

CONSTRICTIONS IN A PLASMA OF FINITE CONDUCTIVITY

A. A. RUKHADZE and S. A. TRIGER

P. N. Lebedev Physics Institute, USSR Academy of Sciences

Submitted September 5, 1968

Zh. Eksp. Teor. Fiz. 56, 1029-1036 (March, 1969)

The problem of appearance of axially-symmetric instabilities in a plasma discharge of arbitrary conductivity is solved. Two configurations are investigated: a simple cylindrical discharge (Z pinch) and a discharge with an inverse axial current. The increments of the most dangerous long-wave instability that disturb a considerable part of the plasma are found, and it is shown that the multimode instability in an infinite-conductivity plasma revealed in Trubnikov's work^[1] changes into a single-mode instability with decrease of conductivity. The finite conductivity does not affect the fundamental unstable mode, whereas the higher modes are stabilized by the finite conductivity of the plasma. The present analysis makes it possible to rigorously justify the two approximations which have hitherto been employed in investigations of stability of plasma discharges, viz., the limiting case of high conductivity (see^[1,3] and the literature cited there) and the case of low conductivity, which is of special interest in connection with the use of self-compressed discharges as light sources^[4,5]. The analysis also points to the correctness of such limiting approaches in investigation of spiral and kink modes.

1. SIMPLE CYLINDRICAL DISCHARGE

UNDER conditions of strong radiant thermal conductivity, the plasma temperature changes little over the cross section of the discharge in the equilibrium state, and temperature fluctuations with a wavelength exceeding the average photon range in the plasma can attenuate within times much shorter than the characteristic frequencies of the oscillations. The corresponding conditions were obtained earlier^[4]. When these conditions are satisfied, the system of equations for the axially symmetrical oscillations of a cylindrical plasma discharge of finite conductivity is written in the form

$$\begin{aligned}
 & -i\omega\rho_1 + \frac{1}{r} \frac{\partial}{\partial r} (r\rho_0 v_r) + ik_z v_z \rho_0 = 0, \\
 & i\omega\rho_0 v_r = \frac{\partial}{\partial r} (E_1 + B_0 B_\varphi / 4\pi) + B_0 B_\varphi / 2\pi r, \\
 & i\omega\rho_0 v_z = ik_z (p_1 + B_0 B_\varphi / 4\pi), \\
 & \omega B_\varphi - \frac{ic^2}{4\pi\sigma_0\omega} \left(\Delta B_\varphi - \frac{B_\varphi}{r^2} \right) = k_z v_z B_0 - i \frac{\partial}{\partial r} (v_r B_0), \\
 & p_1 = v_s^2 \rho_1, \quad v_s^2 = \kappa T_0 (1+z) / M.
 \end{aligned} \tag{1}$$

Here v_s is the velocity of isothermal sound, T_0 is the temperature of the plasma, which does not change during the oscillation, and B_0 and ρ_0 are the equilibrium magnetic field of the current and the density of the cylindrical discharge^[1]:

$$B_0 = \sqrt{4\pi p_0(0)} \frac{r}{r_p}, \quad \rho_0 = \rho_0(0) \left(1 - \frac{r^2}{r_p^2} \right), \quad r_p^2 = \frac{p_0(0)c^2}{\pi j_0^2} \tag{2}$$

where r_p is the equilibrium radius of the discharge.

Using (2), it is convenient to reduce the system (1) to two equations for the quantities u and v , defined by the relations

$$u = \rho_1 v_s^2 + \frac{B_0 B_\varphi}{4\pi}, \quad v = \frac{B_0 B_\varphi}{2\pi r} + \frac{\partial u}{\partial r} \tag{3}$$

The corresponding equations are

$$\frac{\omega^2}{v_s^2} \left[\frac{r}{2} \left(\frac{\partial u}{\partial r} - v \right) + u \right] + \frac{1}{r} \frac{\partial}{\partial r} v r - k_z^2 u = 0,$$

$$\left[\omega^2 - \frac{B_0}{r} \frac{i\omega c^2}{4\pi\sigma_0} \left(\Delta - \frac{1}{r^2} \right) \frac{r}{B_0} \right] \left(v - \frac{\partial u}{\partial r} \right) = \frac{2k_z^2 B_0^2}{4\pi r \rho_0} u - \frac{2B_0}{4\pi r} \frac{\partial}{\partial r} \frac{B_0}{\rho_0} v. \tag{4}$$

The boundary conditions of this system follow directly from the equations of motion for v_r and v_z , if it is recognized that the equilibrium density vanishes on the discharge boundary, $\rho(r_p) = 0$. Hence, taking into account the boundedness of the perturbations of the velocities v_r and v_z on the plasma boundary, we obtain

$$u(r_p) = v(r_p) = 0. \tag{5}$$

We shall show that the condition $u(r_p) = 0$ has an illustrative physical meaning of the conservation of the total current in the case of oscillations in the discharge. Obviously, the conservation of the total current at small perturbations means vanishing of the expression

$$\int_0^{r_p} (\text{rot } B_1)_z r dr + \int_{r_p}^{r_p+\xi_r} (\text{rot } B_0)_z r dr = 0, \tag{6}$$

where ξ_r is the perturbation of the plasma surface, the equation of which is written in the form

$$F(r) = r - r_p - \xi_r(\varphi, z) = 0. \tag{7}$$

If it is recognized that $\xi_r \ll r_p$, then we get from (6)

$$\frac{4\pi}{c} j_0 \xi_r r_p + B_\varphi r_p = 0. \tag{8}$$

On the other hand, this relation follows from the condition $u(r_p) = 0$, with allowance for the continuity equation, written on the plasma boundary, and the equation of motion of the boundary itself

$$i\omega\rho_1 + \frac{2}{r_p} v_r \rho_0(0) = 0, \tag{9}$$

$$v_r = -i\omega \xi_r. \tag{10}$$

Substituting ρ_1 from the condition $u(r_p) = 0$ and (10) in Eq. (9), we obtain the condition (8), proving the statement made above.

Proceeding to solve the system (4), we introduce the dimensionless quantities

$$x = \frac{r^2}{r_p^2}, \quad \kappa = k_z r_p, \quad \lambda = \frac{\omega^2 r_p^2}{v_s^2}, \quad \eta = -\frac{i\omega c^2}{4\pi\sigma_0 v_s^2}. \quad (11)$$

Solving the first equation of (4) with respect to v in substituting the solution in the second equation, we reduce the system (4) to a single differential equation of fourth order in u , or else we represent it in integro-differential form

$$\begin{aligned} & \left[\lambda + \eta \left(4 \frac{\partial}{\partial x} x \frac{\partial}{\partial x} - \frac{1}{x} - \kappa^2 \right) \right] \left[\frac{e^{\lambda x/4}}{2\sqrt{x}} \int_0^x dx' e^{-\lambda x'/4} \left(\kappa^2 u - \lambda \frac{\partial u x'}{\partial x'} \right) \right. \\ & \left. - 2\sqrt{x} \frac{\partial u}{\partial x} \right] - \frac{2\kappa^2 \sqrt{x}}{1-x} u + 4\sqrt{x} \frac{\partial}{\partial x} \left[\frac{e^{\lambda x/4}}{2(1-x)} \right. \\ & \left. \times \int_0^x dx' e^{-\lambda x'/4} \left(\kappa^2 u - \lambda \frac{\partial u x'}{\partial x'} \right) \right] = 0. \end{aligned} \quad (12)$$

The boundary conditions (3) are then written in the form

$$u(1) = 0, \quad \int_0^1 dx' e^{-\lambda x'/4} \left(x - \frac{4\kappa^2}{\lambda^2} \right) u(x) = 0. \quad (13)$$

We consider throughout the most dangerous long-wave perturbations, for which $\kappa \ll 1$, with $\lambda \ll 1$ (if $\kappa \ll 1$ and $\lambda \gg 1$ there are no unstable oscillations). In this case it is easy to show that for $\kappa^2 \ll \lambda$ and for arbitrary η Eq. (12) has a unique solution, finite everywhere, satisfying the boundary condition $u(1) = 0$.

$$u(x) = \text{const} \cdot (1-x). \quad (14)$$

Substituting this solution in the second boundary condition (13), we obtain the spectrum of the unstable oscillations

$$\gamma^2 = -\omega^2 = 2\sqrt{3} |k_z| v_s^2 / r_p. \quad (15)$$

This expression coincides exactly with the growth increment of the fundamental mode, obtained by Trubnikov^[1] for the case of a plasma of ideal conductivity. The derivation of the solution (14) shows that there exists an unstable oscillation mode that is independent of the conductivity and has the character of a constriction of the plasma pinch.

It should be noted that the growth increment of the fundamental mode of the oscillations can be estimated directly from the second boundary condition (13). Since the eigenfunction of the fundamental mode has no nodes inside the segment $[0, 1]$, this boundary condition can be satisfied only when $\lambda^2 \approx \kappa^2$, or, what is the same, $\gamma \sim \sqrt{|k_z| r_p v_s} / r_p$, as is confirmed by exact calculation.

We shall continue the analysis separately for the cases $\eta \kappa^2 < 1$ and $\eta \kappa^2 > 1$. If $\eta \kappa^2 < 1$, Eq. (12) reduces to the form

$$u^{IV} - \frac{\kappa^2}{\lambda(1-x)} u + \frac{\eta}{\lambda} [4x(1-x)u^{IV} + 4(3-5x)u^{III} - 16u^{II}] = 0. \quad (16)$$

When $\eta/\lambda \rightarrow 0$ ($\sigma_0 \rightarrow \infty$), it goes over into the equation obtained and investigated in^[1]. Besides the considered fundamental mode $\kappa^2 \ll \lambda$, Eq. (16) contains also higher unstable oscillation modes, whose growth increments cannot be estimated simply from the boundary condition, since their eigenfunctions have an oscillatory character. To investigate these modes, let us analyze

(16) in the geometrical optics approximation, developed for similar fourth-order equations in^[6].

We seek a solution of (16) in the

$$u = A \exp \left(i \int_0^x k(x) dx \right). \quad (17)$$

In the zeroth geometrical-optics approximation we then obtain the following eikonal equation:

$$k^2 + \frac{\kappa^2}{\lambda(1-x)} - 4 \frac{\eta}{\lambda} x(1-x)k^4 = 0, \quad (18)$$

$$k_{1,2} = \frac{1 \pm \sqrt{1 + 16\eta\kappa^2 x/\lambda^2}}{8\eta x(1-x)/\lambda}. \quad (19)$$

We are interested in unstable aperiodic solutions that increase in time. We therefore introduce $\gamma = -i\omega > 0$. With this, $\eta > 0$ and $\lambda < 0$. One of the roots of (19) is always negative, $k_1^2 < 0$, and the corresponding wave cannot exist in the plasma. The second root, to the contrary, is positive in the entire region occupied by the plasma. Therefore, using the results of^[6], we can write the dispersion equation for the spectrum of the higher modes in the form of the quantization rule

$$\int_0^1 dx k_2(x) = \int_0^1 dx \left\{ \frac{1 - \sqrt{1 + 16\eta\kappa^2 x/\lambda^2}}{8\eta x(1-x)/\lambda} \right\}^{1/2} = \pi \left(n + \frac{3}{4} \right). \quad (20)$$

Here n is an integer (number of oscillation mode) much larger than unity. It is precisely because of this circumstance that the function $u(x)$ oscillates rapidly on the segment $[0, 1]$ and satisfies automatically the second boundary condition (13).

The dispersion equation (20) can be easily analyzed in two limiting cases.

a) High-conductivity plasma: $16 \eta \kappa^2 / \lambda^2 \ll 1$. In this case we get from (20) the following spectrum

$$\lambda = -\frac{4\kappa^2}{\pi^2(n+3/4)^2}, \quad \text{or} \quad \gamma^2 = \frac{4k_z^2 v_s^2}{\pi^2(n+3/4)^2}, \quad (21)$$

which coincides exactly with that obtained in^[1] for high modes in an ideally conducting plasma. The region of applicability of this spectrum is determined by the inequality

$$1 \gg \frac{\pi^2 n^3 c^2}{2\sigma_0 r_p^2 k_z v_s} \gg \frac{\pi^2 c^2}{2\sigma_0 r_p v_s}. \quad (22)$$

This inequality corresponds to skin-penetration of the field perturbations into the plasma, and is the inverse of the inequality used as the basis for the stability analysis in^[5].

b) Low-conductivity plasma: $16 \eta \kappa^2 / \lambda^2 \gg 1$. Equation (20) yields in this case¹⁾

$$\eta \approx \frac{4\kappa^2}{\pi^4(n+3/4)^4}, \quad \text{or} \quad \gamma \approx \frac{4k_z^2 r_p^2}{\pi^4(n+3/4)^4} 4\pi\sigma_0 \frac{v_s^2}{c^2}. \quad (23)$$

The condition for the applicability of formula (23) is the inverse of (22) and coincides with that used in the analysis of the oscillations of a plasma discharge of low conductivity^[5].

Finally, in the plasma-parameter region where $\eta \kappa^2 \gg 1$, i.e., in the case of exceedingly low conduc-

¹⁾The integral appearing in this case is equal to

$$I = \int_0^1 \frac{dx}{x^{1/4} \sqrt{1-x}} = 2 \int_0^1 \frac{\sqrt{u} du}{\sqrt{1-u^2}} = B \left(\frac{3}{4}, \frac{1}{2} \right) \approx 2.$$

tivity, which essentially is the one considered in^[5], the equation for the oscillations is of the form

$$4x(1-x)u^{IV} + 4(3-5x)u^{III} - 16u^{II} = 0 \quad (24)$$

and contains a unique bounded solution satisfying the boundary condition—the fundamental mode considered above.

Thus, we see that the growth increment of the fundamental mode of the unstable oscillations of the constriction type does not depend on the conductivity of the plasma and is determined by expression (15). On the other hand, the growth increment of higher modes decreases monotonically with increasing conductivity of the plasma, in accord with formula (23).

2. DISCHARGE WITH INVERSE AXIAL CURRENT

We now consider the pinch effect in a discharge with inverse current. For this case, the equilibrium solution was obtained earlier^[5]. The kinetic equation of a plasma in such a discharge is balanced by the magnetic field of the current in the plasma and by the magnetic field of the inverse current. The plasma fills a cylindrical layer bounded by radii r_1 and r_2 . An analysis of the plasma oscillation is based on the same equations (1) as in a simple cylindrical pinch, but using the equilibrium state for a discharge with an inverse axial current. Boundary conditions analogous to those obtained for a simple cylindrical pinch are formulated in the case of an inverse pinch on the two boundaries r_1 and r_2 (or $R_0 \pm x_p$ for the case $R_0 \gg x_p$). R_0 is the point at which the maximum plasma pressure is reached^[5]:

$$p_0(r) = p_m + \frac{\pi R_0^2 j_0^2}{c^2} \left(1 - \frac{r^2}{R_0^2} + \ln \frac{r^2}{R_0^2} \right), \quad \rho_0(r) = \frac{p_0(r)}{v_s^2},$$

$$R_0^2 = r_1^2 + \frac{I_0}{\pi j_0}, \quad x_p^2 = \frac{p_m c^2}{2\pi j_0^2}. \quad (25)$$

Introducing the dimensionless variables

$$x = \frac{r^2}{R_0^2}, \quad \lambda = \frac{\omega^2 R_0^2}{v_s^2}, \quad \kappa = k_z R_0, \quad \eta = \frac{-i\omega c^2}{4\pi\sigma_0 v_s^2}, \quad (26)$$

we can reduce the system (1), describing the oscillations in the discharge, to a single integro-differential equation for $u(x)$

$$\left[\lambda + \eta \frac{x-1}{x} \left(4 \frac{\partial}{\partial x} x \frac{\partial}{\partial x} - \frac{1}{x} - \kappa^2 \right) \frac{x}{x-1} \right] \left\{ \frac{e^{\lambda x/4}}{2\sqrt{x}} \int_{x_1}^x dx' e^{-\lambda x'/4} \right. \\ \left. \times \left(\kappa^2 u - \lambda \frac{\partial u x'}{\partial x'} \right) - 2\sqrt{x} \frac{\partial u}{\partial x} \right\} - 2\lambda \frac{v_A^2}{v_s^2} \frac{1}{\sqrt{x}} \frac{\partial u x}{\partial x} \\ + 2 \frac{v_A^2}{v_s^2} \frac{e^{\lambda x/4}}{\sqrt{x}} \left(\frac{\partial}{\partial x} \ln \frac{B_0}{\rho_0 \sqrt{x}} \right) \cdot \int_{x_1}^x dx' e^{-\lambda x'/4} \left(\kappa^2 u - \lambda \frac{\partial u x'}{\partial x'} \right) = 0, \quad (27)$$

$$u = \rho_1 v_s^2 + B_0 B_\varphi / 4\pi. \quad (28)$$

Here $x_{1,2} = r_{1,2}^2 / R_0^2$, and the quantities $v_A^2 = B_0^2 / 4\pi\rho_0$ and ρ_0 / B_0 are functions of x , determined from the equilibrium state (25), with

$$B_0 = \frac{2\pi}{c} j_0 r \left(1 - \frac{R_0^2}{r^2} \right) = \frac{2\pi}{c} j_0 R_0 \sqrt{x} \left(1 - \frac{1}{x} \right). \quad (29)$$

The boundary conditions for Eq. (27) are written in the form

$$u(x_1) = u(x_2) = 0, \quad \int_{x_1}^{2\alpha} dx u(x) e^{-\lambda x/4} \left(x - \frac{4\kappa^2}{\lambda^2} \right) = 0. \quad (30)$$

An analysis of the fundamental mode of the oscillations for $|\lambda| \ll 1$ is even simpler in this case than for the case of a simple Z pinch. Its development increment for the case $R_0 \gg x_p$ can be obtained directly from the second relation in (30), if it is recognized that in this case $x_{1,2} = 1 \pm 2x_p/R_0$. Indeed, for the fundamental mode ($n=0$) the function $u(x)$ has no nodes and is smooth in the region $x_1 \leq x \leq x_2$ occupied by the plasma (this is proved rigorously below), and since $|x_{1,2} - 1| \ll 1$, we have

$$\lambda^2 \cong 4\kappa^2, \quad \text{or } \gamma^2 = -\omega^2 = 2 \frac{|k_z| v_s^2}{R_0} < \frac{v_s^2}{R_0^2}. \quad (31)$$

To find the eigenfunction of the fundamental mode, and also to analyze the higher modes ($n > 1$), we go in (27) and (30) to the limit $\kappa^2 \ll 1$, $\lambda \ll 1$, assuming that κ^2/λ is a finite quantity. We introduce the variable $y = x - 1$ and, assuming $R_0 \gg x_p$ (i.e., $y \ll 1$), we substitute in (27) the equilibrium values for v_A^2 and B_0/ρ_0 . As the result, the equation reduces to the following:

$$\left(1 + 4y \frac{\eta}{\lambda} \frac{\partial^2}{\partial y^2} \frac{1}{y} \right) \frac{\delta u}{\delta y} + 2y \frac{\partial}{\partial y} \frac{uy}{4\alpha^2 - y^2} \\ - 2y \frac{\kappa^2}{\lambda} \frac{4\alpha^2 + y^2}{(4\alpha^2 - y^2)^2} \int_{-2x}^{y+1} u(y') dy' = 0, \quad (32)$$

and the boundary conditions assume the form

$$u(\pm 2\alpha) = 0, \quad \int_{-2\alpha}^{2\alpha} dy u(y) \left(1 + y - \frac{4\kappa^2}{\lambda^2} \right) = 0, \quad \alpha \equiv \frac{x_p}{R_0} \ll 1. \quad (33)$$

For the fundamental mode of the oscillations, as seen from (31), we have $\kappa^2/\lambda \rightarrow 0$. In this case the last term in (32) can be neglected, and a solution satisfying the first of the boundary conditions (33) is written in the form

$$u(y) = \text{const} \cdot (y^2 - 4\alpha^2). \quad (34)$$

This function actually has no nodes in the region occupied by the plasma, i.e., when $y^2 < 4\alpha^2$. Substitution of this solution in the second condition (33) leads to the spectrum (31).

We consider now the high modes ($n > 1$), from which κ^2/λ is finite. An analysis of such modes can be carried out in the geometrical-optics approximation. We then obtain from (32) the following eikonal equation

$$k^2 + \frac{2\kappa^2}{\lambda} \frac{y}{4\alpha^2 - y^2} + 4 \frac{\eta}{\lambda} k^2 \frac{4\alpha^2 - y^2}{4\alpha^2 + y^2} = 0, \quad (35)$$

$$k_{1,2}^2 = \left\{ 1 \pm \sqrt{1 + 32\eta \frac{\kappa^2}{\lambda^2} \frac{y}{4\alpha^2 + y^2}} \right\} \left| 8 \frac{\eta}{\lambda} \frac{4\alpha^2 - y^2}{4\alpha^2 + y^2} \right|. \quad (36)$$

We are interested in solutions that grow aperiodically in time, for which $\gamma = -i\omega > 0$, and therefore $\eta > 0$ and $\lambda < 0$. Taking this into account, we can easily see that the root k_1^2 is either negative or has a large imaginary part in the region occupied by the plasma. No solution corresponding to this root can exist in the form of a wave in the plasma. The second root k_2^2 turns out to be positive in the region $0 \leq y \leq 2\alpha$. Therefore oscillations are possible only in this part of the plasma, and their spectrum, according to^[6], is determined by the dispersion equation

$$\int_0^{2\alpha} k_2 dy = \pi \left(n + \frac{1}{2} \right), \quad (37)$$

where k_2 is given by formula (36), and n is an integer much larger than unity (number of the mode). Because of this, the function $u(y)$ oscillates rapidly on the segment $[0, 2\alpha]$ and the second boundary condition of (33) is automatically satisfied with a good degree of accuracy.

Let us analyze the dispersion equation (37) in the limiting cases of high and low plasma conductivity in the discharge.

a) High-conductivity plasma: $4\eta\kappa^2/\alpha\lambda^2 \ll 1$. In this limit, we obtain from (37)²⁾

$$\lambda = -\frac{4\kappa^2\alpha J_1^2}{\pi^2(n + 1/2)^2}, \quad \text{or}$$

$$\gamma^2 = -\omega^2 = \frac{4k_z^2 v_s^2 J_1^2}{\pi^2(n + 1/2)^2 R_0} \frac{x_p}{R_0} < \frac{x_p v_s^2}{R_0 R_0^2}. \quad (38)$$

The condition for the applicability of the obtained spectrum is of the form

$$1 \gg 4 \frac{c^2}{4\pi\sigma_0 x_p^2} \frac{1}{k_z v_s} \sqrt{\frac{R_0}{x_p}} > \frac{c^2}{\pi\sigma_0 x_p v_s} \left(\frac{R_0}{x_p}\right)^{3/2}. \quad (39)$$

b) Low-conductivity plasma: $4\eta\kappa^2/\alpha\lambda^2 \gg 1$. In this limiting case, Eq. (37) leads to the following oscillation spectrum³⁾

$$\eta = \frac{4\kappa^2\alpha^3 J_2^2}{\pi^4(n + 1/2)^4}, \quad \text{or}$$

$$\gamma = 4\pi\sigma_0 \frac{v_s^2}{c^2} \frac{4k_z^2 x_p^2 J_2^4}{\pi^4(n + 1/2)^4} \frac{x_p^3}{R_0^3} < \frac{v_s^2}{c^2} 4\pi\sigma_0 \left(\frac{x_p}{R_0}\right)^5. \quad (40)$$

The condition for the applicability of this formula is the inverse of the condition (39).

²⁾Here

$$J_1 = \int_0^1 dt \sqrt{\frac{t}{1-t^2}} = \frac{2}{3} \int_0^1 \frac{du}{\sqrt{1-u^{3/2}}} = \frac{\sqrt{\pi} \Gamma(3/4)}{2 \Gamma(5/4)} \approx \frac{\pi}{2}.$$

³⁾Here

$$J_2 = \int_0^1 dt \left[\frac{t(1+t^2)}{(1-t^2)^2} \right]^{1/4} \approx \frac{\pi}{2}.$$

From the foregoing analysis of perturbations of the constriction type it follows that in an inverse pinch, just as in a simple Z pinch, the growth development of the fundamental mode of axially symmetrical oscillations does not depend on the conductivity of the plasma, and may become of the order $\gamma \lesssim v_s/R_0$ (i.e., smaller by a factor R_0/r_p than in the case of a Z pinch). The development increment of the high modes ($n > 1$) decreases with decreasing plasma conductivity, and at a faster rate than the corresponding modes in the Z pinch. Thus, the inverse pinch has at $R_0 \gg r_p$ a much greater stability with respect to very dangerous long-wave instabilities, thus confirming the conclusions made in^[5] on the basis of an analysis of a plasma with "zero" conductivity.

¹B. A. Trubnikov, V sb. Fizika plazmy i problema upravlyaemykh termoyadernykh reaktsii (Plasma Physics and the Problem of Controlled Thermonuclear Reactions) 1, AN SSSR, 1958, p. 289.

²T. R. Volkov, *ibid.* 2, AN SSSR, 1958, p. 144.

³B. B. Kadomtsev, Voprosy teorii plazmy (Problems of Plasma Theory) 2, Atomizdat, 1963, p. 132.

⁴A. A. Rukhadze and S. A. Triger, Theory of Equilibrium and Stability of Strong-current Discharges in a Dense Plasma under Conditions of Radiant Heat Conduction, Preprint FIAN No. 71, 1967; PMTF No. 3, 11 (1968).

⁵A. A. Rukhadze and S. A. Triger, Theory of Equilibrium and Stability of a Strong-current Discharge in a Low-conductivity Plasma, Preprint FIAN No. 29, 1968; PMTF No. 5, 31 (1968).

⁶A. A. Rukhadze, V. S. Savodchenko, and S. A. Triger, PMTF No. 6, 58 (1965).

Translated by J. G. Adashko