conductivity, they are both expressed in terms of the distribution function of the particles and the Green's function of the kinetic equation. Knowledge of the latter two quantities suffices for calculation of the fluctuations in the nonequilibrium electron gas.

2. CORRELATION FUNCTIONS OF THE FLUCTUATING EXTRANEOUS FLUXES

We consider a gas of electrons scattered by phonons or (and) by impurities (if the problem is one of an electron gas in a semiconductor), or by molecules (in a weakly ionized plasma). We assume that the interelectronic collisions, and also the generation and recombination of electrons are unimportant. We limit ourselves to conditions under which the scattering system can be regarded as a "thermostat." As applied to lattice vibrations, this means that the relaxation time of the phonons (with which the electrons interact), associated with nonelectron processes of scattering (for example, with scattering on the basic mass of phonons) is rather small.

If the field $\mathbf{E}(\mathbf{r}t)$ which acts on the electron is quasiclassical and the interaction with the scattering system is weak, the mean value of the occupation number (the distribution function) $n_{\mathbf{p}}(\mathbf{r}t)$ satisfies the kinetic equation:

$$\hat{\mathscr{L}} n_{\mathbf{p}}(\mathbf{r}t) - S_{\mathbf{p}}\{n\} = 0, \tag{2.1}$$

where

$$\hat{\mathscr{L}} = \partial/\partial t + \mathbf{v}_{\mathbf{p}}\partial/\partial \mathbf{r} + (e\mathbf{E}/\hbar)\partial/\partial \mathbf{p}, \tag{2.2}$$

$$S_{\mathbf{p}}\{n\} = \sum_{\mathbf{p}'} (\bar{J}_{\mathbf{p}'\mathbf{p}} - \bar{J}_{\mathbf{p}\mathbf{p}'}) = \bar{J}_{\mathbf{p}}^{+} - \bar{J}_{\mathbf{p}}^{-}, \tag{2.3}$$

$$\bar{J}_{\mathbf{p}\mathbf{p}'} = W_{\mathbf{p}\mathbf{p}'} n_{\mathbf{p}}(\mathbf{r}t)[1 - n_{\mathbf{p}'}(\mathbf{r}t)], \tag{2.4}$$

$W_{\mathbf{p}\mathbf{p}'}$ is the transition probability of the electron (per unit time) from state $\mathbf{p}$ to state $\mathbf{p}'$. (Unless otherwise mentioned, $\mathbf{p}$ also includes the spin quantum number.)

The flux of particles in each given state $\mathbf{p}$ fluctuates. The flux fluctuations consist of two parts. The first arises directly from the fluctuations of the occupation numbers $\delta n_{\mathbf{p}}(\mathbf{r}t)$ and is equal to the fluctuation change of the collision integral:

$$S_{\mathbf{p}'}\{\delta n\} = \sum_{\mathbf{p}'} [W_{\mathbf{p}'\mathbf{p}}(1 - n_{\mathbf{p}}) + W_{\mathbf{p}\mathbf{p}'} n_{\mathbf{p}}]\delta n_{\mathbf{p}'} - \delta n_{\mathbf{p}} \sum_{\mathbf{p}'} [W_{\mathbf{p}\mathbf{p}'}(1 - n_{\mathbf{p}'}) + W_{\mathbf{p}'\mathbf{p}} n_{\mathbf{p}'}] \equiv \sum_{\mathbf{p}'} K(\mathbf{p}\mathbf{p}')\delta n_{\mathbf{p}'}. \tag{2.5}$$

The second part reflects the random character of the electron scattering and would have existed even if the occupation number had been regarded as given. We shall consider this part of the fluctuation flux as the extraneous source of fluctuations in the kinetic equation. We denote the extraneous flux of particles in state $\mathbf{p}$ by $\delta \mathbf{J}_{\mathbf{p}}(\mathbf{r}t)$:

$$\delta J_{\mathbf{p}}(\mathbf{r}t) = \sum_{\mathbf{p}'} (\delta J_{\mathbf{p}'\mathbf{p}} - \delta J_{\mathbf{p}\mathbf{p}'}) \equiv \delta J_{\mathbf{p}}^{+} - \delta J_{\mathbf{p}}^{-}. \tag{2.6}$$

The equation for the fluctuations of occupation numbers takes the form

$$\hat{\mathscr{L}}\delta n_{\mathbf{p}}(\mathbf{r}t) - S_{\mathbf{p}'}\{\delta n\} = -\frac{e}{\hbar}\frac{\partial n_{\mathbf{p}}}{\partial \mathbf{p}}\left\{\delta \mathbf{E} + \frac{1}{c}[\mathbf{v}_{\mathbf{p}}\delta \mathbf{H}]\right\} + \delta J_{\mathbf{p}}(\mathbf{r}t). \tag{2.7}^{*}$$

Here $\delta\mathbf{E}(\mathbf{r}t)$ and $\delta\mathbf{H}(\mathbf{r}t)$ are the field fluctuations, which are connected by means of the equations of electrodynamics (or Kirchhoff's law in a quasistationary circuit) with the fluctuations of current density and charge, and in the final analysis with $\delta n_{\mathbf{p}}$. Because of the linearity of the equations describing the fluctuations, all the necessary correlation functions are expressed in terms of the correlation function of the extraneous fluxes $\delta \mathbf{J}_{\mathbf{p}}$, which we shall also compute.

If we limit ourselves to quasiclassical fluctuations, which change rather slowly in space (characteristic $k \ll p$) and in time (frequency ω small in comparison with the characteristic energy divided by Planck's constant), then the scattering acts at different points of space and different moments of time are statistically independent and the correlation function of the extraneous fluxes $\langle \delta J_{\mathbf{p}_1}(\mathbf{r}_1 t_1)\delta J_{\mathbf{p}_2}(\mathbf{r}_2 t_2)\rangle$ must be regarded as proportional to $\delta(\mathbf{r}_1 - \mathbf{r}_2)\delta(t_1 - t_2)$ (only such fluctuations can be described by the quasiclassical equation). It follows from (2.6) that (the variables $\mathbf{r}$ and t are

*$[\mathbf{v}_{\mathbf{p}}\delta\mathbf{H}] \equiv \mathbf{v}_{\mathbf{p}} \times \delta\mathbf{H}$.

omitted for brevity):

$$\langle \delta J_{\mathbf{p}_1} \delta J_{\mathbf{p}_2} \rangle = \sum_{\mathbf{p}_1' \mathbf{p}_2'} [\langle \delta J_{\mathbf{p}_1' \mathbf{p}_1} \delta J_{\mathbf{p}_2' \mathbf{p}_2} \rangle + \langle \delta J_{\mathbf{p}_1 \mathbf{p}_1'} \delta J_{\mathbf{p}_2 \mathbf{p}_2'} \rangle - \langle \delta J_{\mathbf{p}_1' \mathbf{p}_1} \delta J_{\mathbf{p}_2 \mathbf{p}_2'} \rangle - \langle \delta J_{\mathbf{p}_1 \mathbf{p}_1'} \delta J_{\mathbf{p}_2' \mathbf{p}_2} \rangle]. \quad (2.8)$$

The flow of particles between the states, $J_{\mathbf{p}_1\mathbf{p}_1'}$ and $J_{\mathbf{p}_2\mathbf{p}_2'}$ are correlated when and only when the initial and final states are identical (V is the volume of the crystal):

$$\langle \delta J_{\mathbf{p}_1\mathbf{p}_1'}(\mathbf{r}_1 t_1) \delta J_{\mathbf{p}_2\mathbf{p}_2'}(\mathbf{r}_2 t_2) \rangle = V \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(t_1 - t_2) \bar{J}_{\mathbf{p}_1\mathbf{p}_1'} \delta_{\mathbf{p}_1\mathbf{p}_2} \delta_{\mathbf{p}_1'\mathbf{p}_2'}. \quad (2.9)$$

Here we have used the fact that the fluctuations of collisions numbers are analogous to shot noise, and have assumed $\langle \delta J^2_{\mathbf{p}_1\mathbf{p}_2} \rangle = \bar{J}_{\mathbf{p}_1\mathbf{p}_2}$ (a Poisson process). The property (2.9) is based on the same independence of the separate acts of scattering, which lie at the basis of the collision integral of the kinetic equation (2.3).

From (2.8) and (2.9), we get

$$\langle \delta J_{\mathbf{p}_1}(\mathbf{r}_1 t_1) \delta J_{\mathbf{p}_2}(\mathbf{r}_2 t_2) \rangle = V \delta(\mathbf{r}_1 - \mathbf{r}_2) \delta(t_1 - t_2) \{\delta_{\mathbf{p}_1\mathbf{p}_2}(\bar{J}^+_{\mathbf{p}_1} + \bar{J}^-_{\mathbf{p}_1}) - \bar{J}_{\mathbf{p}_1\mathbf{p}_2} - \bar{J}_{\mathbf{p}_2\mathbf{p}_1}\}. \quad (2.10)$$

This expression has a simple meaning. The fluxes $\delta J^+_{\mathbf{p}_1}$ and $\delta J^+_{\mathbf{p}_2}$ are correlated only when $\mathbf{p}_1 = \mathbf{p}_2$ and their correlation functions are proportional to the mean flux $J^+_{\mathbf{p}_1}$. In similar fashion, the correlations of fluxes following from $\mathbf{p}_1$ and $\mathbf{p}_2$ are proportional to $\delta_{\mathbf{p}_1\mathbf{p}_2} \bar{J}^-_{\mathbf{p}_1}$. The flux of the particles coming into $\mathbf{p}_1$ and the flux of particles coming from $\mathbf{p}_2$ are correlated only if $\mathbf{p}_1 \neq \mathbf{p}_2$ owing to the fact that the entry of a particle into $\mathbf{p}_1$ from $\mathbf{p}_2$ is at the same time the emission of a particle from $\mathbf{p}_2$ (into $\mathbf{p}_1$). The latter reflects the conservation of the number of particles in collisions.

We use the same reasoning for the derivation of the correlation of extraneous fluxes in a gas with pair collisions. We assume that particles with momenta $\mathbf{p}_1$ and $\mathbf{p}_2$ and identical projections of spins (σ) collide and take on the momenta $\mathbf{k}_1$ and $\mathbf{k}_2$. We denote the number of such collisions by $J^{\uparrow\uparrow}(\mathbf{p}_1\mathbf{p}_2; \mathbf{k}_1\mathbf{k}_2)$, and the number of collisions of particles with opposite spins by $J^{\uparrow\downarrow}(\mathbf{p}_1\mathbf{p}_2 \mathbf{k}_1\mathbf{k}_2)$. The extraneous fluctuation flux is equal to

$$\delta J(\mathbf{p}\sigma) = \sum_{\mathbf{p}'\mathbf{k}_1\mathbf{k}_2} \left\{ \frac{1}{2} \delta J^{\uparrow\uparrow}(\mathbf{k}_1\mathbf{k}_2; \mathbf{p}'\mathbf{p}) + \delta J^{\uparrow\downarrow}(\mathbf{k}_1\mathbf{k}_2; \mathbf{p}'\mathbf{p}) - \frac{1}{2} \delta J^{\uparrow\uparrow}(\mathbf{p}\mathbf{p}'; \mathbf{k}_1\mathbf{k}_2) - \delta J^{\uparrow\downarrow}(\mathbf{p}\mathbf{p}'; \mathbf{k}_1\mathbf{k}_2) \right\}. \quad (2.11)$$

The factors $^1/_2$ take into account the identical nature of the particles (see, for example, [20]). In the calculation of the correlation functions we again use the fact that the extraneous fluxes between the states are correlated only each with itself. Therefore (spin indices are again omitted):

$$\langle \delta J_{\mathbf{p}_1}(\mathbf{r}_1 t_1) \delta J_{\mathbf{p}_2}(\mathbf{r}_2 t_2) \rangle = V \delta(t_1 - t_2) \delta(\mathbf{r}_1 - \mathbf{r}_2) \{\delta_{\mathbf{p}_1\mathbf{p}_2}[\bar{J}^+_{\mathbf{p}_1} + \bar{J}^-_{\mathbf{p}_1}] + \bar{J}^+(\mathbf{p}_1\mathbf{p}_2) + \bar{J}^-(\mathbf{p}_1\mathbf{p}_2) - \bar{J}_{\mathbf{p}_1\mathbf{p}_2} - \bar{J}_{\mathbf{p}_2\mathbf{p}_1}\}. \quad (2.12)$$

Here $\bar{J}^-(\mathbf{p}_1\mathbf{p}_2)$ is the mean number of collisions of particles from $\mathbf{p}_1$ with particles from $\mathbf{p}_2$, $\bar{J}^+(\mathbf{p}_1\mathbf{p}_2)$ is the number of collisions ending in the simultaneous appearance of particles in $\mathbf{p}_1$, and $\mathbf{p}_2$, $\bar{J}_{\mathbf{p}_1\mathbf{p}_2}$ is in each case equal to the number of collisions, at the input of which there is a particle from $\mathbf{p}_1$ and at the output a particle in $\mathbf{p}_2$:

$$\bar{J}_{\mathbf{p}_1\mathbf{p}_2} = \sum_{\mathbf{k}_1\mathbf{k}_2} \{\delta_{\sigma_1\sigma_2} \overline{J^{\uparrow\uparrow}}(\mathbf{p}_1\mathbf{k}_1; \mathbf{p}_2\mathbf{k}_2) + \overline{J^{\uparrow\downarrow}}(\mathbf{p}_1\mathbf{k}_1; \mathbf{p}_2\mathbf{k}_2)\}.$$

By virtue of the conservation of the number of particles in the collisions $\sum_{\mathbf{p}} \delta J_{\mathbf{p}} = 0$, and we should have

$$\sum_{\mathbf{p}_1} \langle \delta J_{\mathbf{p}_1}(\mathbf{r}_1 t_1) \delta J_{\mathbf{p}_2}(\mathbf{r}_2 t_2) \rangle = \sum_{\mathbf{p}_2} \langle \delta J_{\mathbf{p}_1}(\mathbf{r}_1 t_1) \delta J_{\mathbf{p}_2}(\mathbf{r}_2 t_2) \rangle = 0. \quad (2.13)$$

It is easy to prove that Eqs. (2.10) and (2.12) satisfy this condition.

If the gas is nondegenerate and the exchange effects unimportant, it is convenient to use the expression for the correlation function of extraneous fluxes, which is obtained from (2.12) after summation over the spins:

$$\langle \delta J_{\mathbf{p}_1}(\mathbf{r}_1 t_1) \delta J_{\mathbf{p}_2}(\mathbf{r}_2 t_2) \rangle = V \delta(t_1 - t_2) \delta(\mathbf{r}_1 - \mathbf{r}_2) \times \Big\{ \delta_{\mathbf{p}_1\mathbf{p}_2} \sum_{\mathbf{k}_1\mathbf{k}_2\mathbf{p}} [W(\mathbf{k}_1\mathbf{k}_2; \mathbf{p}_1\mathbf{p}) n_{\mathbf{k}_1} n_{\mathbf{k}_2} + W(\mathbf{p}_1\mathbf{p}; \mathbf{k}_1\mathbf{k}_2) n_{\mathbf{p}_1} n_{\mathbf{p}}] + \sum_{\mathbf{k}_1\mathbf{k}_2} [W(\mathbf{k}_1\mathbf{k}_2; \mathbf{p}_1\mathbf{p}_2) n_{\mathbf{k}_1} n_{\mathbf{k}_2} + W(\mathbf{p}_1\mathbf{p}_2; \mathbf{k}_1\mathbf{k}_2) n_{\mathbf{p}_1} n_{\mathbf{p}_2}] - 2 \sum_{\mathbf{k}_1\mathbf{k}_2} [W(\mathbf{k}_1\mathbf{p}_1; \mathbf{k}_2\mathbf{p}_2) n_{\mathbf{k}_1} n_{\mathbf{p}_1} + W(\mathbf{k}_2\mathbf{p}_2; \mathbf{k}_1\mathbf{p}_1) n_{\mathbf{k}_2} n_{\mathbf{p}_2}] \Big\}. \quad (2.14)$$

Here $W(\mathbf{k}_1\mathbf{k}_2; \mathbf{p}_1\mathbf{p}_2)$ is the probability of collision of two particles with momenta $\mathbf{k}_1$ and $\mathbf{k}_2$, after which the emerging particles have momenta $\mathbf{p}_1$ and $\mathbf{p}_2$.

The expression (2.14) corresponds to the formula (6) of the work of Kadomtsev.[15] In thermodynamic equilibrium, there is detailed balance and Eq. (2.13) goes over into the correlation function of the extraneous fluxes, found by a different method in the work of Gor'kov, Dzyaloshinskiĭ, and Pitaevskiĭ.[16]

3. GREEN'S FUNCTION OF THE KINETIC EQUATION

The calculation of the correlation function of the fluctuations of the occupation numbers includes, according to (2.7), the solution of the inhomogeneous linearized kinetic equation. It is convenient to represent the desired quantity in terms of the Green's function, which satisfies the equation

$$\hat{\mathscr{L}}_{\mathbf{p}}(\mathbf{r}t) G_{\mathbf{p}\mathbf{p}'}(\mathbf{r}t, \mathbf{r}'t') - S_{\mathbf{p}'}\{G\} = V \delta(\mathbf{r} - \mathbf{r}') \delta(t - t') \delta_{\mathbf{p}\mathbf{p}'} \quad (3.1)$$

and the condition $G = 0$ for $t \leq t'$. It has the meaning of the change in the mean occupation number of state $\mathbf{p}$ at the point $\mathbf{r}$ at the time t as a result of the introduction, at the point $\mathbf{r}'$ and time t' of a single particle with momentum $\mathbf{p}'$. We now enumerate the properties of the Green's function.

The limiting value of G as $t - t' \to \infty$ is equal to the change in the distribution function $n_{\mathbf{p}}(\mathbf{r}t)$ associated with the increase in the total number of particles in the system

$$N = V^{-1} \sum_{\mathbf{p}} \int d^3 r n_{\mathbf{p}}(\mathbf{r}t), \quad (3.2)$$

by one:

$$\lim_{t - t' \to \infty} G_{\mathbf{p}\mathbf{p}'}(\mathbf{r}t, \mathbf{r}'t') = \left(\frac{\partial n_{\mathbf{p}}(\mathbf{r}t)}{\partial N} \right)_E \quad (3.3)$$

Here the field is regarded as fixed. The derivative $(\partial n_{\mathbf{p}}/\partial N)_E$ satisfies Eq. (2.7) without the right-hand part. In a nondegenerate gas without pair collisions,

$$\left(\frac{\partial n_{\mathbf{p}}(\mathbf{r}t)}{\partial N} \right)_E = \frac{n_{\mathbf{p}}(\mathbf{r}t)}{N}. \quad (3.3a)$$

It follows from (3.1) that

$$V^{-1}\sum_{\mathbf{p}}\int d^3r G_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t') = \Theta(t-t') \tag{3.4}$$

(conservation of the number of particles).

By means of (3.1), it is easy to prove that the following sum rule holds:

$$V^{-1}\sum_{\mathbf{p}'}\int d^3r' G_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t')\frac{\partial n_{\mathbf{p}'}(\mathbf{r}'t')}{\partial N} = \Theta(t-t')\frac{\partial n_{\mathbf{p}}(\mathbf{r}t)}{\partial N}. \tag{3.5}$$

The equation for $G_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t')$ as a function of the arguments $\mathbf{p}'$, $\mathbf{r}'$, t' is (here K is the kernel of the integral equation $S_{\mathbf{p}'}$)

$$-\hat{\mathscr{L}}_{\mathbf{p}'}(\mathbf{r}'t')G_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t') - \sum_{\mathbf{p}''}G_{\mathbf{pp}''}(\mathbf{r}t, \mathbf{r}'t')K(\mathbf{p}''\mathbf{p}') = V\delta(\mathbf{r}-\mathbf{r}')\delta(t-t')\delta_{\mathbf{pp}'}. \tag{3.6}$$

Inasmuch as $\sum_{\mathbf{p}}\delta J_{\mathbf{p}} = 0$ (see (2.13)), we actually need the function

$$g_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t') = G_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t') - \Theta(t-t')\frac{\partial n_{\mathbf{p}}(\mathbf{r}t)}{\partial N}, \tag{3.7}$$

which, in accord with (3.3), can be called the relaxing part of the Green's function. It follows from (3.4), (3.5), and (3.2) that

$$\sum_{\mathbf{p}}\int d^3r g_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t') = 0, \tag{3.4a}$$

$$\sum_{\mathbf{p}'}\int d^3r' g_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t')\frac{\partial n_{\mathbf{p}'}(\mathbf{r}'t')}{\partial N} = 0. \tag{3.5a}$$

For stationary systems, we can write down the sum rule over the frequencies relative to the Fourier components $g(\omega)$ over the time difference $t-t'$. From the analyticity of $g(\omega)$ in the upper half-plane of ω we obtain the dispersion relations between Re $g(\omega)$ and Im $g(\omega)$ in the usual way. Furthermore, using the fact that as $\omega \to \infty$

$$g_{\mathbf{pp}'}(\mathbf{r}\mathbf{r}', \omega) \approx i\omega^{-1}\left[V\delta_{\mathbf{pp}'}\delta(\mathbf{r}-\mathbf{r}') - \frac{\partial n_{\mathbf{p}}(\mathbf{r})}{\partial N}\right] \tag{3.8}$$

(this follows from (3.1) and (3.7)), we can easily establish the following sum rule over the frequencies:

$$\frac{1}{\pi}\int_{-\infty}^{\infty} d\omega\, \mathrm{Re}\, g_{\mathbf{pp}'}(\mathbf{r}\mathbf{r}', \omega) = V\delta_{\mathbf{pp}'}\delta(\mathbf{r}-\mathbf{r}') - \frac{\partial n_{\mathbf{p}}(\mathbf{r})}{\partial N}. \tag{3.9}$$

We find the Green's function for the homogeneous stationary nondegenerate gas under the conditions that the inelasticity of the collisions of the electrons (with phonons, impurities, molecules). In this case, as is known, the distribution function in a strong electric field is calculated by the method of Davydov,[21, 22] which is based on the smallness of the quantities $eE\tau_1/mv \sim \delta^{1/2} \ll 1$ and $kv\tau_1 \ll 1$, where τ_1 is the scattering time of the momentum, and δ the characteristic parameter of inelasticity.

Let us represent the Fourier component of the Green's function[1)]

$$G_{\mathbf{pp}'}(\mathbf{k}\omega) = V^{-1}\int d(\mathbf{r}-\mathbf{r}')\,d(t-t')\,e^{i\omega(t-t')-i\mathbf{k}(\mathbf{r}-\mathbf{r}')}G_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t') \tag{3.10}$$

in the form of an expansion in spherical harmonics:

$$G_{\mathbf{pp}'}(\mathbf{k}\omega) = \sum_{lm;\, l'm'} G_{lm;\, l'm'}(pp';\, \mathbf{k}\omega)\, Y_{lm}(\theta, \varphi)\, Y^{*}_{l'm'}(\theta', \varphi'), \tag{3.11}$$

which we agree to normalize to 4π. We assume that the scattering probability $W_{\mathbf{pp}'}$ is a function of p, p'' and the angle $\theta = \mathbf{p}\cdot\mathbf{p}''$. The equations for $G_{lm;\, l'm'}$ have the form

$$\begin{aligned}(-i\omega+\eta)G_{lm;\,l'm'} + i(\mathbf{k}v_{\mathbf{p}}G)_{lm;\,l'm'} + (2l+1)^{-1/2}\frac{eE}{N(\varepsilon)} \\ \times\left\{\left[\frac{(l+1)^2-m^2}{2l+3}\right]^{1/2}\varepsilon^{-l/2}\frac{d}{d\varepsilon}[\varepsilon^{l/2}vN(\varepsilon)G_{l+1,\,m;\,l'm'}]\right. \\ \left.+\left(\frac{l^2-m^2}{2l-1}\right)^{1/2}\varepsilon^{l/2}v\frac{d}{d\varepsilon}[\varepsilon^{-l/2}N(\varepsilon)G_{l-1,\,m;\,l'm'}]\right\} + \sum_{\mathbf{p}''}[W_{\mathbf{pp}''}G_{lm;\,l'm'}(pp') \\ - W_{\mathbf{p}''\mathbf{p}}P_l(\cos\theta'')G_{lm;\,l'm'}(p''p')] = [\delta(\varepsilon-\varepsilon')/N(\varepsilon)V]\delta_{ll'}\delta_{mm'}.\end{aligned} \tag{3.12}$$

in this case. Here $\epsilon = \hbar^2p^2/2m$, the density of electron states without account of the spin $N(\epsilon) = (m^3\epsilon/2)^{1/2}/\pi^2\hbar^3$, the polar axis is directed along the field E, $\eta \to +0$.

As usual in the case of weakly inelastic scattering, we can neglect, for all $l \neq 0$, the differences between p and p'' in the collision integral and introduce the relaxation time $\tau_l(\epsilon)$, where

$$\tau_l^{-1}(\varepsilon) = \sum_{\mathbf{p}''}W_{\mathbf{pp}''}[1-P_l(\cos\theta'')], \quad l \geqslant 1. \tag{3.13}$$

It is known that for small inelasticity of collisions, the harmonic functions of the electron distribution fall off with increase in l even in a strong electric field.[21] For harmonic Green's functions with $l = l'$ and $m = m'$, this does not generally hold, especially for small times $t-t'$. For example, we give the expression for $G_{lm;\,l'm'}$ with $l \neq 0$ in the absence of an electric field and for $k = 0$ (it follows from (3.12)):

$$G_{lm,\,l'm'}(pp';\,0\omega) = (-i\omega+\tau_l^{-1})^{-1}[\delta(\varepsilon-\varepsilon')/N(\varepsilon)V]\delta_{ll'}\delta_{mm'}. \tag{3.14}$$

It is seen that the harmonic Green's functions $G_{lm;\,lm}$ with different l and m are generally of the same order. Nevertheless the Davydov approximation can be used in the definite form and in the calculation of the Green's function. It follows from (3.12) that for each given l' the harmonics with $l > l'$ are less than the diagonal $G_{l'l'}$ and fall off with increase of l, just as the corresponding harmonics of the distribution function. This makes it possible to cut off the chain of equations at an arbitrary $l = l' + 1$.

For the calculation of the various physical quantities in terms of the Green's function, only its harmonics with l, $l' = 0, 1$ are usually necessary. For brevity, we represent $g_{\mathbf{pp}'}(\mathbf{k}\omega)$ in the form of a sum of symmetric (g_0) and antisymmetric (g_a) parts (in $\mathbf{p}$), meaning thereby only the harmonics with $l = 0$ and $l = 1$, respectively. It follows from the equation for $g_{\mathbf{pp}'}(\mathbf{k}\omega)$ (see (3.1), (3.6)) and

$$\left[-i\omega+\eta+i\mathbf{k}v_{\mathbf{p}}+\frac{e\mathbf{E}}{\hbar}\nabla_{\mathbf{p}}\right]g - S_{\mathbf{p}}\{g\} = \left[\delta_{\mathbf{pp}'}-\frac{n_{\mathbf{p}}}{N}\delta_{\mathbf{k}0}\right] \tag{3.15}$$

that

$$\begin{aligned}g_{a\mathbf{pp}'}(\mathbf{k}\omega) = (-i\omega+\tau_1^{-1})^{-1}\{(\delta_{\mathbf{pp}'}-\delta_{-\mathbf{p},\,\mathbf{p}'})/2 - i\mathbf{k}vg_0 \\ -(e\mathbf{E}/\hbar)\nabla_{\mathbf{p}}g_0 - (n_a/N)\delta_{\mathbf{k}0}\},\end{aligned} \tag{3.16}$$

[1)] We shall not consider further in this section scattering with spin reversal; therefore G will be diagonal in the spin indices and any of the diagonal components of the Green's function is denoted below by $G_{\mathbf{pp}'}(\mathbf{r}t, \mathbf{r}'t')$, while $N = nV/2$, where n is the concentration.

$$\frac{d}{d\varepsilon}\left[\frac{N(\varepsilon)\varepsilon}{\tau_0}\left(T^*\frac{d}{d\varepsilon}+1\right)g_0\right]+N(\varepsilon)\left[i\omega-\frac{k^2v^2\tau_1}{3}+ie\mathbf{kE}\frac{v^2\tau_1}{3}\frac{d}{d\varepsilon}\right]g_0$$
$$+ie\frac{\mathbf{kE}}{3}\frac{d}{d\varepsilon}[N(\varepsilon)\tau_1v^2g_0]=\delta_{\mathbf{k}0}n_0(\varepsilon)\frac{N(\varepsilon)}{N}$$
$$-V^{-1}\left[1-i\mathbf{kv}'\tau_1(\varepsilon')-e\mathbf{E}v'\tau_1(\varepsilon')\frac{d}{d\varepsilon}\right]\delta(\varepsilon-\varepsilon'). \quad (3.17)$$

Here n_0 and n_a are the symmetric and antisymmetric parts of the distribution function, τ_0 the scattering time of the energy,

$$N(\varepsilon)\varepsilon\tau_0^{-1}(\varepsilon)=(VT)^{-1}\sum_{\mathbf{p}'\mathbf{p}''}W^{\mathrm{abs}}_{\mathbf{p}'\mathbf{p}''}(\varepsilon_{\mathbf{p}''}-\varepsilon)^2\delta(\varepsilon_{\mathbf{p}'}-\varepsilon). \quad (3.18)$$

where $W^{\mathrm{abs}}_{\mathbf{p}'\mathbf{p}''}$ is the probability of collision with absorption of energy, T the temperature of the scattering system,

$$T^*(\varepsilon)=T+2e^2E^2\tau_1(\varepsilon)\tau_0(\varepsilon)/3m, \quad (3.19)$$

$\mathbf{v}'\equiv\mathbf{v}_{\mathbf{p}'}$. It is assumed in (3.17) that $\omega\tau_1\ll 1$.

We seek such a solution of Eq. (3.17) which is normalized, i.e., the solution itself and also its derivatives tend to zero necessarily as $\epsilon\to\infty$ and satisfy the continuity equation

$$\int_0^\infty d\varepsilon N(\varepsilon)\left[-i\omega+\frac{k^2v^2\tau_1}{3}-\frac{ie\mathbf{kE}v^2\tau_1}{3}\frac{d}{d\varepsilon}\right]g_0(\varepsilon\mathbf{p}';\mathbf{k}\omega)$$
$$=V^{-1}[1-\delta_{\mathbf{k}0}-i\mathbf{kv}_{\mathbf{p}'}\tau_1(\varepsilon')]. \quad (3.20)$$

The latter follows from (3.15) and reflects the conservation of the number of particles (see (3.4)).

We first consider the case $\mathbf{k}=0$. For $\omega\tau_0\ll 1$ we can neglect the component with ω in the equation for g_0 ($g(\omega)$, in contrast with $G(\omega)$, does not have a pole singularity as $\omega\to 0$). Equation (3.17) can easily be integrated with account of (3.20):

$$g_0(\varepsilon\mathbf{p}';0,0)=\frac{n_0(\varepsilon)}{V}\int_0\frac{dx\tau_0(x)}{N(x)xT^*n_0(x)}[\Theta(\varepsilon-x)-f(x)]$$
$$\times[\Theta(\varepsilon'-x)-f(x)+e\mathbf{E}v_{\mathbf{p}'}\tau_1(\varepsilon')\delta(x-\varepsilon')]. \quad (3.21)$$

Here

$$f(\varepsilon)=\frac{2}{n}\int_\varepsilon^\infty dx\,N(x)n_0(x), \quad (3.22)$$

where n is the concentration of electrons.

At high frequencies, $\omega\tau_0\gg 1$, the imaginary part of $g_0(\epsilon\mathbf{p}';0\omega)$ falls off with frequency as $(\omega\tau_0)^{-1}$ while the real part falls off at $(\omega\tau_0)^{-2}$ and they can easily be computed by iterating (3.17).

For $\mathbf{k}\neq 0$, the function $g_0(\epsilon\mathbf{p}';\mathbf{k}\omega)$ describes not only the relaxation of the energy of the particles, but also the relaxation in coordinate space. Equation (3.17) for g_0 can be solved in general form only when the following characteristic parameters are small in comparison with unity (or, conversely, are large): $\omega\tau_0$, $k^2l_e^2$, and kl_{de}. Here $l_e=(v^2\tau_1\tau_0/3)^{1/2}$ is the energy scattering length, $l_{de}=eE\tau_1\tau_0/3m$ is the drift distance in the time τ_0. We note that the parameters $k^2l_e^2$ and kl_{de} can be represented in the form

$$k^2l_e^2\sim\frac{T^*-T}{T^*}\left(\frac{kT^*}{eE}\right)^2,\quad kl_{de}\sim\frac{T^*-T}{T^*}\frac{kT^*}{eE}. \quad (3.23)$$

If the heating up of the electron gas is large, i.e., $T^*-T\sim T^*$, then the parameters shown are small only for such k for which[2)]

$$kT^*\ll eE. \quad (3.24)$$

This restriction does not apply to k if the heating is insignificant and the nonequilibrium character of the system reduces only to the presence of electron drift in it.

We now calculate g_0 for the case of weak spatial and temporal dispersion, i.e., under the conditions

$$\omega\tau_0\ll 1,\quad k^2l_e^2\ll 1,\quad kl_{de}\ll 1. \quad (3.25)$$

For compactness, we introduce the notation

$$\hat{I}g_0=\frac{N(\varepsilon)\varepsilon}{\tau_0(\varepsilon)}\left(T^*\frac{d}{d\varepsilon}+1\right)g_0, \quad (3.26)$$

$$F(\varepsilon\mathbf{p}';\mathbf{k})=\left[1-i\mathbf{kv}_{\mathbf{p}'}\tau_1(\varepsilon')+e\mathbf{E}v_{\mathbf{p}'}\tau_1(\varepsilon')\frac{d}{d\varepsilon'}\right]\Theta(\varepsilon'-\varepsilon)-\delta_{\mathbf{k},0}f(\varepsilon),$$
$$\hat{\lambda}g_0=N(\varepsilon)\left[-i\omega-ie\mathbf{kE}\frac{2e\tau_1}{3m}\frac{d}{d\varepsilon}+k^2\frac{2e\tau_1}{3m}\right]g_0 \quad (3.27)$$
$$-ie\mathbf{kE}\frac{d}{d\varepsilon}\left[\frac{2e\tau_1}{3m}N(\varepsilon)g_0\right]. \quad (3.28)$$

The quantity $\hat{I}g_0$ has the meaning of a flux of particles caused by the field and collisions, out of the energy region bounded by the isoenergetic surface ϵ.

Taking into account the normalizability condition for g_0, Eq. (3.17) can be integrated once and, using (3.26)–(3.28), the result written in the form

$$\hat{I}g_0+\int_\varepsilon^\infty dx\hat{\lambda}g_0=V^{-1}F(\varepsilon\mathbf{p}';\mathbf{k}). \quad (3.29)$$

Taking the inequalities (3.25) into account, we find the particular solution (3.29) and the solution of the corresponding homogeneous equation by the method of successive approximations in $\hat{\lambda}$. We require that the desired function g_0 satisfy the condition (3.20). As a result we obtain the following, with the required degree of accuracy:

$$g_0(\varepsilon\mathbf{p}';\mathbf{k}\omega)=\frac{2n_0(\varepsilon)}{nV}\left\{C\left[1+\int_\varepsilon^\infty\frac{dx\,\tau_0(x)}{N(x)xT^*n_0}\int_x^\infty dx'\hat{\lambda}n_0\right]\right.$$
$$\left.-\frac{n}{2}\int_\varepsilon^\infty\frac{dx\,\tau_0(x)F(x\mathbf{p}')}{N(x)xT^*n_0(x)}\right\}, \quad (3.30)$$

$$C=\left\{1-\delta_{\mathbf{k},0}-i\mathbf{kv}_{\mathbf{p}'}\tau_1(\varepsilon')+\int_0^\infty dx\hat{\lambda}n_0\int_x^\infty dx'\frac{\tau_0(x')F(x'\mathbf{p}')}{N(x')x'T^*n_0(x')}\right\}$$
$$\times\left\{-i(\omega-\mathbf{ku})+k^2D+\frac{2}{n}\int_0dx\hat{\lambda}n_0\int_x dx'\frac{\tau_0(x')\int_{x'}^\infty dx''\hat{\lambda}n_0(x'')}{N(x')x'T^*n_0(x')}\right\}^{-1}. \quad (3.31)$$

Here

$$\mathbf{u}=\frac{4e\mathbf{E}}{3nm}\int_0^\infty dx\,N(x)x\tau_1(x)\left(-\frac{dn_0}{dx}\right) \quad (3.32a)$$

is the drift velocity in a constant field, and

$$D=\frac{4}{3mn}\int_0dx\,N(x)x\tau_1(x)n_0(x) \quad (3.32b)$$

is the diffusion coefficient in the direction perpendicular to the constant electric field.

We note that the solution (3.30) satisfying the physical requirements (in particular, (3.20)) contains small parameters in the denominator. Therefore, it is impossible to obtain g_0 directly by iterations of Eq. (3.9). We also note that from the inequality (3.24), which follows

[2)] A similar restriction arises, naturally, in the solution of the problem in the electron temperature approximation. [23]

from (3.25) and is also the condition of weakness of spatial dispersion in strong heating, it follows that $k^2D \ll ku$.

Equation (3.30) for the Green's function allows us to compute (see Sec. 4) the differential conductivity $\sigma(\mathbf{k}\omega)$ and the spatially inhomogeneous fluctuations of an electron gas located in a strong electric field. Expressions for the functions $g_0(\epsilon\mathbf{p}', \mathbf{k}\omega)$, $\sigma(\mathbf{k}\omega)$ and the fluctuation correlation functions deserve a detailed consideration and a separate paper will be devoted to this question. Here we shall only show that if a constant electric field creates only electron drift, and heating can be neglected, then we must omit in (3.3) all terms proportional to τ_0 in order to obtain g_0 in this case. Then ($\mathbf{k} \neq 0$):

$$g_0(\varepsilon\mathbf{p}'; \mathbf{k}\omega) = \frac{2n_0(\varepsilon)}{nV} \frac{1 - i\mathbf{k}\mathbf{v}_{\mathbf{p}'}\tau_1(\varepsilon')}{-i(\omega - \mathbf{k}\mathbf{u}) + k^2D}. \quad (3.33)$$

4. CORRELATION FUNCTIONS OF THE FLUCTUATION OF THE OCCUPATION NUMBERS AND THE CURRENT FLUCTUATIONS IN A NONDEGENERATE GAS

The fluctuation change in the occupation number $\delta n_{\mathbf{p}}(\mathbf{r}t)$ is made up, according to (2.6), of the response to the fluctuation change of field (to the first component at the right) and of the response to the extraneous fluctuation flux $\delta J_{\mathbf{p}}(\mathbf{r}t)$. We shall call this second part of δn the extraneous fluctuations of the occupation number and denote it by $\delta n_{\mathbf{p}}^{ex}(\mathbf{r}t)$. Such would be the fluctuations of δn if there were no field fluctuations: $\delta\mathbf{E} = \delta\mathbf{H} = 0$. The observed fluctuations of the current density, like δn, consist of two parts:

$$\delta j_\alpha(\mathbf{r}t) = \int \sigma_{\alpha\beta}(\mathbf{r}t; \mathbf{r}'t')\delta E_\beta(\mathbf{r}'t')\,d\mathbf{r}'\,dt' + \delta j_\alpha^{ex}(\mathbf{r}t). \quad (4.1)$$

The first part is the response to the fluctuations of the field ($\sigma_{\alpha\beta}(\mathbf{r}t; \mathbf{r}'t')$ is the electrical conductivity tensor (see below), the second component, equal to

$$\delta j_\alpha^{ex}(\mathbf{r}t) = \frac{e}{V}\sum_{\mathbf{p}} v_{\alpha\mathbf{p}}\delta n_{\mathbf{p}}^{ex}(\mathbf{r}t), \quad (4.2)$$

is known in the theory of electromagnetic fluctuations as the extraneous fluctuation current.[13]

We compute the correlation function $\langle \delta n_{\mathbf{p}_1}^{ex}(\mathbf{r}_1 t_1) \times \delta n_{\mathbf{p}_2}^{ex}(\mathbf{r}_2 t_2)\rangle$ for a nondegenerate electron gas in the absence of interelectronic collisions. In accord with (2.6) and (3.1),

$$\langle \delta n^{ex}(1)\delta n^{ex}(2)\rangle = V^{-2}\int d1'\,d2'\,G(1,1')G(2,2')\langle \delta J(1')\delta J(2')\rangle \quad (4.3)$$

(for brevity, we let $\mathbf{p}_1, \mathbf{r}_1, t_1 \equiv 1$). We substitute Eq. (2.9) for the flux correlation function $J_{\mathbf{p}}$ and take into account Eq. (3.6) for G in the primed variables and (2.1). We obtain

$$\langle \delta n^{ex}(1)\delta n^{ex}(2)\rangle = G(1,2)n(2) + G(2,1)n(1) - V^{-1}\int d^3r' \sum_{\mathbf{p}'} n_{\mathbf{p}'}(\mathbf{r}', -\infty)G(1; \mathbf{p}'\mathbf{r}', -\infty)G(2; \mathbf{p}'\mathbf{r}', -\infty). \quad (4.4)$$

By virtue of (3.7), (3.2), and (3.3a), this expression reduces finally to the form

$$\langle \delta n^{ex}(1)\delta n^{ex}(2)\rangle = g(1,2)n(2) + g(2,1)n(1). \quad (4.5)$$

In the presence of pair collisions in the fluctuation correlation function of the occupation numbers (4.4), the component

$$V^{-1}\int d^3r'\,dt' \sum_{\mathbf{p}_1'\mathbf{p}_2'} G(1; \mathbf{p}_1'\mathbf{r}'t')G(2; \mathbf{p}_2'\mathbf{r}'t')[\overline{J^+}(\mathbf{p}_1'\mathbf{p}_2') - \overline{J^-}(\mathbf{p}_1'\mathbf{p}_2')] \quad (4.4a)$$

appears in addition to the terms contained on the right side of (4.4) (see (2.12)). This naturally changes the correlation function (4.5). In the state of thermal equilibrium, (4.4a) vanishes and Eq. (4.5) is seen to be valid for a gas with pair collisions.

It follows from (4.5) that the correlation function of extraneous currents is equal to

$$\langle \delta j_\alpha^{ex}(\mathbf{r}_1 t_1)\delta j_\beta^{ex}(\mathbf{r}_2 t_2)\rangle = V^{-2}\sum_{\mathbf{p}_1\mathbf{p}_2} e^2 v_{\alpha\mathbf{p}_1}v_{\beta\mathbf{p}_2}\{g(1,2)n(2) + g(2,1)n(1)\}. \quad (4.6)$$

If the electron gas is stationary and homogeneous and, moreover, if the fluctuations considered are homogeneous ($\mathbf{k} = 0$), then one can also write[3)]

$$\langle \delta j_\alpha^{ex}(t_1)\delta j_\beta^{ex}(t_2)\rangle_\omega = \frac{e}{V}\sum_{\mathbf{p}}[v_{\alpha\mathbf{p}}\gamma_{\beta\mathbf{p}}(\omega) + v_{\beta\mathbf{p}}\gamma_{\alpha\mathbf{p}}(-\omega)], \quad (4.7)$$

where

$$\gamma_{\alpha\mathbf{p}}(\omega) = \frac{e}{V}\sum_{\mathbf{p}'} g_{\mathbf{p}\mathbf{p}'}(0\omega)v_{\alpha\mathbf{p}'}n_{\mathbf{p}'}. \quad (4.7a)$$

Equation (4.7) corresponds to the formula for the fluctuation spectral density (FSD) of the currents, obtained by Gurevich and Katilyus[10] by the method of moments (the function γ_α is the same as the solution of Eq. (3.14) in [10]).

We compare (4.7) with the expression for the differential electrical conductivity tensor, which connects the linear changes in current and field:

$$\delta j_\alpha(\mathbf{r}_1 t_1) = \int dt_2\,d\mathbf{r}_2\,\sigma_{\alpha\beta}(\mathbf{r}_1 t_1, \mathbf{r}_2 t_2)\delta E_\beta(\mathbf{r}_2 t_2). \quad (4.8)$$

According to (2.6), (3.1), and (3.7) (we neglect the contribution from $\delta\mathbf{H}$ here),

$$\sigma_{\alpha\beta}(\mathbf{r}_1 t_1; \mathbf{r}_2 t_2) = \left(\frac{e}{V}\right)^2 \sum_{\mathbf{p}_1\mathbf{p}_2} v_{\alpha\mathbf{p}_1}g(1,2)\left[-\frac{\partial n(2)}{\partial\hbar\, p_{2\beta}}\right]. \quad (4.9)$$

In the state of thermodynamic equilibrium (the contribution to σ from $\delta\mathbf{H}$ is strictly equal to zero) $\partial n_{\mathbf{p}}/\partial\hbar p_\beta = -v_{\beta\mathbf{p}}n_{\mathbf{p}}/T$ and, by comparing (4.6) with (4.9), we establish the fact that the fluctuation-dissipation theorem is satisfied.[13, 14] In a nonequilibrium electron gas, there is no such general connection between the fluctuation spectral density and the electrical conductivity. However, it is seen that each is expressed by one and the same quantity: the particle distribution function and the Green's function of the kinetic equation.

Now let us compute the FSD of the current in a homogeneous nondegenerate gas. For $\mathbf{k} = 0$ and in the absence of a magnetic field, it is possible to put it in diagonal form in the indices α and β. The current fluctuations in a direction normal to the strong field, for $\omega\tau_1 \ll 1$, are simply equal to (see (4.6), (3.16), and (3.26))

$$\langle \delta j_\alpha^{ex}(t_1)\delta j_\beta^{ex}(t_2)\rangle_\omega = 2V^{-1}ne^2D\delta_{\alpha\beta}, \quad \alpha, \beta \equiv x, y. \quad (4.10)$$

[3)] The transition to the fluctuation spectral density over positive frequencies $\nu = \omega/2\pi$ is carried out by multiplication of (4.7) by 2.

The longitudinal fluctuations of the current have the same spectral density:

$$\langle \delta j_z^{ex}(t_1)\delta j_z^{ex}(t_2)\rangle_\omega = 2V^{-1}ne^2D, \quad \tau_0^{-1} \ll \omega \ll \tau_1^{-1}. \quad (4.11)$$

At low frequencies ($\omega\tau_0 \ll 1$), the contribution from $g_0(\epsilon\mathbf{p}'; 0\omega)$ (energy fluctuation) becomes important in the current fluctuations. This contribution is calculated by means of (4.6), (3.16), and (3.21). This addition can be positive or negative, and leads to the result that the low frequency value of the FSD of the current is correspondingly greater or less than the high frequency. However, it is more convenient to study the sign of the integral over the frequency of the contribution of $g_0(\epsilon\mathbf{p}'; 0\omega)$ to the FSD of the current. Thanks to the monotonic dependence of $g_0(\epsilon\mathbf{p}'; 0\omega)$ on ω, the sign of this integral is identical with the sign of the difference $\langle \delta j_z^{ex}(t_1)\delta j_z^{ex}(t_2)\rangle_\omega - 2ne^2DV^{-1}$. With the help of the sum rule over frequencies, we get for $g_0(\epsilon\mathbf{p}'; 0\omega)$:

$$\frac{1}{\pi}\int_{-\infty}^{+\infty} d\omega \left\{ \langle \delta j_z^{ex}(t_1)\delta j_z^{ex}(t_2)\rangle_\omega - \frac{2ne^2D}{V} \right\} = \frac{ne^4E^2}{m^2V}\left\{ \overline{\tau_1^2} - \bar{\tau}_1^2 + \frac{1}{3}\overline{\frac{\varepsilon d\tau_1^2}{d\varepsilon}} \right\}. \quad (4.12)$$

This follows from (3.9). The bar here indicates averaging of the form

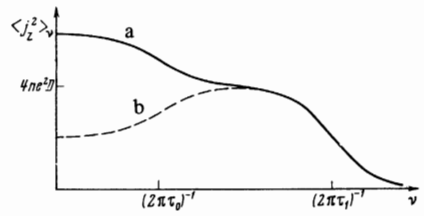
$$\bar{\tau}_1 = \frac{4}{3n}\int_0 d\varepsilon\, N(\varepsilon)\varepsilon\left(-\frac{dn_0}{d\varepsilon}\right)\tau_1(\varepsilon). \quad (4.13)$$

It follows from (4.12) that the sign of the difference between the low frequency and high frequency plateaus in the spectrum of current fluctuations depends on whether τ_1 rises or falls with the energy ϵ, i.e., on the form of the nonlinearity of the volt-ampere characteristic. This conclusion had been reached earlier in [8], in which the electron temperature was used.[4)] If τ_1 does not depend on the energy, then there is no frequency dispersion of the FSD of the current (and voltage) at $\omega\tau_0 \sim 1$. If $d\tau_1/d\epsilon > 0$ ("superlinear" volt-ampere characteristic), then the FSD of the current falls on going from low to high frequencies (see curve a in the drawing). In scattering by the deformation potential of acoustic phonons (as also in the scattering on molecules in a weakly ionized plasma) $\tau_1 \propto \epsilon^{-1/2}$, $\tau_0 \propto \epsilon^{-1/2}$ and the characteristic is sublinear. Analysis shows that in this case the low-frequency plateau is lower than the high-frequency one (this conclusion is contained in [10]) (see curve b in the drawing).

The frequencies $\omega \lesssim \tau_0^{-1}$ make a contribution to the integral (4.12). Therefore, the relative value of the excess of the low-frequency plateau over the high-frequency one is

$$\frac{|\langle \delta j_z^{ex}(t_1)\delta j_z^{cr}(t_2)\rangle_0 - 2ne^2D/V|}{2ne^2D/V} \sim \frac{T^* - T}{T^*}, \quad (4.14)$$

i.e., of the order of the relative heating of the electron gas.



Schematic representation of the spectral density of the current fluctuations in a short-circuited specimen; a—$d\tau_1/d\epsilon > 0$, spuerlinear volt-ampere characteristic; b—$d\tau_1/d\epsilon < 0$ (scattering by the deformation potential of acoustical phonons or by neutral molecules), sublinear volt-ampere characteristic.

Let us make clear whether or not one can connect with a semiconductor in which a nonequilibrium distribution of current carriers has been created by a strong constant field, certain EMF fluctuations independent of the lead $Z_{ext}(\omega)$ in the circuit. Doubts arise as to the existence of such a possibility (they have been expressed by Lax[24] for example; see also the book of Rytov,[12] p. 155) because, in a semiconductor with hot electrons and a nonlinear volt-ampere characteristic, the external circuit influences the relaxation of the energy of the electron gas and the current (in particular, on the relaxation time).[8]

It follows from (4.1) that the current fluctuations in the circuit are equal to

$$\delta I(\omega) = Z^{-1}(\omega)\delta U(\omega) + \delta I^{cr}(\omega), \quad (4.15)$$

where $Z^{-1}(\omega)$ is the differential conductivity of the semiconductor (see (4.9)). $\delta U(\omega)$ is the voltage fluctuation, and δI^{ex} the external current fluctuation, the FSD of which we found above. According to (4.15), δI^{ex} is the external current fluctuation in the short-circuited case, when $\delta U = 0$. It follows from (4.15) and Kirchhoff's laws that the FSD of the current in the presence of a load is equal to

$$\langle \delta I^2\rangle_\omega = \langle \delta U_I^2\rangle_\omega |Z(\omega) + Z_{ext}|^{-2}. \quad (4.16)$$

Here

$$\langle \delta U_I^2\rangle_\omega = \langle (\delta I^{cr})^2\rangle_\omega |Z(\omega)|^2 \quad (4.17)$$

is the FSD of the voltage in the given current regime. It plays the role of the Nyquist noise EMF and can be used completely analogously to the latter in the analysis of electrical fluctuations. Here the non-ohmic semiconductor in the corresponding equivalent circuit must be replaced by its impedance $Z(\omega)$ (see (4.16)).

Let us consider current fluctuations with $\mathbf{k} \neq 0$. If the constant field is such that it only creates an electron drift, and the heating is small, then it suffices, in the calculation of the FSD of the current, to use only (3.33). Substituting (3.33) and (4.6), we obtain (V = 1)

$$\langle \delta j_\alpha^{cr}(\mathbf{r}_1 t_1)\delta j_\beta^{cr}(\mathbf{r}_2 t_2)\rangle_{\mathbf{k}\omega} = 2ne^2D\{\delta_{\alpha\beta} + [(\omega - \mathbf{ku})(k_\alpha u_\beta + u_\alpha k_\beta) + k^2(u_\alpha u_\beta - k_\alpha k_\beta D^2)][(\omega - \mathbf{ku})^2 + (Dk^2)^2]^{-1}\}. \quad (4.18)$$

We compare (4.18) with the expression for the real part of the differential conductivity, which is easily obtained with the help of (3.16), (3.33), and (4.9):

$$\mathrm{Re}\,\sigma_{zz} = \sigma\omega(\omega - \mathbf{ku})[(\omega - \mathbf{ku})^2 + (Dk^2)^2]^{-1}. \quad (4.19)$$

Since $ne^2D = T\sigma$ in the absence of heating, we have established that the fluctuation-dissipation theorem is

[4)] To avoid misunderstanding, we note that in [8] we studied the FSD of the voltage (see (4.17) below), the frequency dispersion of which in the region $\omega \sim \tau_0^{-1}$ has a sign opposite that of the frequency dispersion of the FSD of the current.

not valid in systems with drift but without heating.

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