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SPIN WAVES IN AN ANTIFERROMAGNETIC METAL IN THE PRESENCE OF A MAGNETIC FIELD. II

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The magnetic spectrum of an antiferromagnetic metal in magnetic fields, corresponding to the ground state of sublattices with a non-mirror configuration of magnetic moments, is investigated within the framework of the s-d exchange model. As in the case of antiparallel orientation of the moments, an interaction occurs between the paramagnetic and antiferromagnetic excitation branches. The appearance of a large splitting between the Fermi surfaces of electrons with different spin polarizations leads to a substantial modification of the paramagnetic branch. Longitudinal spin waves are also considered.

IN a previous article[1] (cited below as I) we considered the magnetic excitation spectrum of an antiferromagnetic metal in magnetic fields which do not exceed the critical field requisite for violation of the collinearity of the sublattice magnetic moments. It was shown that the interaction between lattice spin waves and conduction electrons leads to a strong coupling between the paramagnetic and antiferromagnetic frequencies and, to a substantial degree, determines the value of the critical field. Here we consider magnetic fields corresponding to the ground state of sublattices with a non-mirror configuration of moments within the framework of the model for an antiferromagnetic metal used in I. This is of interest not only from a theoretical point of view but also from an experimental point of view since the actual fields at which changes of magnetic structure occur are not too large.[2]

In Sec. 1 the ground state energy and the equilibrium configurations of the magnetic sublattices are calculated, with their interaction with the conduction electrons taken into consideration. The s–d interaction on the one hand leads to a renormalization of the constant for homogeneous exchange between sublattices, and on the other hand it leads to the appearance of a splitting of the Fermi surfaces for s-electrons with oppositely-oriented spins; this splitting is of the order of $2\mu_0 H\epsilon_F/J$ (ϵ_F is the Fermi energy, J is the exchange integral of the s–d bands, H is the magnitude of the applied field, and μ_0 is the Bohr magneton). Expressions for the Green's functions of the lattice spin waves are derived in Sec. 2. In Sec. 3 the spectrum of the resonance frequencies is investigated for the two cases of longitudinal and transverse orientations of the magnetic field relative to the axis of antiferromagnetism. Spin waves associated both with transverse and with longitudinal oscillations of the magnetization are considered. Whereas the first, as a consequence of the s–d interaction, turn out to be associated with spin waves in the electronic subsystem, similar to those considered in the previous work, the longitudinal spin waves are intermingled with the branches of zero-sound type spin excitations.

In view of the smallness of the cyclotron frequency in comparison with the mentioned splitting of the Fermi surfaces, we did not take the orbital motion of the s-electrons into consideration. As in the previous work, the case of zero temperature is considered.

Below we shall follow the notation used in article I.

1. GROUND STATE ENERGY AND SPECTRUM OF THE CONDUCTION ELECTRONS

1. We shall be interested in the part of the ground state energy which depends on the configuration of equilibrium magnetic moments of the sublattices. Let us write the Hamiltonian of an antiferromagnetic metal in the form

$$H = H_e + H_m + H_{sd}, \tag{1}$$

where H_m is the Hamiltonian of the subsystem of magnetic lattices:[3]

$$H_m = \int d\mathbf{r}\left\{\frac{\alpha}{2}\left[\frac{\partial \mathbf{M}_1}{\partial x_i}\frac{\partial \mathbf{M}_1}{\partial x_i} + \frac{\partial \mathbf{M}_2}{\partial x_i}\frac{\partial \mathbf{M}_2}{\partial x_i}\right] + \alpha_{12}\frac{\partial \mathbf{M}_1}{\partial x_i}\frac{\partial \mathbf{M}_2}{\partial x_i} + \delta \mathbf{M}_1\mathbf{M}_2 - \frac{\beta}{2}[(\mathbf{M}_1\mathbf{n})^2 + (\mathbf{M}_2\mathbf{n})^2] - (\mathbf{M}_1 + \mathbf{M}_2, \mathbf{H})\right\}; \tag{2}$$

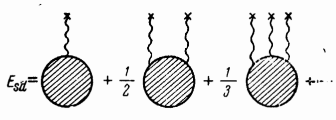
the Hamiltonian describing the exchange interaction between the magnetic sublattices and the conduction electrons is taken in the form[4]

$$H_{sd} = \frac{g_0}{\sqrt{\mu_0 M_0}}\int d\mathbf{r}\,\psi_\alpha^+(r)\mathbf{S}_{\alpha\beta}\mathbf{M}\psi_\beta(r); \tag{3}$$

H_e is the Hamiltonian of the system of conduction electrons, whose explicit form is not essential for our purposes. Here $M_{1,2}$ are the magnetic moments of the sublattices; $M = M_1 + M_2$; n is the axis of antiferromagnetism; ψ_α^+ and ψ_β^+ are electron creation and annihilation operators; $S_{\alpha\beta}$ are spin matrices. In order of magnitude the s–d exchange constant g_0 is equal to $(\mu_0/M_0)^{1/2}J$ ($J \ll \epsilon_F$). The meaning of the remaining coefficients is the same as in[3]. The ground state energy corresponding to the Hamiltonian H_m was found by Kaganov and Tsukernik,[5] and by Turov.[6]

Assuming that the atomic spin is much larger than unity, we shall use the classical approximation in order to describe the lattice.

As is well known, the correction to the ground state energy, due to the interaction existing in the system, is determined by the total set of connected vacuum diagrams for the S-matrix.[7] We note that in a classical investigation of magnetic sublattices the magnetic moment **M** plays the role of a classical external field; therefore the correction to the ground state energy of an antiferromagnet, caused by the s—d interaction, is determined by the following diagrams:



(4)

Here the crosses denote **M**, the wavy lines denote $(g_0/\sqrt{\mu_0 M_0})\mathbf{S}_{\alpha\beta}$, and the shaded blocks represent the totality of all possible graphs formed by electron lines and having the appropriate number of s—d interaction vertices. The blocks do not contain inside them quantities pertaining to s—d exchange, but include all interactions in the electronic subsystem.

In what follows, as in the previous work, we do not consider relativistic interactions in the electronic system. On the strength of this, since each of the diagrams shown in expression (4) is proportional to the corresponding power of the vector **M**, and the quantity E_{sd} is a scalar, it is obvious that all diagrams containing an odd number of s—d interaction vertices are equal to zero.

Let us estimate the contributions from diagrams of different order. Since a factor $(JM/M_0)^l$ is associated with each diagram of order l containing l vertices of the s—d interaction, it follows from dimensional considerations that in order of magnitude it is equal to $(M/M_0)^l J(J/\epsilon_0)^{l-1}$, where ϵ_0 is the characteristic energy of the electronic subsystem, i.e., $\epsilon_0 \sim \epsilon_F$.[1)]

Thus, expression (4) is an expansion in powers of the small parameter J/ϵ_F. Therefore, in order to calculate the corrections to the ground state energy it is sufficient to confine our attention to the diagram with two vertices. The order of the discarded term $E^{(4)} \sim J(J/\epsilon_F)^3(M/M_0)^4$.

In the approximation adopted here E_{sd} is given by[2)]

$$E_{sd} = -\frac{1}{2}\frac{g_0^2}{\mu_0 M_0}(\mathbf{M}\mathbf{S}_{\alpha\gamma})\,Q_{\gamma\delta\alpha\beta}\,(\mathbf{S}_{\beta\delta}\mathbf{M}), \qquad (5)$$

where we have denoted the expression for the electronic diagram block of second order by $Q_{\gamma\delta\alpha\beta}$. In the absence of relativistic interactions in the electronic subsystem, $Q_{\gamma\delta\alpha\beta}$ may be represented in the form

$$Q_{\gamma\delta\alpha\beta} = A\delta_{\alpha\gamma}\delta_{\beta\delta} + B\mathbf{S}_{\alpha\gamma}\mathbf{S}_{\beta\delta}. \qquad (6)$$

Substituting expression (6) into formula (5), we obtain

$$E_{sd} = \frac{1}{8}\frac{g_0^2 M^2}{\mu_0 M_0}B. \qquad (7)$$

On the other hand, as is not difficult to see, the quantity $(1/2)g_0^2 B$ is the limit, as ω and **k** tend to zero, of the polarization operator $\Pi(\omega, \mathbf{k})$ for a transverse spin wave. In this connection it exists in the form of the limit $\omega = 0$, $\mathbf{k} \to 0$, i.e., Π^k, since one is talking about an evaluation of the energy of a state of thermodynamical equilibrium. Thus, from formula (7) it follows that

$$E_{sd} = \frac{1}{4}\frac{\Pi^k \mathbf{M}^2}{\mu_0 M_0} = \frac{\Pi^k \mathbf{M}_1\mathbf{M}_2}{2\mu_0 M_0} + \frac{\Pi^k M_0^2}{2\mu_0 M_0}. \qquad (8)$$

The second term in expression (8) is simply an additive constant which does not depend on the configuration of the moments. Comparing expressions (2) and (8), one can see that taking account of the s—d interaction is equivalent to a renormalization of the constant δ for homogeneous exchange between the sublattices[3)]

$$\delta \to \tilde{\delta} = \delta + \frac{\Pi^k}{2\mu_0 M_0}. \qquad (9)$$

Therefore the equilibrium configurations of the moments in an antiferromagnetic metal will be the same as in an antiferromagnetic dielectric with the indicated renormalization of δ taken into consideration. In particular, the boundary of metastability for the state with antiparallel moments in the presence of a magnetic field parallel to the axis of antiferromagnetism is determined by the field

$$H_{c1} = \left[\beta\left(2\delta + \frac{\Pi^k}{\mu_0 M_0}\right)\right]^{1/2} M_0.$$

Using formula (35) from I and the expression given in article[9] for the frequencies ω_0 contained in it, it is easy to see that this field coincides with the field H_c introduced in the previous article from the condition that the static, transverse susceptibility of a metal tends to infinity. Thus, $\tilde{\delta} \approx \delta(1-\xi)$ in terms of the notation used in I.

The equilibrium configuration in a state with nonmirror sublattices will, as in a dielectric, be determined by the angle between the directions of the sublattice moments and the magnetic field:

$$\cos\theta = \frac{H}{H_{c2}}, \quad H_{c2} = \begin{cases}(2\tilde{\delta} - \beta)\,M_0, & \mathbf{H}\parallel\mathbf{n}\\ (2\tilde{\delta} + \beta)\,M_0, & \mathbf{H}\perp\mathbf{n}\end{cases}; \qquad (10)$$

H_{c2} is the critical field associated with the transition to a ferromagnetic structure.

2. Let us consider the change δG of the electron Green's function in the presence of an external magnetic field. In an antiferromagnetic metal the change in the electron Green's function is due to both the interaction of the electron spin with the external magnetic field and the interaction of the electrons with the sublattice magnetic moments (we recall that we shall be interested in those fields such that the total moment does not vanish).

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1) In principle a situation is possible in which ϵ_0 is much smaller than ϵ_F (as in an "almost ferromagnetic" Fermi liquid owing to the paramagnetic susceptibility enhancement factor [8]). However, as is evident from the expressions given below for the ground state energy, in this case the antiferromagnetic ordering does not correspond to an energy minimum even in an arbitrarily weak magnetic field.

2) In connection with the calculation of the free energy (4), we did not write down the terms stipulated by the interaction of the electron spins with the magnetic field. Taking this interaction into consideration leads to the appearance in the expression for the free energy of both which are independent of **M** as well as cross terms containing the interaction of electron spins with the lattice magnetic moment. Since, as was indicated, we are interested in the contribution to the ground state energy which depends on the configuration of magnetic moments, terms of the first type are not essential for us. As far as the cross terms are concerned, to the lowest order in the s—d interaction they are proportional to $(J/\epsilon_F)\mathbf{M}\cdot\mathbf{H}$ and represent a small correction to the last term in the expression (2) for H_m.

3) Taking the relativistic interactions in the electronic subsystem into consideration would also lead to a renormalization of the anisotropy constant β, which we neglect for the sake of simplicity

The interaction energy corresponding to this has the form

$$U_{\alpha\beta} = S_{\alpha\beta}{}^z\left(2\mu_0 H + \frac{g_0 M}{\sqrt{\mu_0 M_0}}\right). \tag{11}$$

The z axis is chosen along the direction of H. As is well known,[7]

$$(\delta G^{-1}(p))_{\alpha\beta} = -\delta U_{\alpha\beta} + i\int\frac{d^4p'}{(2\pi)^4}\tilde{\Gamma}^k_{\alpha\gamma\beta\delta}(p,p')\delta U_{\delta\gamma}. \tag{12}$$

Using the definition of the total electron-magnon interaction vertex contained in I (formula (4)), we write δG^{-1} in the form

$$\delta G_{\alpha\beta}{}^{-1} = \frac{g^k}{g_0} U_{\alpha\beta}. \tag{13}$$

Thus, the expression for the total Green's function as $\epsilon \to 0$, $|p| \to p_0$

$$G_{\alpha\beta}{}^{-1} = G^{-1}_{0_{\alpha\beta}} + \delta G_{\alpha\beta}{}^{-1}$$

takes the form

$$G_{\alpha\beta}{}^{-1} = \frac{\varepsilon - v(|p| - p_0)}{a}\delta_{\alpha\beta} - \frac{g_0}{a(1+B_0)}\left(\frac{M}{\sqrt{\mu_0 M_0}} + \frac{2\mu_0 H}{g_0}\right) S_{\alpha\beta}{}^z. \tag{14}$$

(We used the definition of the electron Green's function without taking account of the interaction or formula (A.4) of the Appendix.)

From here it follows that the pole expressions for the Green's functions of electrons with oppositely polarized spins have the form

$$G_\pm = a / [\varepsilon - v(|p| - p_\pm)], \tag{15}$$

where $p_\pm$ denote the Fermi momenta of electrons with spins along and against the field. The splitting of the Fermi surfaces is given by $v(p_+ - p_-)$. Taking into consideration that $M = 2M_0H/H_{c2}$ for $H < H_{c2}$ and $M = 2M_0$ for $H > H_{c2}$, we find

$$v(p_+ - p_-) = \begin{cases} \dfrac{\varkappa}{1+B_0} 2\mu_0 H, & H < H_{c2} \\ \dfrac{2\mu_0}{1+B_0}[(\varkappa - 1)H_{\kappa 2} + H], & H > H_{c2} \end{cases}, \tag{16}$$

where

$$\varkappa = \left(\frac{M_0}{\mu_0}\right)^{1/2} \frac{g_0}{\mu_0 M_0 (2\tilde{\delta} + \beta)}. \tag{17}$$

Let us estimate the order of magnitude of the parameter κ:

$$\varkappa \sim \frac{J}{\mu_0 M_0 \delta(1-\xi)} \sim \frac{\varepsilon_F}{J}(1-\xi)^{-1},$$

where we have used the estimate for the s–d interaction exchange integral contained in[4], $J \sim (J_{dd}\epsilon_F)^{1/2}$. From here it follows that $|\kappa| \gg 1$, i.e., the emergence of a nonvanishing total moment of the sublattices leads to a large value for the splitting of the Fermi levels. We note that depending on the sign of the exchange integral, κ may be either positive or negative.

2. GREEN'S FUNCTIONS OF THE MAGNONS

In this section we shall derive expressions for the bare Green's functions of the spin waves:

$$\begin{aligned} D_+{}^0(x, x') &= -i\langle Tm_+(x) m_-(x')\rangle / 4\mu_0 M_0, \\ D_z{}^0(x, x') &= -i\langle Tm_z(x) m_z(x')\rangle / 2\mu_0 M_0, \end{aligned} \tag{18}$$

where $m_\pm = m_x \pm im_y$, $m_i = m_{1i} + m_{2i}$ (i = x, y, z) are the deviations of the magnetic moments from their equilibrium values.

In expression (18) the averaging is carried out with respect to the actual ground state of the interacting "electrons plus lattice" system. Meanwhile the ground state of the lattice Hamiltonian H_m does not correspond to the actual configuration (10) of equilibrium sublattice moments. In this connection it is necessary to redefine the Hamiltonian H_m, having added the exchange energy (8) to it. Correspondingly, it is necessary to subtract the same term from expression (3) for H_{sd}.

One can obtain the equations for the D-functions by using the classical equations of motion and the commutation relations for the magnetic moments. Omitting the simple but rather cumbersome calculations, we present the final expressions for the D-functions: for $H < H_{c2}$ one finds

$$\begin{aligned} D_+{}^0(\omega, \mathbf{k}) &= (2\mu_0 H_{c2})^{-1}\frac{\omega_1{}^2(\mathbf{k}) + 4\mu_0 H\omega + (2\mu_0 H)^2}{\omega^2 - \omega_1{}^2(\mathbf{k})}, \\ D_z{}^0(\omega, \mathbf{k}) &= (\mu_0 H_{c2})^{-1}\frac{\omega_2{}^2(\mathbf{k})}{\omega^2 - \omega_2{}^2(\mathbf{k})}; \end{aligned} \tag{19}$$

for $H > H_{c2}$

$$D_+{}^0(\omega, \mathbf{k}) = \frac{2}{\omega - \omega_3(\mathbf{k})}, \quad D_z{}^0(\omega, \mathbf{k}) = 0. \tag{20}$$

The poles of the functions D_+^0 and D_z^0 determine the resonance frequencies of the system of antiferromagnetic sublattices and, to within the renormalization δ, coincide with the results cited in the review article[3]. For convenience we shall write down their expressions here.

In the case H ∥ n

$$\begin{aligned} \omega_1{}^2(\mathbf{k}) &= (2\mu_0)^2(H^2 - H_{c1}'^2 + 2\tilde{\delta}M_0{}^2(\alpha - \alpha_{12})\mathbf{k}^2), \\ \omega_2{}^2(\mathbf{k}) &= (2\mu_0)^2 \cdot 2\tilde{\delta}M_0{}^2(\alpha - \alpha_{12})\mathbf{k}^2, \\ \omega_3(\mathbf{k}) &= 2\mu_0 H + 2\mu_0 M_0(\alpha + \alpha_{12})\mathbf{k}^2, \end{aligned} \tag{21}$$

where $H_{c1}' = H_{c1}(2\tilde{\delta} - \beta)/(2\tilde{\delta} + \beta)$ is the field corresponding to the lower boundary of metastability for the state with nonvanishing total equilibrium moment of the sublattices.[5]

In the case H ⊥ n one finds

$$\begin{aligned} \omega_1{}^2(\mathbf{k}) &= (2\mu_0)^2(H^2 + H_{c1}{}^2 + 2\tilde{\delta}M_0{}^2(\alpha - \alpha_{12})\mathbf{k}^2), \\ \omega_2{}^2(\mathbf{k}) &= (2\mu_0)^2\left[H_{c1}{}^2\left(1 - \left(\frac{H}{H_{c2}}\right)^2\right) + 2\tilde{\delta}M_0{}^2(\alpha - \alpha_{12})\mathbf{k}^2\right], \\ \omega_3(\mathbf{k}) &= 2\mu_0 H + 2\mu_0 M_0(\alpha + \alpha_{12})\mathbf{k}^2. \end{aligned} \tag{22}$$

3. SPECTRUM OF THE RESONANCE FREQUENCIES

1. First let us consider transverse oscillations of the magnetization. Just as in the case $H < H_{c1}$, for $\mathbf{k} = 0$ the eigenfrequencies of an antiferromagnetic metal consist of a two-parameter family of harmonics $\omega_{lm}(H)$ ($l \neq 0$), representing spin waves in the electronic subsystem,[10] and three resonance frequencies determined from the equation relating the paramagnetic and antiferromagnetic frequencies. Whereas the frequencies ω_{lm} ($l \neq 0$) are determined by the poles of the vertex part $\tilde{\Gamma}$ (see[1]), the last three frequencies correspond to singularities of the total magnon Green's function $D(\omega, 0)$.

The appearance of a large splitting (17) of the Fermi surfaces for $H > H_{c1}$ leads to the following modification of the frequency spectrum ($l \neq 0$):

$$\omega_{lm} = \frac{2\varkappa\mu_0 H}{1+B_0}(1+B_l). \quad (23)$$

In formula (23) we neglected the cyclotron frequency in comparison with the splitting of the Fermi surfaces, as a consequence of which the spectrum, to within terms of order $|\kappa|^{-1}$, is degenerate in m. We note that the sign of κ determines the sign of the frequency ω_{lm} and corresponds to right-hand or left-hand polarizations of the wave. In order to be definite, we shall consider positive values of κ.

The equation for the determination of the three remaining frequencies may be obtained from Dyson's equation (given by formula (12) in I) with the redefinition of the Hamiltonian H_{sd}, made in the second Section, taken into consideration. The renormalization of the s–d interaction Hamiltonian leads to the result that the difference $\Pi - \Pi^k$ appears in Dyson's equation instead of the polarization operator Π. Using expression (A.10), derived in the Appendix, for $\Pi - \Pi^k$ and formula (19), we obtain a dispersion equation relating the frequencies of the transverse spin waves and the paramagnetic frequency for $H < H_{c2}$:

$$(\omega^2 - \omega_1^2)(\omega - 2\varkappa\mu_0 H) = \frac{\xi\omega}{2(1-\xi)}[(\omega + 2\mu_0 H)^2 + \omega_1^2 - \omega^2], \quad (24)$$

where

$$\xi = \frac{\vartheta g_0^2}{(1+B_0)\cdot 2\delta\mu_0 M_0}$$

is the exchange-interaction parameter which we introduced earlier in I.

Let us consider the case of longitudinal orientation of the magnetic field with respect to the axis of antiferromagnetism. In this case the frequency ω_1 is determined by formula (21).

Since $\kappa \gg 1$ and the frequency ω_1 is always smaller than $2\mu_0 H$, it is not difficult to see that Eq. (24) has $\omega = 2\kappa\mu_0 H$ (to lowest order in κ^{-1}) as one of its solutions. The two other solutions of this equation correspond to $|\omega| < 2\mu_0 H$. Actually they represent the frequencies of the antiferromagnetic resonance, renormalized by the s–d interaction. Near $H = H'_{c1}$ this renormalization is essential. For $H^2 - H'^2_{c1} \ll H'^2_{c1}$ one finds

$$\omega_\pm = 2\mu_0\left\{-\frac{\xi}{4(1-\xi)\varkappa}H \pm \left[\left(\frac{\xi H}{4(1-\xi)\varkappa}\right)^2 + H^2 - H_{c1}'^2\right]^{1/2}\right\}. \quad (25)$$

If $H - H'_{c1} \ll H\xi/\kappa(1-\xi)$, the expressions for these frequencies take the form

$$\omega_+ = 2\mu_0\frac{4(1-\xi)\varkappa}{\xi}(H - H_{c1}'),$$
$$\omega_- = -2\mu_0 H\frac{\xi}{2(1-\xi)\varkappa}. \quad (26)$$

We note that in contrast to the case of an antiferromagnetic dielectric, only one of the resonance frequencies tends to zero at $H = H_{c1}$, having in this connection a finite derivative $d\omega/dH$. For $\xi = 0$, as is evident from formula (25), both frequencies tend to zero at the transition point, just as happens in the case of a dielectric.

Finally, in fields $H \gg H'_1$ the frequencies approach their limiting values

$$\omega_\pm = \pm 2\mu_0 H + O(\varkappa^{-1}). \quad (27)$$

The dependence of ω on H for H ∥ n is schematically shown in Fig. 1.

Now let us consider the case when the orientation of the magnetic field is perpendicular to the axis of antiferromagnetism. Equation (24) remains in force, and the frequency ω_1 is determined by formula (22). Since for transverse orientation of the magnetic field the antiparallel nature of the moments is violated even in an arbitrarily weak field, the paramagnetic frequency of the electrons is equal to $2\kappa\mu_0 H$ over the entire range $H < H_{c2}$. From the form of the dependence of ω_1 on H it follows that, without taking account of the interaction, crossing of the bare antiferromagnetic and paramagnetic branches will occur. This crossing takes place in the frequency region $\omega \sim 2\mu_0 H_{c1}$. Taking the interaction into consideration leads, as usual, to an intermingling of the frequencies.

Let us consider the asymptotic behavior of the resonance frequencies. For $\kappa H \ll H_{c1}$ it follows from the dispersion equation (24), with formula (22) taken into consideration, that the antiferromagnetic frequencies have the form

$$\omega_\pm = \pm\sqrt{\frac{2-\xi}{2(1-\xi)}}2\mu_0 H_{c1} + \frac{\xi}{2(2-\xi)}\sqrt{\frac{2-\xi}{2(1-\xi)}}2\varkappa\mu_0 H, \quad (28)$$

but the paramagnetic frequency is given by

$$\omega = \frac{2(1-\xi)}{2-\xi}2\varkappa\mu_0 H. \quad (29)$$

Both the paramagnetic and the antiferromagnetic frequencies are strongly renormalized in weak magnetic fields. For large fields, $\kappa H \gg H_{c1}$, as one can easily see from Eq. (24), the branches approach the asymptotic curves $2\kappa\mu_0 H$ and $\pm 2\mu_0\sqrt{H_{c1}^2 + H^2}$.

In connection with the excitation branches with negative polarization, we note the following. For small values of H, as is evident from formula (28), $d\omega_-/dH > 0$. On the other hand, for $H \gg \kappa^{-1}H_{c1}$ the curve $\omega_-(H)$ approaches the asymptote $-2\mu_0\sqrt{H_{c1}^2 + H^2}$. In this connection, an analysis of the dispersion equation (24) shows that the branch $\omega_-(H)$ is always located below the curve $\omega = -\omega_1(H)$. Thus, the magnetic field dependence of the frequency of a spin wave with negative polarization has an extremum at a value of the field on the order of $\kappa^{-1}H_{c1}$. For H ⊥ n the magnetic field dependence of the resonance frequencies is shown schematically in Fig. 2.

The picture described above is maintained down to the field $H = H_{c2}$ at which collapse of the magnetic moments takes place, and the metal goes over into a ferromagnetic state. Using expression (20) and formula (A.10), from Dyson's equation (given by Eq. (12) of I) by using standard methods we obtain the dispersion equation in the ferromagnetic region:

$$(\omega - 2\mu_0 H)[\omega - 2\mu_0((\varkappa - 1)H_{c2} + H)] = 4\mu_0 H_{c2}\frac{\xi}{1-\xi}\omega. \quad (30)$$

In the most accessible region of fields, $H \ll \kappa H_{c2}$, it follows from here that the paramagnetic and ferromagnetic frequencies turn out to be independent, correct to within terms of order κ^{-1}:

$$\omega = 2\mu_0 H, \qquad \omega = 2\mu_0\varkappa H_{c2}. \quad (31)$$

Finally we note that to within terms of relative order $|\kappa|^{-1}$ Eq. (24) is invariant with respect to simultaneous change of sign of κ and ω. Therefore, in the case of negative values of the parameter κ, one should change the sign of ω in the formulas obtained above, pertaining to the field region $H < H_{c2}$.

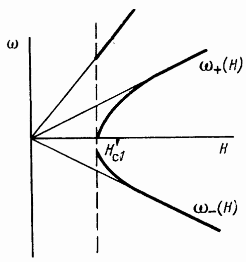

FIG. 1

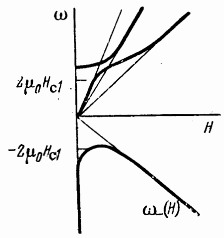

FIG. 2

2. Now let us go on to a consideration of the spectrum of longitudinal spin waves. In contrast to the situation with transverse spin waves, the longitudinal oscillations of the sublattice magnetic moments interact both with the oscillations of the spin density and with the spinless oscillations of the conduction electrons. This happens as a consequence of the fact that since in an external magnetic field only the components in the direction of the field of the total spin of the interacting particles are preserved, transformations of excitations with $S = 1$, $S_z = 0$ (which are longitudinal spin waves) into excitations with $S = 0$ are possible, that is, transformations into spinless oscillations of the electron density. Denoting by Γ_z, $\tilde{\Gamma}_z$, g_z the set of longitudinal with respect to the field spin components of the quantities $\Gamma_{\alpha\beta\gamma\delta}$, $\tilde{\Gamma}_{\alpha\beta\gamma\delta}$, $g_{\alpha\gamma}$ (they exist in the form of the components $\Gamma_{\alpha\beta\gamma\delta}$, $\tilde{\Gamma}_{\alpha\beta\gamma\delta}$, $g_{\alpha\gamma}$ with $\alpha = \gamma$, $\beta = \delta$), let us write down an expression for Γ_z:

$$\Gamma_z(p_1, p_2; k) = \tilde{\Gamma}_z(p_1, p_2; k) + g_z(p_1, k) \cdot \times \frac{\omega_2^2(\mathbf{k})}{\mu_0 H_{c2}} \left\{ \omega^2 - \omega_2^2(\mathbf{k}) \left(1 + \frac{\Pi_z(k) - \Pi^k}{\mu_0 H_{c2}}\right)\right\}^{-1} g_z(p_2, k). \quad (32)$$

(Here formula (19) is used for the bare Green's function of the longitudinal spin waves.)

The components $\tilde{\Gamma}_z$ satisfy a system of equations of the Fermi-liquid type for the spectrum of zero-sound oscillations (see, for example, [11]).

Let $\mathbf{H} \parallel \mathbf{n}$. Using the characteristics of $\tilde{\Gamma}_z$ and formula (21), one can see that Γ_z contains singularities corresponding to two noninteracting spin wave branches. In fact, for $\omega \ll v|\mathbf{k}|$ (where v is the Fermi velocity of the conduction electrons) $\tilde{\Gamma}_z \cong \tilde{\Gamma}_z^k$, $g_z \cong g_z^k$, $\Pi \cong \Pi^k$ and the pole of Γ_z is determined by the pole $\omega = \omega_2(\mathbf{k})$ of the bare Green's function for longitudinal magnons. In the opposite case, $\omega \gg \omega_2(\mathbf{k})$, one can neglect the second term in (32) and the singularity of Γ_z coincides with the singularity of $\tilde{\Gamma}_z$, i.e., determines the spectrum for longitudinal spin waves of the zero-sound type.

Now let us consider $\mathbf{H} \perp \mathbf{n}$. In this connection the only difference from the previous case is contained in the nature of the dependence of the bare magnon frequency $\omega_2(\mathbf{k})$, which is given by Eq. (22). It is not difficult to see that since $\omega_2(0) \neq 0$, the situation arising here is analogous to the case of transverse spin waves in an antiferromagnetic metal in the absence of any magnetic field, considered in article[9], taking into account the dependence of ω on H, which is determined by formula (22).

For $H > H_{c2}$ the longitudinal spin waves, just as in the case of antiparallel orientations of the magnetic moments, only lead to oscillations of the zero-sound type in the system of electrons. This is associated with the fact that in states with a collinear configuration of the moments, the absolute magnitude of the moment is preserved and, consequently, longitudinal spin waves are not present in the lattice.

In conclusion we note that an investigation of the spectrum of transverse spin waves for nonvanishing wave vectors is completely analogous to the consideration given in the previous article, with the large value of the paramagnetic splitting of the Fermi surfaces for electrons with different spin polarizations taken into consideration.

APPENDIX

We shall prove a number of relations for the total vertex characterizing the interaction of electrons with spin waves and for the polarization operator.

First of all let us show that if the relativistic interactions in the electronic subsystem are negligible, then

$$g^{\omega} = \lim_{H=0,\ \mathbf{k}=0,\ \omega\to 0} g(p, k) = \frac{g_0}{a}. \quad (A.1)$$

We shall start from the relation[11]

$$\left[\hat{S}^z \frac{\partial}{\partial \varepsilon} \hat{G}^{-1}(p)\right]_{\alpha\beta} = S^z_{\alpha\beta} - i \int \frac{d^4 p'}{(2\pi)^4} \tilde{\Gamma}^{\omega}_{\alpha\varkappa_1\beta\varkappa_2}(p, p') [\hat{G}(p') \hat{S}^z \hat{G}(p')]^{\omega}_{\varkappa_2\varkappa_1}. \quad (A.2)$$

In the absence of a magnetic field

$$\tilde{\Gamma}^{\omega}_{\alpha\beta\gamma\delta} = \tilde{\Gamma}^{\omega}_{*} \delta_{\alpha\gamma}\delta_{\beta\delta} + 2\tilde{\Gamma}^{\omega} S_{\alpha\gamma} S_{\beta\delta}. \quad (A.3)$$

Substituting (A.3) into (A.2) and using the definition of the total vertex g(p, k)[9] and the pole expression for the electron Green's function, we obtain the required relation (A.1).

In the case of an isotropic electron spectrum, from the equation for g(p, k) (formula (34) of article[9]) with Eq. (A.1) taken into consideration, one can easily find an expression for $g^k = \lim_{\omega = 0,\ \mathbf{k} \to 0} g(p, k)$:

$$g^k = g_0 / (1 + B_0) a. \quad (A.4)$$

Let us introduce the tensor

$$\Pi_{ij}(k) = -i \int \frac{d^4 p}{(2\pi)^4} S_{\gamma\alpha}{}^i G_{\alpha\varkappa_1}(p + k) G_{\varkappa_2\gamma}(p) \times \left[S^j_{\varkappa_1\varkappa_2} - i \int \frac{d^4 p'}{(2\pi)^4} \tilde{\Gamma}_{\varkappa_1\beta\varkappa_2\delta}(p, p', k) (\hat{G}(p' + k) \hat{S}^j \hat{G}(p'))_{\delta\beta} \right]. \quad (A.5)$$

It is obvious that in the system of coordinates with axis $Oz \parallel \mathbf{H}$ the combination of components $\Pi_{11} + \Pi_{22} \pm (\Pi_{12} - \Pi_{21})$ corresponds to the polarization operator for the transverse spin waves (± corresponds to the two possible directions of polarization for circularly-polarized waves), and the component Π_{33} is equal to the polarization operator $\Pi_z(k)$ for longitudinal spin waves. Let us multiply (A.2) by

$$-\frac{2ig_0^2}{(2\pi)^4} (\hat{G}(p) \hat{S}_z \hat{G}(p))_{\beta\alpha}{}^{\omega}$$

and integrate with respect to d^4p. The left-hand side of the equation takes the form

$$\frac{ig_0^2}{4} \int \frac{d^4 p}{(2\pi)^4} \frac{\partial}{\partial \varepsilon} G_{\alpha\alpha}(p)$$

and tends to zero since $G(p)|_{\epsilon \to \pm\infty} \to 0$. On the right-hand side we have

$$\Pi_z^\omega = \lim_{k=0,\ \omega\to 0} \Pi_z(k).$$

Thus

$$\Pi_z^\omega = 0. \quad (A.6)$$

Since $\Pi_{ij} \backsim \delta_{ij}$ in the absence of a magnetic field, this means that, neglecting the relativistic interactions in the electronic subsystem for H = 0 and also as ω tends to zero, the limit of the polarization operator for transverse spin waves becomes:

$$\Pi^\omega = 0. \quad (A.7)$$

Using the equation for $\Pi(k)$ (formula (35) of[9]) and formulas (A.1) and (A.4), we obtain the following result for Π^k, the limit of the polarization operator for H = 0:

$$\Pi^k = -\frac{g_0^2\vartheta}{1+B_0}. \quad (A.8)$$

Finally, taking formulas (A.1), (A.7), and the expression obtained in I for the polarization operator of transverse spin waves for k = 0 and H ≠ 0 (formula (31) of I) into account, we find

$$\Pi(\omega, 0) = \frac{g_0^2\vartheta}{1+B_0}\frac{2\varkappa\mu_0 H}{\omega - 2\varkappa\mu_0 H}. \quad (A.9)$$

Finally, the formula used in the text has the form

$$\Pi(\omega, 0) - \Pi^k = \frac{g_0^2\vartheta}{1+B_0}\frac{\omega}{\omega - 2\varkappa\mu_0 H}. \quad (A.10)$$

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