TURBULENT DIFFUSION AND ION HEATING IN THE CURRENT-DRIVEN INSTABILITY

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The development of turbulence and the plasma state associated with the current-driven instability are investigated. It is shown that anomalous transverse diffusion occurs and is caused by the turbulent fluctuations. The excitation of the instability also leads to heating of the ions.

CURRENT-DRIVEN instabilities in a potasium plasma have been investigated earlier by the present authors.[1-4] In this earlier work two instabilities, ion-cyclotron and "ion-acoustic," were observed. The ion-cyclotron instability was observed in the passage of current over a small cross-section along the axis of the plasma column (case I); the "ion-acoustic" instability was observed when current flowed over the entire cross-section of a plasma (case II). However, the ion-acoustic instability could not be established definitely since it was found that the critical drift velocity for both instabilities was essentially the same: $u_c \sim 0.1 v_{Te}$. For the ion cyclotron instability this value coincides with the calculated value, but for the ion-acoustic instability the value is very different from the calculated value ($u_c \sim v_{Te}$).

In the present work we present a model for the current-driven instability according to which the primary instability in case II is the ion-cyclotron instability. As a consequence of the random nature of the initial fluctuations the ion-cyclotron waves are excited in different "current tubes" (the cross-section of a "tube" is smaller than the cross-section of a plasma column). The oscillations in different tubes are uncorrelated. As a consequence there arise random transverse potential differences which lead to the diffusion of plasma across the magnetic fields.

The enhanced diffusion leads to a reduction of the plasma density, and consequently, a reduction in the current, which is determined by the density. When the current is reduced below a critical value the instability is terminated. After the density is restored the instability is excited again and the cycle repeats. Thus, relaxation oscillations are excited in the system.

In the present work we describe experiments that support the model that has been proposed here; it is shown that the oscillations which had earlier been thought to be ion-acoustic oscillations are relaxation oscillations. We have investigated the development of turbulence in the excitation of the current-driven instability, the final turbulent state of the plasma, the diffusion mechanism, and the heating of the ions that follows the development of the instability.

DESCRIPTION OF THE APPARATUS AND METHOD

These experiments have been carried out in a Q-machine, in which the plasma is produced by thermal ionization of potasium on a tungsten ionizing plate (Fig. 1) of radius R = 2 cm which is heated to a temperature T $\sim 2000°$ K. The length of the plasma column L is 36 cm.

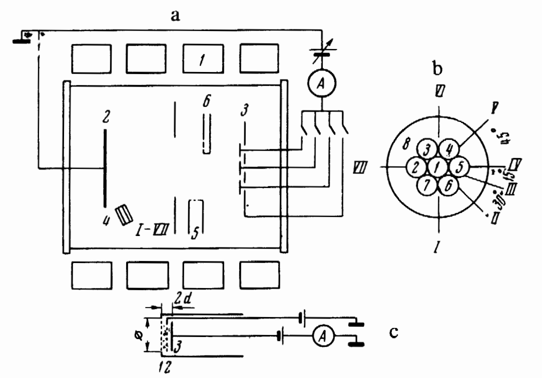

FIG. 1. Diagram of the apparatus (a): 1) solenoid, 2) ionizer, 3) electrodes, 4) potasium oven, 5) transverse-energy analyzer, 6) longitudinal-energy analyzer; I-VII) probes. (b) Diagram showing the arrangement of probes and electrodes. (c) Diagram of the transverse-energy analyzer: 1) first grid and frame, 2) second grid, 3) collector; 2d = 4mm; $\varnothing$= 8mm.

The magnetic field is along the axis of the chamber and is 1000 Oe. The plasma is essentially collisionless ($n \sim 10^9$–10^{10} cm^{-3}). The cold electrodes 1–7 (Fig. 1b), which are provided for the formation of "current tubes," are insulated from each other so that a desired potential can be applied to any one of them as well as to the larger electrode 8. The diameter of the small electrodes 1–7 is 9.5 mm and the diameter of the large electrode is 65 mm.

The density and plasma oscillations are measured with single probes. The probes I, IV, VI, and VII are separated from each other by 90° and can be moved in the radial direction. There is also a probe that can be moved along the axis of the chamber. Probes II, III and V are located at r = 10 mm. The probe length is 1 mm. and the diameter is 0.25 mm. The oscillation spectrum is observed by means of selective voltmeters (S5-2 and S5-3) which record the effective amplitude in a passband of 200 Hz. The diffusion coefficient for diffusion across the magnetic field is determined from the transverse plasma flux and is also estimated from the longitudinal density gradient.[2]

The ion energies are analyzed by means of two-grid analyzers which can be moved in the parallel or perpendicular directions (with respect to the magnetic field). A diagram of an analyzer is shown in Fig. 1c. The grid 1 and the frame of the analyzer are at the floating potential. The grid 2 is maintained at a negative potential sufficient to block electrons. A retarding potential is applied to the collector. We note that in work with an alkali metal plasma the surface of the collector is subject to the formation of a film of alkali metal. This leads to the production of a contact potential (which can reach values of approximately 2–3 V) and consequently, to a bias on the volt-ampere characteristic of the collector in the negative region. It should be noted that the accuracy in the reference for the energy is not changed significantly in this case if the origin is taken to be the beginning of the retardation curve. In working with the transverse-energy analyzer the volt-ampere characteristic can be distorted by an inhomogeneity in the plasma density if particles with different values of the Larmor radius (different energy) come from regions of different density. As a consequence the quantitative analysis of the volt-ampere characteristic (determination of the distribution function) is subject to error.

In investigating the turbulent state of the plasma we have measured the time correlation $F(\tau)$ and the space correlation $F(\xi)$ of the density oscillations. Use has been made of a correlator which records the one-bit correlation function.[2] In the measurements of $F(\tau)$ we make use of a delay line in which τ varies from 0 to 100 μsec in steps of 2.5 μsec. The function $F(\xi)$ is measured for $\tau = 0$ by varying ξ the distance between the probes that record the oscillations. In measuring the azimuthal correlation function $F(\xi_\varphi)$ we have used probes I–VI and in measuring the radial function $F(\xi_r)$ we have used probes IV, VII, one of which is located at r = 10 mm; the others can be moved in the radial direction. The longitudinal correlation function is measured by means of a probe which can be moved in the parallel direction.

In measuring the correlation function in the noisy portion of the spectrum $f \gtrsim 40$ kHz the signal from the plasma is passed through a low-frequency RC-filter which allows us to filter out low-frequencies without distorting the spectrum in the region $f \gtrsim 40$ kHz. The functions $F(\tau)$ and $F(\xi)$ measured in the experiment can be used to determine the randomization time for the signal phase and the correlation length.

We note that the conventional normalized correlation functions $\rho(\tau)$ and $\rho(\xi)$, which can be applied to a normal distribution of probability density p(x) and signal x(t), are related to the one-bit correlation functions $F(\tau)$ and $F(\xi)$ by the expressions

$$\rho(\tau) = -\cos[2\pi F(\tau)], \qquad \rho(\xi) = -\cos[2\pi F(\xi)]. \qquad (1)$$

The distribution p(x) is determined by analysis of the oscillogram signals that are processed through the low-frequency RC-filter.

EXPERIMENTAL RESULTS

Two series of experiments have been carried out. In the first series we have verified the model described above: we have investigated the development of turbulence, the appearance of diffusion and the excitation of low-frequency oscillations as the number of "current tubes" is increased. The current tubes are simulated by passing current to the electrodes with small cross-section (electrodes 1–7, Fig. 1b). In the second series of experiments we have investigated the turbulent state of the plasma, the diffusion, and the acceleration of ions in the passage of current over the entire cross-section of the plasma column, that is to say, in the development of the current-driven instability.

1. In accordance with the earlier results[4] and theory[5] at a critical electron drift velocity $u_c \sim 40\, v_{Ti}$ in the axial "tube" (current flowing to electrode 1) an ion-cyclotron instability is excited. The instability is manifest in the excitation of radial waves with frequencies $\sim nf_{Hi}$ (n = 1, 2, 3). Oscillations of this kind were excited in each of the current tubes 2–7.

It is found that in simultaneous excitation of ion-cyclotron waves in different tubes the oscillations are not correlated. As a result, potential differences appear which lead to the drift of plasma across the magnetic field. This feature is found in the fact that outside the tubes there are density oscillations caused by the displacement of plasma due to the drift in the electric and magnetic fields. In Fig. 2 we show oscillograms of the density oscillations at r = 12 mm taken with probe 1 (cf. Fig. 1b) with different numbers of current tubes being operated. It is evident from the oscillogram that switching on tubes 6 or 7 alone does not lead to appreciable density oscillations (curve b). (Curve a corresponds to the case in which there is no current in the plasma, that is to say, all of the tubes are unconnected.) If two tubes are switched on simultaneously one expects that the amplitude of the density oscillations at a given point will be equal to the sum of the amplitudes when the tubes are activated individually. This would indicate a simple superposition of the waves from the two tubes. However, it is evident from oscillogram c in Fig. 2 that this is not the case: simultaneous activation of two tubes 6 and 7 leads to the appearance of oscillations whose amplitude is much larger than the sum of the amplitudes of the oscillations caused by activating tubes 6 or 7 individually. This result verifies the assumption of the displacement of plasma due to drift in the electric fields between the tubes. Increasing the number of current tubes leads to an increase in the amplitude of the density oscillation (in curve d we have activated tubes 1, 6, and 7). We note that the critical electron drift velocity required for excitation of the ion-cyclotron oscillations for one or for several current tubes is essentially the same.

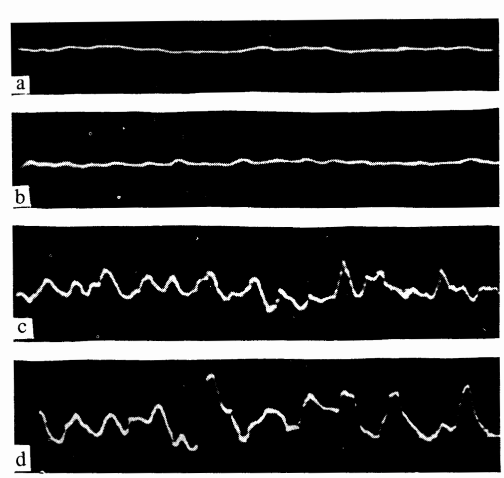

FIG. 2. Oscillograms showing the density oscillations at r = 12mm taken with probe I: a) in the absence of current; b) with electrode 6 connected; c) with electrodes 6 and 7 connected; d) with electrodes 1,6 and 7 connected. The sweep width is 200 μsec.

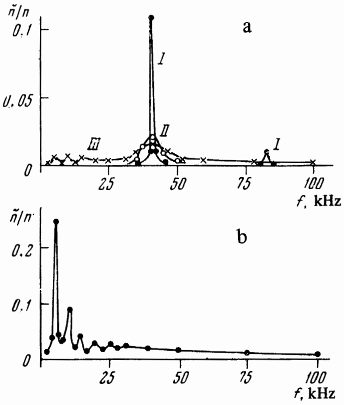

FIG. 3. Spectrum measured at the axis: a) I) with electrode 1 connected; II) with electrodes 1 and 3 connected; III) with electrodes 1 – 5 connected; b) with current flowing over the entire plasma cross-section.

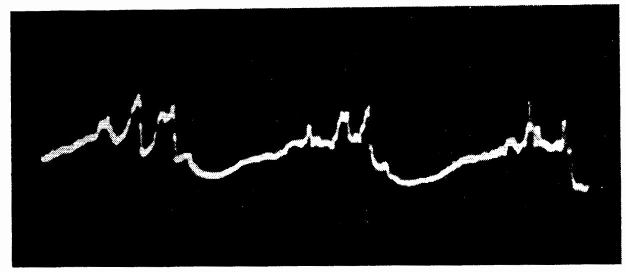

FIG. 4. Density oscillations at the axis. The sweep width is 500 μsec.

The development of turbulence can be viewed from the spectrum (Fig. 3a) which is measured at the axis of the plasma column as more and more current tubes are activated. The spectrum marked I corresponds to the oscillations excited by the axial tube 1 alone. The narrow peak in the spectrum indicates regular oscillations. Increasing the number of current tubes leads to a broadening of the peak in the spectrum corresponding to random oscillations (in spectrum II tubes 1, 2, and 3 are activated and in spectrum III tubes 1–5 are activated). Under these conditions the oscillations in different tubes are found to be uncorrelated, that is to say, the plasma becomes turbulent and the ion-cyclotron oscillations are manifest in the form of irregular fluctuations.

2. When current flows over a large cross-section of the plasma, low-frequency oscillations are observed in the instability spectrum (Fig. 3a, spectrum III). Current flowing over the entire cross-section of the plasma leads to the excitation of low-frequency oscillations with large amplitude in which case that portion of the spectrum in which $f \sim f_{Hi}$ is lost in the noise and the ion-cyclotron frequency cannot be distinguished (spectrum III,b). The critical velocity necessary for excitation of low-frequency oscillations is essentially the same as u_c for the ion-cyclotron instability.

It is found that irregular oscillations with frequency $\sim f_{Hi}$ are observed at a definite phase of the low-frequency oscillations. As is evident from Fig. 4, irregular oscillations are excited at the phase of the maximum and are terminated at the phase of minimum density and exist for a time interval $\tau_0 \sim 4f_{Hi}^{-1}$. As will be shown below (Fig. 7) it is precisely during this time interval that enhanced diffusion is observed. The excitation of irregular oscillations occurs simultaneously over the entire length of the column. The growth in density occurs later at larger values of z.

On the basis of these results it can be assumed that the low-frequency oscillations are the result of a relaxation process, the detailed mechanism of which is the following. When the current exceeds the critical value ($u > u_c$) uncorrelated ion-cyclotron oscillations are excited in the plasma and these lead to diffusion.

The higher diffusion causes a longitudinal density gradient $n \sim n_0 e^{-\alpha z}$ (n_0 is the density near the ionizer) and a reduction in the current as determined by the density n_L near the electrode. When the current is reduced to a value such that $j/n_0 < u_c$ the instability is terminated at the ionizer and with a further reduction in current it is gradually terminated over the entire length of the plasma column. If the diffusion is large the instability is terminated essentially simultaneously over the entire length of the column. After the instability is terminated a front of unperturbed plasma propagates out from the ionizer. When this front reaches the electrode (in which case $n_L = n_0$ and $u > u_c$ over the entire length of the system) the instability is excited and the process repeats itself.

Thus, time-periodic oscillations are established in the system and the frequency is determined by τ_0, the time in which u falls below u_c, and the velocity of the front v_0: $f \sim (\tau_0 + L/v_0)^{-1}$. An estimate of f on the basis of the value of τ_0 as determined from the oscillogram (cf. Fig. 4), and v_0 as found from the phase shift in the density jump for different values of z, yields the value f = 4.5 kHz which coincides with the experimental value. We note that $v_0 \sim 2 \times 10^5$ cm/sec is close to the ion-acoustic velocity. It is evident from the foregoing that the low-frequency oscillations are relaxation oscillations and that the initiating cause for their excitation is the ion-cyclotron instability, which causes diffusion. Thus, the assumption as to the ion-acoustic nature of the instability observed in the excitation of low-frequency oscillations[1-3] appears to be in error.

3. It is of interest to investigate the state of a plasma in the time interval in which irregular oscillations of the density characterized by a period $\sim f_{Hi}^{-1}$ are observed against the background of low-frequency oscillations. For this purpose we have measured the correlation functions $F(\tau)$ and $F(\xi)$ for signals that are filtered by the low-frequency RC-filter.

As we have indicated above, $F(\tau)$ and $F(\xi)$ are related to $\rho(\tau)$ and $\rho(\xi)$ by the expressions in (1) in the case in which the distribution of the probability density p(x) and the signal x(t) are of normal form. We have measured the distribution of the probability density of the signal as transmitted through the low-frequency filter. It is found that it can be rather well approximated by a Gaussian curve, that is to say, Eq. (1) can be used in the present case.

In Fig. 5 we show time correlation functions measured with different distances between probes. It is found that these are symmetric with respect to the delay time τ, that is to say, $\rho(\tau, \xi) = \rho(-\tau, \xi)$. This means that the oscillations being studied do not have a favored direction of propagation across the column. The absence of a favored direction of propagation might appear to be strange since the wave excited at the axis

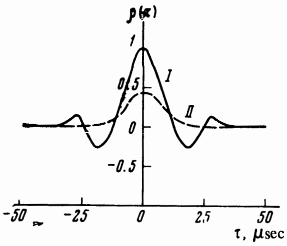

FIG. 5

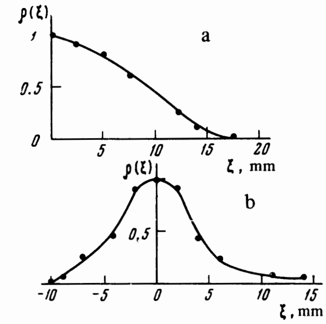

FIG. 6

FIG. 5. Time correlation functions. I) autocorrelation function (ξ=0); II) cross correlation function for the signals from probes I and II.

FIG. 6. Space correlation functions: a) azimuthal, b) radial.

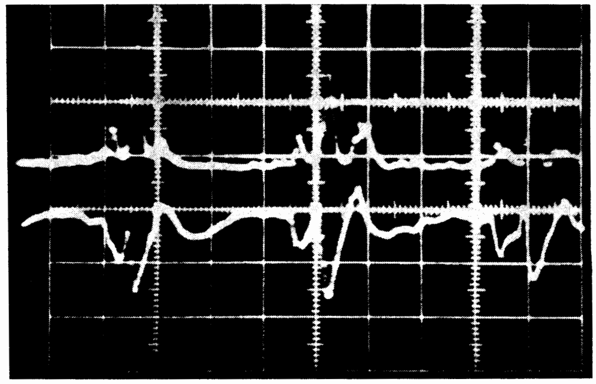

FIG. 7. Oscillograms showing the loss of plasma across the magnetic field: upper trace—density oscillations at r = 25mm (probe IV); lower trace—potential difference between probes II and V; sweep width 500 μsec.

propagates in the radial direction.[4] However, account should be taken of the fact that in the flow of current over the entire cross-section at different instants of time, waves are excited at arbitrary points over the cross-section so that on the average there will not be any preferred directions in the transverse plane. In Fig. 6 we show the azimuthal and radial spatial correlation functions. It is evident that the correlation between oscillations at different points over the cross section of the plasma column is lost in a length of approximately 1 cm. Thus, the fully developed instability appears in the form of uncorrelated fluctuations with dimensions of approximately 1 cm.

The measurement of the longitudinal correlation function $F(\xi_z)$ shows that a longitudinal correlation is maintained over a length of approximately 15 cm., that is to say, the fluctuations are in the form of tubes that are elongated along the magnetic field. Thus, it can be stated that in the case being investigated the plasma exhibits a highly anisotropic turbulence which is characterized by a long longitudinal correlation length and a small transverse correlation length. The anisotropy of the turbulence is associated primarily with the nature of the instability that leads to the turbulent state. The ion-cyclotron instability is characterized by the inequality $\lambda_z \gg \lambda_r$,[4] so that one expects that the longitudinal correlation length will be larger than the transverse correlation length.

It has been shown in [4] and [5] that the ion-cyclotron wave is electrostatic, that is to say $\tilde{n}/n \sim e\tilde{\varphi}/T$ where $\tilde{n}$ and n are the fluctuating and constant components of the density and $\tilde{\varphi}$ is the fluctuating component of the potential. Assuming that the oscillations are electrostatic it then follows that irregular fluctuations in density should lead to irregular potential differences between different points over the cross-section of the column. Actually, measurement of the potential differences $\Delta\varphi$ by means of an S 1-17 oscilloscope with a differential input shows that the average $\Delta\varphi$ with a distance between probes of 0.7, 1.5, and 2 cm. is respectively 0.05, 0.1, and 0.1 V. Thus, at distances larger than the correlation length $e\varphi/T \sim 0.5$ ($T \sim 0.2$ eV). The maximum value $\Delta\varphi \sim 0.15$ that is to say $e\tilde{\varphi}/T \sim 0.7$. In order to estimate the intensity of the fluctuating electric field we can make use of the expression $\tilde{E} \sim \Delta\tilde{\varphi}/l_0$ where l_0 is the transverse dimension of the fluctuation, which is approximately 1 cm, so that $\tilde{E} \sim 0.1$ V/cm.

It should be noted that the experiments are carried out in a plasma that is inhomogeneous in density in the radial direction. In this case one expects that the turbulent spectrum will be influenced by drift waves as well as ion-cyclotron waves.

4. The loss of plasma across the magnetic field occurs in phase with the low frequency oscillations when the plasma is subject to the density fluctuations or the fluctuations in potential difference is associated with the density fluctuations. This can be seen from the oscillogram in Fig. 7 where we show the oscillations in the potential between probes II and V and the oscillations in the density outside the plasma column at r = 2.5 cm. We note that in [2] an investigation was not made of the detailed nature of the loss of plasma across the magnetic field and from an oscillogram taken in the multiple sweep mode it was concluded that there was an azimuthally symmetric loss of plasma. The detailed pattern of the plasma bursts outside the plasma column shows that at different azimuthal points these bursts appear in the same time intervals at which there exist potential differences at the given azimuthal point. Since the oscillations of potential are irregular and exist at different points at different times the flux across the field will be azimuthally symmetric on the average.

Turbulent diffusion for an anisotropic case similar to ours has been considered by Spitzer.[6] Spitzer gives the following diffusion coefficients:

$$D_\perp = \frac{2c^2\overline{E^2}}{H^2}\tau_S, \quad \tau_S = \int_0^\infty \rho_E(\tau)\,d\tau, \tag{2}$$

where $\overline{E^2}$ is the mean-square electric field, $\rho_E(\tau)$ is the correlation function for the electric field. The final expression for $D_\perp$ is obtained under the following assumptions:

$$(\overline{E^2})^{1/2} = k_1\frac{\varphi}{\rho_i}, \quad e\tilde{\varphi} = k_2 T, \quad \tau_S = \frac{k_3}{\omega_{Hi}}, \tag{3}$$

where k_1, k_2, and k_3 are unknown constants and $\rho_i = \omega_{Hi}^{-1}\sqrt{T/M}$. Substituting (3) in (2) we have

$$D_\perp = 2k_1^2k_2^2k_3\frac{cT}{eH}. \tag{4}$$

The coefficients k_1, k_2, and k_3 can be estimated on the basis of the experimental data and in this way it is possible to obtain the expected value of $D_\perp$ from Eq. (4) for the plasma state being investigated. As we have indicated above, the transverse correlation length $l_0 \sim 1$ cm. This dimension is used in estimating $\tilde{E}$ from the measured value of $\tilde{\varphi}$. Since $l_0 \sim 3\rho_i$, in accordance with Eq. (3) we find $k_1 \sim 0.3$. Starting from the result that $e\tilde{\varphi} \sim 0.5\,T$, we have $k_2 \sim 0.5$. In estimating τ_S, assuming that the oscillations are electrostatic, we can replace $\rho_E(\tau)$ by the correlation function for the density oscillations $\rho_n(\tau)$, which is measured experimentally. The value of τ_S as estimated from $\rho_n(\tau)$ (Fig. 7) is approximately $0.25\, f_{Hi}^{-1}$ i.e., in accordance with (3), $k_3 \sim 1.5$. Finally, we have

$$D_\perp \sim 8 \cdot 10^{-2}\, cT / eH. \qquad (5)$$

In our case ($T \sim 0.2$ eV; $H = 1000$ Oe) $D_\perp \sim 1.6 \times 10^3$ cm^2/sec. This result is found to be in satsifactory agreement with the experimental values of the diffusion coefficient[2] $D_\perp \sim (1.5 \pm 0.3) \cdot 10^3$ cm^2/sec and $(3 \pm 1) \cdot 10^3$ cm^2/sec (the first value refers to the diffusion coefficient as determined by measuring the transverse flux and the second as determined by measuring the longitudinal density gradient). The functional relation $D_\perp(H)$ as observed experimentally is also in agreement with the relation in (5). We note that the diffusion coefficient in (5) is essentially the same as the Bohm value $\frac{1}{16} cT/eH$.

Thus, it is concluded that the diffusion of plasma across the magnetic field is of a turbulence nature.

5. The excitation of the instability is accompanied by the appearance of accelerated ions. It is evident from the volt-ampere characteristics of the transverse energy-analyzer (Fig. 8) that excitation of the instability leads to a reduction in the slope of the volt-ampere characteristic and a shift toward the region of higher energy.

In Figs. 9 and 10 we show oscillograms of the current for the ion transverse-energy analyzer (upper trace) and the density fluctuations in the plasma. The oscillogram in Fig. 9 is taken at a collector voltage $U_C = -3$V, in which case the collector accepts essentially all of the accelerated ions. It will be evident that these appear essentially simultaneously with the irregular fluctuations in the plasma (lower trace) and disappear when these fluctuations disappear. The oscillogram in Fig. 10 is taken with $U_C = +3$ V, that is to say, when the collector accepts ions only with energies ≥ 6 eV. It will be evident from the figure that these fast ions appear with a delay with respect to the excitation of the fluctuations.

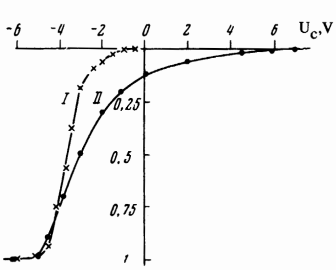

FIG. 8. Volt-ampere characteristics of the transverse-energy analyzers: I) in the absence of the instability; II) with the instability present.

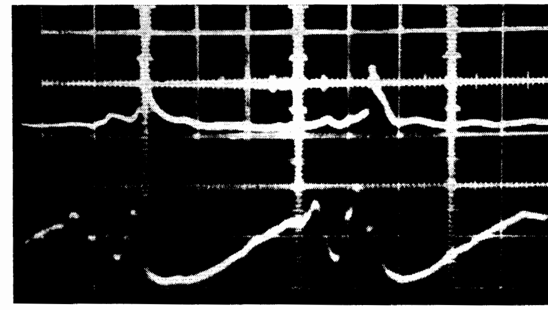

FIG. 9. Upper trace—collector current in the transverse energy analyzer, $U_C = -3$V; lower trace—density fluctuations at r = 10mm.

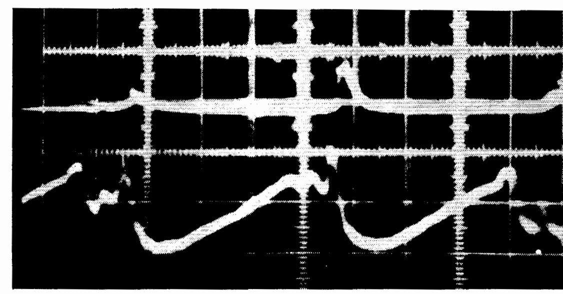

FIG. 10. Upper trace collector current $U_C = +3$V; lower trace—density fluctuations at r = 10mm.

It follows from a comparison of Figs. 9 and 10 that the acceleration of ions is caused by the turbulence fluctuation and that a finite time is required for the ions to be accelerated to a given energy, the ion energy increasing with increasing acceleration time.

The fact that excitation of the instability causes a modification of the entire volt-ampere characteristic (Fig. 8) leads to the conclusion that the acceleration of ions is in the nature of a heating process. Assuming that the distribution function is Maxwellian, we can find the temperature from the collector potential at which the current amounts to a given fraction (for example 10^{-2}) of the initial value. This estimate yields $T_{\perp i} \sim 1{-}2$ eV, which is significantly greater than the initial value $T_{0i} \sim 0.2$ eV.

We consider possible mechanisms for ion acceleration or heating: 1) resonance acceleration due to ion-cyclotron waves; 2) Fermi acceleration; 3) stochastic heating.

The resonance mechanism is eliminated in the present experiments since the oscillations are irregular and the correlation time is smaller than the cyclotron period.

Fermi acceleration for the case of anisotropic turbulence, similar to the present case, has been considered by A. V. Gurevich. The mechanism responsible for this acceleration is essentially collisions between fast ions ($\rho_i > l_0$ where l_0 is the correlation length) with plasma inhomogeneities that move with a velocity $\tilde{v}_d = c\tilde{E}/H \sim c\tilde{\varphi}/l_0 H$. The coefficient found by Gurevich for diffusion in velocity space (cf. Appendix) is given by

$$D_{v\perp} = \frac{c^2 e^2 (\tilde{\varphi})^4}{M^2 H^2 v_\perp{}^3 l_0{}^3}, \qquad (6)$$

where $v_\perp$ is the ion velocity. Thus this mechanism can lead to acceleration only for fast ions in a limited velocity region ($D_{v_\perp} \sim v_\perp^{-3}$).

The stochastic mechanism considered by Bass, Faĭnberg, and Shapiro[7] causes heating of all of the ions in the plasma. In this case the diffusion coefficient in velocity space is given by[7]

$$D_{v\perp} = \frac{e^2}{4M^2} \sum E_k{}^2 \frac{\tau_c}{1 + (\omega_k - \Omega_H)^2 \tau_c{}^2}, \tag{7}$$

where τ_c is the correlation time while E_k and ω_k are the electric field and the frequency of a wave characterized by wave vector k. For the case in which the oscillation spectrum is localized around Ω_H, neglecting terms containing the factor $(\omega_k - \Omega_H)$ and assuming that $E \sim \tilde{\varphi}/l_0$, we can write Eq. (7) in the form

$$D_{v\perp} \sim \frac{e^2 (\tilde{\varphi})^2 \tau_c}{4M^2 l_0{}^2}. \tag{8}$$

A comparison of the diffusion coefficients given by Eq. (6) and (8) indicates that under the present experimental conditions $D_{v_\perp}$ for stochastic heating is always much greater (at least two orders of magnitude) than $D_{v_\perp}$ as computed for Fermi acceleration. Thus, stochastic heating appears to be the important process in the present case.

The formula for the ion temperature given in [7] under the assumptions made above is

$$T_{\perp i} = T_{0i} + \frac{e^2 (\tilde{\varphi})^2 \tau_c \Delta t}{2M l_0{}^2};$$

where Δt is the ion heating time. For the present case ($\tau_c \sim 1/f_{Hi}$, $\Delta t = 4/f_{Hi}$) Eq. (8) yields $T_{\perp i} \sim 1$ eV, which is in good agreement with experimental result $T_{\perp i} \sim 1-2$ eV. Thus it is assumed that the ions are heated by stochastic heating.

An analysis of the longitudinal ion energy indicates that this energy is also increased when the instability is excited, although to a much smaller extent than the transverse energy. The increase in the longitudinal energy can be explained by ion-ion collisions which lead to the transfer of energy from the transverse degree of freedom into the longitudinal degree of freedom. Under the present experimental conditions the ion-ion collision time ($\sim 0.5 \times 10^{-4}$ sec) is comparable with the heating time ($\sim 1 \times 10^{-4}$ sec).

The authors wish to thank A. V. Gurevich for valuable discussions and for kindly providing unpublished results. The authors are also indebted to I. A. Bogashchenko for help in carrying out the experiments.

APPENDIX

DIFFUSION COEFFICIENT IN VELOCITY SPACE FOR FERMI ACCELERATION

Let us consider a magnetized plasma in which low-frequency electrostatic waves are excited. Assume that the phase of these waves changes in a direction transverse to the magnetic field in a random way in a distance l_0 so that fluctuating electric fields are produced. The presence of these fields leads to a drift of any density inhomogeneity, diffusion of plasma, and acceleration of ions in the direction perpendicular to the magnetic field.

We consider the case in which the correlation radius is greater than the mean ion-Larmor radius and we determine the diffusion coefficient in velocity space for ions whose Larmor radius $\rho_\perp$ is greater than l_0 (the mechanism being considered here does not operate for ions with smaller values of $\rho_\perp$).

The drift velocity of the inhomogeneities in the random fields is

$$v_d = \frac{c\tilde{E}}{H} = \frac{c\tilde{\varphi}}{Hl_0}. \tag{9}$$

The velocity of the fast ions ($\rho_\perp > l_0$) in the presence of inhomogeneities is

$$v_i = \left[(v_\perp + v_d)^2 + \frac{2e\tilde{\varphi}}{M} \right]^{1/2} - v_d$$

Consequently, the fluctuations in ion velocity due to scattering on inhomogeneities is

$$\Delta v_i = \overline{v_i - v_\perp} = \frac{e\tilde{\varphi}}{M(v_\perp + v_d)}. \tag{10}$$

The ion velocity fluctuations due to scattering on fixed inhomogeneities is

$$\Delta v_{i0} = e\tilde{\varphi} / Mv_\perp. \tag{11}$$

We note that the scattering of ions on fixed inhomogeneities does not lead to the diffusion in velocity space since the ion energy is not changed in this process.

The diffusion coefficient in velocity space is

$$D_{v\perp} = \frac{\overline{d(\Delta v_i)^2}}{dt} = \overline{[(\Delta v_i)^2 - (\Delta v_{i0})^2]}\,\nu. \tag{12}$$

Here, $\nu = v_\perp / l_0$ is the frequency of collisions between ions and inhomogeneities. Substituting Eqs. (9)–(11) in Eq. (12) we have

$$D_{v\perp} = \frac{c^2 e^2 \tilde{\varphi}^4}{M^2 H^2 v_\perp{}^3 l_0{}^3}. \tag{13}$$

[1] N. S. Buchel'nikova, R. A. Salimov and Yu. I. Eidel'man, Nuclear Fusion 6, 255 (1966).
[2] N. S. Buchel'nikova, R. A. Salimov, and Yu. I. Edel'man, Zh. Eksp. Teor. Fiz. 52, 387 (1967) [Sov. Phys. JETP 25, 252 (1967)].
[3] N. S. Buchel'nikova, R. A. Salimov, and Yu. I. Eidel'man, Zh. Tekh. Fiz. 37, 1477 (1967) [Sov. Phys.-Tech. Phys. 12, 1072 (1968)].
[4] N. S. Buchel'nikova, R. A. Salimov, and Yu. I. Eidel'man, PMTF No. 4, 149 (1968).
[5] W. E. Drummond and M. N. Rosenbluth, Phys. Fluids 5, 1507 (1962).
[6] L. Spitzer, Phys. Fluids 3, 659 (1960).
[7] F. G. Bass, Ya. B. Faĭnberg, and V. D. Shapiro, Zh. Eksp. Teor. Fiz. 49, 329 (1965) [Sov. Phys. JETP 22, 230 (1966)].

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