SURFACE IMPEDANCE OF METALS IN A WEAK MAGNETIC FIELD

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The dependence of the surface impedance of metals in a weak magnetic field on the field strength and on the coefficient of electron reflection from the surface is studied on the basis of the solution of the Boltzmann kinetic equation under the assumption of a quadratic dispersion law. It is shown that even when $\Omega\tau \ll 1$, where Ω is the Larmor frequency and τ the relaxation time, the surface impedance continues to depend on the magnetic field, provided the reflection of the electrons from the surface is almost specular. The derived formulas can be used to obtain information on some properties of the surface on the basis of the experimental dependence of the surface impedance on the magnetic field.

1. INTRODUCTION

We investigate in this paper, assuming a quadratic dispersion law, the dependence of the surface impedance of metals on the surface properties in the presence of a weak magnetic field parallel to the surface. In Sec. 2 we establish the boundary condition on the surface of the metal and discuss the cases under which the surface can be regarded as specular and diffuse. In Sec. 3, starting from the kinetic and wave equations, we obtain an integral equation for the Fourier component of the field in a metal under the condition

$$\Omega\tau \ll 1, \tag{1.1}$$

where $\Omega = eH/mc$ is the Larmor frequency of the electron and τ is the relaxation time. We confine ourselves to such a weak magnetic field, because under condition (1.1) the surface impedance may depend on the magnetic field only as a result of the specular surface of the metal. This dependence is derived in Sec. 4. With the aid of the derived formulas we can deduce from the experimental data certain properties of the metal surface.

All the calculations are made within the framework of classical mechanics, and consequently do not describe the quantum oscillations in a weak magnetic field, which were discovered by Khaĭkin[1]. Taking into account the periodic electron motion in the skin layer along the z axis in the case of a specular surface, the classical approach is valid under the condition

$$\frac{1}{\hbar}\oint p_z dz \gg 1, \tag{1.2}$$

which can be reduced to the form

$$\hbar\Omega \ll \varepsilon_0(\delta / R)^{3/2}. \tag{1.3}$$

Here ϵ_0 is the Fermi energy, δ the thickness of the skin layer, and R the Larmor radius of the electron.

2. BOUNDARY CONDITION

Most problems involving the penetration of an electromagnetic field in a metal in the presence of a constant and homogeneous magnetic field are solved under the assumption that the surface of the metal is either weakly specular, or diffuse (see, for example,[2,3]). Let us examine in greater detail the question of the applicability of either of the boundary conditions.

If the electron moves at an angle α to the surface of the metal, then it interacts effectively with a section of the surface with dimension of the order of $r_0/\sin\alpha$, where r_0 is the electron wavelength. Let d be the characteristic dimension of the inhomogeneities of the surface. It is clear that if the dimensions of the surface inhomogeneities are large compared with the effective interaction area, i.e.,

$$\delta = \frac{d}{r}\sin\alpha \gg 1, \tag{2.1}$$

and the inhomogeneities themselves have a random character, then the reflection of the electrons from the surface should be regarded as diffuse. In the opposite limiting case

$$\delta \ll 1 \tag{2.2}$$

the surface should be regarded as specular.

For real metals, the dimensions of the surface inhomogeneities are much larger than the electron wavelength:

$$d \gg r_0, \tag{2.3}$$

therefore the reflection is close to specular only for electrons moving at small angles to the surface. Since, furthermore, it is precisely such electrons which make the main contribution to the conductivity in the case of the strongly anomalous skin effect, it is clear that the case in which the reflection is close to specular is of great interest.

We shall assume, following[4], that the fraction p of the electrons reaching the surface of the metal is reflected specularly, and the fraction $(1 - p)$ is reflected diffusely. The parameter p is the coefficient of reflection of the electrons from the surface; as follows from the foregoing, it depends on the properties of the boundary and on the incidence angle, in the form of the combination $\delta = (d/r_0)\sin\alpha$:

$$p = p(\delta). \tag{2.4}$$

When $\delta \to 0$ we have $p(\delta) \to 1$, which corresponds to specular reflection, and when $\delta \to \infty$ we have $p(\delta) \to 0$, and the reflection is diffuse.

In the region of small δ, when the reflection is close to specular, we expand the function $p(\delta)$ in a Taylor series and confine ourselves to two terms:

$$p = 1 - \lambda\alpha, \quad \delta \ll 1, \tag{2.5}$$

where

$$\lambda = -\frac{d}{r_0}\frac{dp(0)}{d\delta}. \tag{2.6}$$

The parameter λ introduced with the aid of (2.6) is the main characteristic of the surface, and we are interested in its effect on the surface impedance.

3. INTEGRAL EQUATION FOR THE FOURIER COMPONENT OF THE FIELD IN A METAL

Let the metal occupy the region $z < 0$. We assume that a constant and homogeneous magnetic field $\mathbf{H}$ is directed along the y axis, and the electric vector $\mathbf{E}$ of the high-frequency field lies in the xy plane. Introducing a spherical coordinate system in velocity space

$$v_x = v\sin\vartheta\sin\varphi, \quad v_y = v\cos\vartheta, \quad v_z = v\sin\vartheta\cos\varphi, \tag{3.1}$$

we write the kinetic equation for the distribution function f_1, which is linearized with respect to the field, in the form

$$\left(i\omega + \frac{1}{\tau}\right) f_1 + v_z\frac{\partial f_1}{\partial z} + \Omega\frac{\partial f_1}{\partial\varphi} = e\mathbf{E}\mathbf{v}\frac{\partial f_0}{\partial\varepsilon}. \tag{3.2}$$

Here

$$f_0 = \left[1 + \exp\left(\frac{\varepsilon - \varepsilon_0}{T}\right)\right]^{-1}$$

is the equilibrium Fermi distribution function of the electrons; ω is the frequency of the electromagnetic field.

The boundary condition for the distribution function of the electrons on the surface of the metal, introduced in Sec. 2, is given by

$$f_1(z = -0, v_z < 0) = pf_1(z = -0, v_z > 0). \tag{3.3}$$

Inside the metal, far enough from the surface, the electromagnetic field and the addition to the equilibrium distribution function vanish:

$$\mathbf{E}(z \to -\infty) = f_1(z \to -\infty, \mathbf{v}) = 0. \tag{3.4}$$

The field inside the metal satisfies the wave equation

$$\mathbf{E}''(z) = \frac{4\pi i\omega}{c^2}\mathbf{j}(z), \tag{3.5}$$

in which we neglect the displacement current compared with the conduction current. The current density $\mathbf{j}(z)$ is expressed in terms of the electron distribution function:

$$\mathbf{j}(z) = -\frac{2e}{h^3}\int \mathbf{v}f_1(z, v)\,d\tau_p, \tag{3.6}$$

where $d\tau_p = dp_x dp_y dp_z$, and h is Planck's constant. The electron gas of the metal is assumed, for simplicity, to be degenerate.

Relations (3.2)–(3.6) constitute a complete system of initial equations and boundary conditions. Solving the kinetic equation and then eliminating the distribution function and the current density from the equations, we arrive at the following integral equation for the Fourier components of the low-frequency field in the metal:

$$\sum_{\nu=x,y}\left\{[k^2\delta_{\mu\nu} + K_{\mu\nu}(k)]\mathscr{E}_\nu(k) - \int_0^\infty Q_{\mu\nu}(k, k')\mathscr{E}_\nu(k')\,dk'\right\} = 2E_\mu'(-0), \quad \mu = x, y. \tag{3.7}$$

The first terms of the expansion in $1/\beta$ of the coefficient $K_{\mu\nu}(k)$ and of the kernel $Q_{\mu\nu}(k, k')$ are of the form:

$$K_{\mu\nu}(k) = \frac{16\pi i\omega e^2}{mc^2\Omega\beta}\left(\frac{mv_F}{h}\right)^3\int_0^\pi \sin\vartheta\,d\vartheta\int_{-\pi/2}^{\pi/2}\frac{n_\mu(\varphi)n_\nu(\varphi)\,d\varphi}{1 + k^2v_{zF}^2/\Omega^2\beta^2}, \tag{3.8}$$

$$Q_{\mu\nu}(k, k') = \frac{16i\omega e^2 v_F}{mc^2\Omega^2\beta^2}\left(\frac{mv_F}{h}\right)^3\int_0^\pi \sin^2\vartheta\,d\vartheta$$

$$\times\int_{-\pi/2}^{\pi/2}\frac{(1-p)[1 + (1-p)e^{-\beta\pi}\operatorname{ch}2\beta\varphi)]\operatorname{th}\beta(\pi/2 - \varphi)n_\mu(\varphi)n_\nu(\varphi)\cos\varphi\,d\varphi}{[1 - pe^{-\beta(\pi-2\varphi)}][1 - pe^{-\beta(\pi+2\varphi)}][1 + (kv_{zF}/\Omega\beta)^2][1 + (k'v_{zF}/\Omega\beta)^2]}, \tag{3.9}$$

where

$$\beta = \frac{i\omega\tau + 1}{\Omega\tau}, \quad n_\mu(\varphi) = \frac{v_\mu(\varphi)}{v}; \tag{3.10}$$

v_F is the Fermi velocity.

Equation (3.7) with coefficient (3.8) and kernel (3.9) describes the penetration of the field into the metal for both the normal skin effect $(kv_F/\sqrt{\omega^2 + 1/\tau^2} \ll 1)$ and the anomalous skin effect.

4. DEPENDENCE OF SURFACE IMPEDANCE ON THE MAGNETIC FIELD IN THE REGION (1.1)

It is easy to see that in the region (1.1) the coefficient $K_{\mu\nu}(k)$ does not depend on the magnetic field at all, but the kernel $Q_{\mu\nu}(k, k')$, continues to depend strongly on the magnetic field. This is explained by the fact that, depending on the magnitude of the electric field and on the coefficient of reflection from the surface, the electrons can either stay for a long time in the skin layer, being multiply reflected from the surface of the metal, or else leave the skin layer rapidly as the result of the collisions. The kernel (3.9) ceases to depend on the magnetic field only if the number of collisions is so much larger than the Larmor frequency that even the electrons entering the skin layer at small angles to the surface go off rapidly to the interior of the metal after being reflected from the surface. This condition, under which the magnetic field can be entirely neglected, is of the form

$$\Omega\tau \ll \sqrt{\omega^2 + 1/\tau^2}/kv_F. \tag{4.1}$$

In the region (4.1) we obtain for $Q_{\mu\nu}(k, k')$

$$Q_{\mu\nu}(k, k') = \frac{16i\omega e^2 v_F}{mc^2\Omega^2\beta^2}\left(\frac{mv_F}{h}\right)^3\int_0^\pi \sin^2\vartheta\,d\vartheta$$

$$\times\int_{-\pi/2}^{\pi/2}\frac{(1-p)n_\mu(\varphi)n_\nu(\varphi)\cos\varphi\,d\varphi}{[1 + (kv_{zF}/\Omega\beta)^2][1 + (k'v_{zF}/\Omega\beta)^2]}. \tag{4.2}$$

In the limiting case of a strongly anomalous skin effect, when

$$\frac{kv_F}{\sqrt{\omega^2 + 1/\tau^2}} \gg 1, \tag{4.3}$$

the main contribution to the integrals with respect to φ in (3.8) and (4.2) is made by the small vicinities of the points $\pm\pi/2$. Assuming the reflection coefficient p to be independent of the angle of incidence, we obtain

$$K_{\mu\nu}(k) = i\frac{k_0^3}{k}\delta_{\mu\nu}, \quad (4.4)$$

$$Q_{\mu\nu}(k,k') = \frac{2i}{\pi^2}(1-p)k_0^3\frac{\ln(k/k')}{k^2-k'^2}\delta_{\mu\nu}, \quad (4.5)$$

$$k_0 = (e^2m^2v_F^2\omega / c^2\hbar^3)^{1/3}. \quad (4.6)$$

Thus, equation (3.7) assumes in the region (4.1) the following form:

$$\left(\xi^2+\frac{i}{\xi}\right)F(\xi) - \frac{2i}{\pi^2}(1-p)\int_0^\infty \frac{\ln(\xi/\xi')}{\xi^2-\xi'^2}F(\xi')d\xi' = 1, \quad (4.7)$$

where

$$\xi = k/k_0, \quad F(\xi) = \frac{k_0^2}{2}\frac{\mathscr{E}_\mu(k_0\xi)}{E_\mu'(-0)}. \quad (4.8)$$

Equation (4.7) were solved by Hartman and Luttinger[5], who obtained for the surface-impedance tensor the expression

$$Z_{\mu\nu} = \frac{4\sqrt{3}\pi\omega}{c^2k_0}e^{i\pi/3}\frac{1-\cos(^2/_3\arccos p)}{1-p}\delta_{\mu\nu}. \quad (4.9)$$

The diagonality of the surface-impedance tensor follows from the quadratic dispersion law. Thus, in the region (4.1) the magnetic field is of no importance.

We now consider another limiting case, when

$$1 \gg \Omega\tau \gg \frac{\sqrt{\omega^2+1/\tau^2}}{k_0v_F}. \quad (4.10)$$

In the region (4.10), the electrons moving at small angles to the surface of the metal can stay in the skin layer for a long time. In this case the main contribution to the integral with respect to φ in (3.9) is made by only a small vicinity of the point $-\pi/2$. Thus,

$$Q_{\mu\nu}(k,k') = \frac{8i\omega e^2v_F}{mc^2\Omega^2\beta^2}\left(\frac{mv_F}{\hbar}\right)^3\int_0^\pi n_\mu\left(-\frac{\pi}{2}\right)n_\nu\left(-\frac{\pi}{2}\right)\sin^2\vartheta d\vartheta$$
$$\times\int_0^\infty\left\{\left[1+\frac{2\beta x}{1-p}\right]\left[1+\left(\frac{kv_F\sin\vartheta}{\Omega\beta}x\right)^2\right]\right.$$
$$\left.\times\left[1+\left(\frac{k'v_F\sin\vartheta}{\Omega\beta}x\right)^2\right]\right\}^{-1}(3-p)x\,dx, \quad (4.11)$$

where $x = \varphi + \pi/2$ when $\varphi \to -\pi/2$.

We assume first that inequality (2.1) holds, so that the reflection from the surface can be regarded as diffuse. In this case

$$Q_{\mu\nu}(k,k') = \frac{3i}{2\pi^2}k_0^3\frac{\ln(k/k')}{k^2-k'^2}\delta_{\mu\nu}, \quad (4.12)$$

and Eq. (3.7) takes the form

$$\left(\xi^2+\frac{i}{\xi}\right)F(\xi) - \frac{3i}{2\pi^2}\int_0^\infty\frac{\ln(\xi/\xi')}{\xi^2-\xi'^2}F(\xi')d\xi' = 1 \quad (4.13)$$

where ξ and $F(\xi)$ are given by formulas (4.8). The solution of (4.13) is best obtained by noting that it coincides with (4.7) when $p = 1/4$. For the surface impedance we get

$$Z_{\mu\nu} = \frac{16}{\sqrt{3}}\left[1-\cos\left(\frac{2}{3}\arccos\frac{1}{4}\right)\right]\frac{\pi\omega}{c^2k_0}e^{i\pi/3}\delta_{\mu\nu}. \quad (4.14)$$

In diffuse reflection of the electrons from the surface, the impedance does not depend on the magnetic field in the region (4.10). If the reflection is close to specular (condition (2.2)), then the reflection coefficient p should be regarded as a function of the incidence angle $p = 1 - \lambda\alpha$ (2.5), where the incidence angle is

$$\alpha \approx \sin\alpha = v_z/v = \sin\vartheta\cos\varphi, \quad (4.15)$$

and λ is a constant characterizing the surface. Substituting (2.5) in (4.11) and integrating, we get

$$Q_{\mu\nu}(k,k') = \frac{2i}{\pi^2}\rho_\mu\left(\frac{2\beta}{\lambda}\right)k_0^3\frac{\ln(k/k')}{k^2-k'^2}\delta_{\mu\nu}, \quad \mu,\nu = x,y, \quad (4.16)$$

where

$$\rho_x\left(\frac{1}{t}\right) \equiv \frac{1}{\pi}\int_0^\pi\frac{\sin^3\vartheta d\vartheta}{\sin\vartheta+1/t} = \frac{1}{2}+\frac{1}{t^2}-\frac{2}{\pi t}\left[1+\frac{\ln(t+\sqrt{t^2-1})}{t\sqrt{t^2-1}}\right], \quad (4.17)$$

$$\rho_y\left(\frac{1}{t}\right) \equiv \frac{1}{\pi}\int_0^\pi\frac{\cos^2\vartheta\sin\vartheta d\vartheta}{\sin\vartheta+1/t}$$
$$= \frac{1}{2}-\frac{1}{t^2}+\frac{2}{\pi t^2}[t-\sqrt{t^2-1}\ln(t+\sqrt{t^2-1})]. \quad (4.18)$$

Substituting (4.16) and (4.4) in (3.7), we arrive at the equation

$$\left(\xi^2+\frac{i}{\xi}\right)F(\xi) - \frac{2i}{\pi^2}\rho_\mu\left(\frac{2\beta}{\lambda}\right)\int_0^\infty\frac{\ln(\xi/\xi')}{\xi^2-\xi'^2}F(\xi')d\xi' = 1, \quad (4.19)$$

from which we get for the surface impedance

$$Z_{\mu\nu} = \frac{4\sqrt{3}\pi\omega}{c^2k_0}e^{i\pi/3}\frac{1-\cos\{^2/_3\arccos[1-\rho_\mu(2\beta/\lambda)]\}}{\rho_\mu(2\beta/\lambda)}\delta_{\mu\nu}. \quad (4.20)$$

Thus, allowance for the dependence of the reflection coefficient on the incidence angle leads to a dependence of the surface impedance of the metal on the magnetic field in the region (4.10). This dependence, however, is small, and the surface impedance changes only by 4–5% in the entire considered region of variation of the magnetic field.

Comparing the experimental dependence of the surface impedance of the metal on the magnetic field in the region (4.10) with formula (4.20), we can determine the constant λ, which characterizes the specular character of the surface.

No oscillations of the surface impedance in a weak magnetic field occur here for two reasons. First, as stated in[6], these oscillations have a quantum origin and consequently are not described within the framework of the classical theory developed above. Second, these oscillations were observed so far only in metals with a cylindrical Fermi surface, whereas we confined ourselves to the case of a quadratic dispersion law.

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