

GALVANOMAGNETIC PHENOMENA IN METALS IN THE QUANTUM LIMIT

A. A. ABRIKOSOV

Institute of Technical Physics, U.S.S.R. Academy of Sciences

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The conductivity tensor in a strong magnetic field is determined for the case when magnetic splitting is of the order or more than the Fermi energy. The case of $T = 0$ and neutral point impurities is considered. Two models are analyzed, the first with a "good" metal with a quadratic isotropic dispersion law, and the second with a semimetal with isotropic and quadratic spectra of electrons and holes separated by a large momentum in momentum space. The calculation method, which makes use of Green's function expanded in the eigenfunctions of the problem, considerably simplifies the calculations for the first model, which has already been considered in previous papers, and refines the results. The conductivity tensor for the second model is found for the limiting quantum case in which, owing to level overlapping, the number of electrons and holes becomes much greater than in the absence of a field. Other possible applications of the calculation method are discussed.

1. INTRODUCTION

As is well known, the galvanomagnetic properties of metals, even in very strong fields, are well described by the quasiclassical approximation, in which the electrons can be regarded as moving in their classical orbits. This is connected with the fact that usually the distance between the Landau levels μ^*H is much smaller than the Fermi energy ϵ_F . The fields required to violate this relation at $\epsilon_F \sim 1$ eV and $\mu^* \sim \mu_{free}$ are of the order of 10^8 Oe, which at any rate are presently not attainable under laboratory conditions. When $\mu^*H \ll \epsilon_F$, allowance for the quantum effects yields all the small oscillatory additions to the conductivity tensor.

The relation $\mu^*H \ll \epsilon_F$ can be violated in semimetals, where, first, the Fermi energy is of the order of several hundred eV, and on the other hand the small effective masses give large values of μ^* . As a result, fields on the order of 10^5 Oe may suffice. In principle, this is the case also for semiconductors. However, inasmuch as the presence of free carriers is ensured in semiconductors by the presence of impurities, the number of carriers is approximately equal to the number of impurities, the condition $\hbar/\tau \ll \epsilon_F$ is violated in semiconductors (τ —free path time), leading to additional difficulties. Such a case will not be considered here.

To study the main features of the phenomenon, we confine ourselves to two simple models. The first is a metal with a dispersion law $p^2/2m_1$, with m_1 of the order of the electron mass and with the usual electron density ($n_e \sim 10^{22}$ cm⁻³). This model will help us develop a calculation method in simple form, and to investigate the role of various contributions to the effect.

The magnetic field is assumed arbitrary. In principle, this case was investigated earlier.^[1, 2] However, the method which is employed here is much simpler and is more general. This has made it possible to refine the results obtained in^[1, 2].

The second model is a simplified variant of a semimetal, which, insofar as we know, was never investigated in the quantum limit.

The calculation method is based on expanding the

Green's function of the electron in terms of the eigenfunctions in the magnetic field. This method was used by Bychkov^[3] to investigate the role of collisions in the de Haas-van Alphen effect. We shall show here that it is also very convenient for the calculation of kinetic phenomena.

2. ISOTROPIC MODEL OF METAL

Let us consider a metal with a dispersion law $\epsilon = p^2/2m_1$, an electron density n_e , and an impurity density N_i . If the magnetic field is directed along the z axis, then the electron energy is described by the formula

$$\epsilon_{n\pm}(p_z) = \frac{eH}{m_1c} \left(n + \frac{1}{2} \right) \mp \frac{eH}{2mc} + \frac{p_z^2}{2m_1} \quad (1)$$

(we put here $\hbar = 1$). For each value of n and for each index "plus" or "minus" there corresponds a separate Fermi momentum $p_{0n\pm}$, determined from the conditions

$$p_{0n\pm} = \left[\left(\zeta - \frac{eH}{m_1c} \left(n + \frac{1}{2} \right) \pm \frac{eH}{2mc} \right) 2m_1 \right]^{1/2}, \quad (2)$$

$$n_e = \sum p_{0n\pm} \frac{eH}{\gamma_{\sigma^2} \sigma^2}, \quad (3)$$

where ζ is the chemical potential. It is defined by condition (3), in which account is taken of the degeneracy of the Landau levels. The sum is over all the real $p_{0n\pm}$.

If we assume the interaction of the electrons with the impurity to be independent of the electron spin, then the projection of the electron spin does not change upon scattering, and we can consider the corresponding currents produced by electrons with spin projections $\pm 1/2$ independently. We therefore consider, say, electrons with spin projection $+1/2$. If a gauge $A_y = Hx$ is chosen, the eigenfunctions take the form

$$\Psi_{p_x, p_y, n} = \exp\{ip_x z + ip_y y\} \psi_n \left(x - \frac{cp_y}{eH} \right), \quad (4)$$

where

$$\psi_n(x) = \left(\frac{eH}{\pi c} \right)^{1/4} \frac{1}{2^{n/2} \sqrt{n!}} \exp\left\{ -\frac{eH}{2c} x^2 \right\} H_n \left(\sqrt{\frac{eH}{c}} x \right)$$

(H_n is a Hermite polynomial).

With the aid of the functions $\psi_{p_z p_y n}$ we can easily write down the Green's function of the free electrons (without interaction with the impurity) at $T = 0$

$$G^{(0)}(p_z, p_y, x, x', \omega) = \sum_n \frac{\psi_n(x - cp_y/eH)\psi_n(x' - cp_y/eH)}{\omega - \xi_n(p_z) + i\delta \text{sign } \omega}, \quad (5)$$

where

$$\xi_n(p_z) = \varepsilon_{n+}(p_z) - \zeta.$$

We now consider electron scattering, assuming the interaction to be pointlike and the scattering amplitude to be isotropic (U_0).¹⁾ Averaging over the random distribution of the impurities, we have in first order in U_0

$$\begin{aligned} \delta^{(1)}G &= \int G^{(0)}(x, x_1) G^{(0)}(x_1, x') dx_1 U_0 N_i \\ &= U_0 N_i \sum_n \frac{\psi_n(x - cp_y/eH)\psi_n(x' - cp_y/eH)}{[\omega - \xi_n(p_z) + i\delta \text{sign } \omega]^2} \end{aligned}$$

(we use here the fact that the ψ_n are orthonormal). This correction corresponds to diagram a of Fig. 1.

We now consider the next-order correction, diagram b of Fig. 1, and assume that both crosses pertain to an atom. We then obtain after averaging over the impurity positions

$$\begin{aligned} \delta^{(2)}G &= N_i U_0^2 \int G^{(0)}(p_z, p_y, x, x_1) G^{(0)}(p_{z1}, p_{y1}, x_1, x_1) \\ &\cdot G^{(0)}(p_z, p_y, x_1, x') \frac{dp_{y1}}{2\pi} \frac{dp_{z1}}{2\pi} dx_1. \end{aligned}$$

Since the ψ_n in the second G -function contain identical arguments, we obtain the factor $eH/2\pi c$ by integrating over p_{y1} and using the normalization condition. We are then left with the expression

$$\sum_n [\omega - \xi_n(p_{z1}) + i\delta \text{sign } \omega]^{-1}.$$

Integration with respect to p_{z1} under the condition $\omega \ll p_{n0}^2/2m_1$ yields $-im_1 \text{sign } \omega \sum_n p_{0n}^{-1}$. Then, integrating with respect to x_1 , we obtain

$$\begin{aligned} \delta^{(2)}G &= -i \text{sign } \omega N_i U_0^2 \frac{eHm_1}{2\pi c} \sum_n \frac{1}{p_{0n}} \\ &\times \sum_n \frac{\psi_n(x - cp_y/eH)\psi_n(x' - cp_y/eH)}{[\omega - \xi_n(p_z) + i\delta \text{sign } \omega]^2} \end{aligned}$$

Thus, the correction $\delta^{(2)}G$, relative to $\delta^{(1)}G$, is of the order of $U_0 eHm_1/cp_0$. If we recognize that $U_0 \sim (p_0^2/m) \times (1/p_0^3)$ (U_0 is the Born scattering amplitude with dimension $\text{erg}\cdot\text{cm}^3$), then the ratio $\delta^{(2)}G/\delta^{(1)}G$ is of the order of $(eH/mc)(p_0^2/m)$. We consider the case when this ratio is of the order of or larger than unity. Therefore all the corrections of this kind should be taken into account.

The sum of all the diagrams of type c in Fig. 1 yields

$$\delta G = \sum_n \frac{\psi_n(x - cp_y/eH)\psi_n(x' - cp_y/eH)}{[\omega - \xi_n(p_z) + i\delta \text{sign } \omega]^3},$$

where

$$\Sigma = N_i U_0 \left[1 + i \text{sign } \omega U_0 \frac{eHm_1}{2\pi c} \sum_n \frac{1}{p_{0n}} \right]^{-1}. \quad (6)$$

¹⁾Such a description of the interaction of the electrons with the impurity in a superstrong magnetic field is in fact far from obvious, and should be regarded as a very simple model assumption.

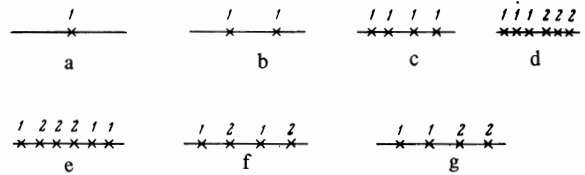


FIG. 1

If we now take into account the diagrams of type d in Fig. 1, then it is easy to see that we obtain as the sum

$$G = \sum_n \frac{\psi_n(x - cp_y/eH)\psi_n(x' - cp_y/eH)}{\omega - \xi_n(p_z) - \Sigma}. \quad (7)$$

We can choose for Σ only its imaginary part, since the real part is a renormalization of the chemical potential. If $\text{Im } \Sigma \ll p_0^2/2m$, then the integral with respect to p_z of the denominator, including Σ , does not differ from the case when we had $i\delta \text{sign } \omega$ in lieu of Σ . In view of this, expression (6) actually takes into account also diagrams of type E in Fig. 1, where all the crosses from atom 2 fall between the two nearest crosses of atom 1.

We have not yet taken into account the diagrams with meshing of the crosses from different atoms. Let us consider the simplest of these diagrams, f of Fig. 1. It contains five Green's functions and integration with respect to two coordinates x , two coordinates p_y , and two coordinates p_z . Since all the $\psi_n(x - cp_y/eH)$ are exponentially decreasing functions, it follows that, as a result of the normalization condition, the integration with respect to x_1 and x_2 annihilates four of these functions, and the integration with respect to p_{y1} and p_{y2} annihilates four more, with a factor $(eH/c)^2$ appearing as a result. We are left with the integrals of the denominators with respect to p_z . These integrals can be readily evaluated. As a result we find that this diagram is in order $\omega/(p_0^2/m)$ relative to diagram g of Fig. 1 with non-meshing crosses. Since we shall henceforth be interested throughout in ω of the order of τ^{-1} , where $1/\tau = -2\text{Im } \Sigma$, and Σ has been defined earlier, we can neglect diagrams with meshing if $\tau p_0^2/m \gg 1$. This will serve as a condition for the applicability of the present results (with respect to limitation on the impurity density).

We now proceed to calculate the conductivity tensor. We describe the external electric field by a vector potential $\mathbf{A}_1(\mathbf{r}, t)$, and we denote the vector potential of the large static magnetic field by $\mathbf{A}_0(\mathbf{r})$. The current operator is of the form

$$\tilde{\mathbf{j}} = -\frac{ie}{2m_1} (\nabla_{\mathbf{r}} - \nabla_{\mathbf{r}'}) \bar{\psi}^+(r', t) \bar{\psi}(\mathbf{r}, t) - \frac{e^2}{m_1 c} \bar{\psi}^+ \bar{\psi} (\mathbf{A}_0 + \mathbf{A}_1). \quad (8)$$

This expression must be averaged and, since there is no current under equilibrium, it is necessary to separate the terms linear in \mathbf{A}_1 . We shall use the formula

$$\delta \mathcal{H}_A = -\frac{1}{c} \int \mathbf{j} \delta \mathbf{A} dV.$$

In this case, the role of $\delta \mathbf{A}$ is played by \mathbf{A}_1 . Expressing

$$\tilde{\mathbf{j}} = \exp\left(i \int_{-\infty}^t \delta \mathcal{H}_A dt\right) \hat{\mathbf{j}} \exp\left(-i \int_{-\infty}^t \delta \mathcal{H}_A dt\right)$$

and separating terms linear in \mathbf{A}_1 , we obtain

$$j_i(\mathbf{r}, t) = \frac{ie^2}{m_1^2 c} \int dt_1 dr_1 \left[\frac{i}{2} \left(\frac{\partial}{\partial x_i} - \frac{\partial}{\partial x_i'} \right)_{r' \rightarrow r} + \frac{e}{c} A_0^i(\mathbf{r}) \right] \times \left[\frac{i}{2} \left(\frac{\partial}{\partial x_{1h}} - \frac{\partial}{\partial x_{1h}'} \right)_{r_1 \rightarrow r_1'} + \frac{e}{c} A_0^h(\mathbf{r}_1) \right] \langle [\psi^+(\mathbf{r}'t)\psi(\mathbf{r}t), \psi^+(\mathbf{r}'_1 t_1)\psi(\mathbf{r}_1 t_1)] \rangle \times \Theta(t - t_1) A_1^h(\mathbf{r}_1 t_1) - \frac{e^2}{m_1 c} A_1^i n_e \quad (9)$$

(where $\Theta(x) = 1$ when $x > 0$ and $\Theta(x) = 0$ when $x < 0$). As is well known, the mean value of a retarded commutator can be expressed in terms of the time-ordered mean value, by simply reversing the sign of the imaginary part of the Fourier component with respect to $t - t_1$ when $\omega < 0$. In view of this we define in lieu of (9) the j_i^t with time-ordered mean value. It is easy to go over in it to Green's function, and in place of $\langle \dots \rangle$ in (9) we obtain $G(\mathbf{r}, \mathbf{r}'_1, t - t_1)G(\mathbf{r}, \mathbf{r}'_1, t_1 - t)$. As usual, this product must be averaged over the positions of the impurity centers.

We begin with the component σ_{ZZ} . We consider first diagrams in which different impurity atoms enter in both G-functions. Then the mean value of the product is equal to the product of the mean values, and changing over to Fourier components we obtain

$$j_{0z}^t(k_y, k_z, x, \omega_0) = \frac{ie^2}{m_1^2 c} \int p_z^2 G(p_y - \frac{k_y}{2}, p_z - \frac{k_z}{2}, x, x_1, \omega) \times G(p_y + \frac{k_y}{2}, p_z + \frac{k_z}{2}, x_1, x, \omega + \omega_0) dx_1 dp_y dp_z d\omega / (2\pi)^3 \times A_1^z(k_y, k_z, x_1, \omega_0) - \frac{n_e e^2}{m_1 c} A_1^z(k_y, k_z, x, \omega_0). \quad (10)$$

This corresponds to a diagram a of Fig. 2. This is the general expression, and we are interested in the static tensor σ_{ik} . We therefore assume the potential A_1 to be independent of x , and put $k = 0$, assuming ω_0 to be small.

We substitute in the integral (10) the expression (7). Integration with respect to x_1 and p_y yields, as a result of the normalization of the functions ψ_n ,

$$j_{0z} = \frac{ie^2}{m_1^2 c} \frac{eH}{2\pi c} \int dp_z d\omega / (2\pi)^2 p_z^2 \left[\sum_n (\omega - \xi_n - \Sigma(\omega))^{-1} \times (\omega + \omega_0 - \xi_n + \Sigma(\omega + \omega_0))^{-1} \right] A_1^z(\omega_0) - \frac{n_e e^2}{m_1 c} A_1^z(\omega_0). \quad (11)$$

As already mentioned, it is sufficient to take into account only the imaginary part of Σ . Putting $\text{Im } \Sigma = -(i/2\tau) \text{ sign } \omega$, and assuming that $\omega_0 > 0$, we write the integral with respect to ω in the form

$$\int_{-\infty}^{-\omega_0} \left[(\omega - \xi_n - \frac{i}{2\tau}) (\omega + \omega_0 - \xi_n - \frac{i}{2\tau}) \right]^{-1} d\omega + \int_0^{\infty} \left[(\omega - \xi_n + \frac{i}{2\tau}) (\omega + \omega_0 - \xi_n + \frac{i}{2\tau}) \right]^{-1} d\omega + \int_{-\omega_0}^0 \left[(\omega - \xi_n + \frac{i}{2\tau}) (\omega + \omega_0 - \xi_n + \frac{i}{2\tau}) \right]^{-1} d\omega.$$

This integral is equal to

$$\frac{i/\tau}{\omega_0(\omega_0 + i/\tau)} \ln \frac{(\xi_n + \omega_0 + i/2\tau)(\xi_n - \omega_0 - i/2\tau)}{(\xi_n + i/2\tau)(\xi_n - i/2\tau)}.$$

We then integrate with respect to p_z . In view of the fact that only the vicinity of $p_z = \pm p_{n0}$ is important, we

can replace p_z^2 by p_{n0}^2 and integrate with respect to ξ , with

$$dp_z \rightarrow \frac{2m_1}{p_{n0}} \int d\xi$$

(the factor 2 is due to the two possible signs of p_0). We then obtain (we omit the symbol t, since j^t coincides with j when $\omega_0 > 0$)

$$j_{0z} = \frac{e^2}{m_1 c} \left[\frac{eH}{2\pi^2 c} \sum_n p_{0n} \frac{i/\tau}{\omega_0 + i/\tau} - n_e \right] A_z(\omega_0).$$

To determine the static conductivity, it is sufficient to put $\omega_0 \ll 1/\tau$. Then the zeroth term in ω_0 vanishes by virtue of (3). Since $E_k = i\omega_0 A_1^k/c$, we obtain from the first approximation, adding the currents from the electrons with spin projections $\pm 1/2$,

$$\sigma_{zz} = (n_+ \tau_+ + n_- \tau_-) e^2 / m_1, \quad (12)$$

where

$$\tau_{\pm} = -(2 \text{Im } \Sigma)^{-1} = \left\{ 1 + \left(U_0 \frac{eHm_1}{2\pi c} \sum \frac{1}{p_{0n}} \right)^2 \right\} / N_i U_0^2 \frac{eHm_1}{\pi c} \sum \frac{1}{p_{0n}} \quad (13)$$

(the summation for τ_+ and τ_- is over the corresponding p_{0n} , see (2)).

We have omitted here the zero index of σ_{ZZ} , since we shall show presently that other diagrams make no appreciable contribution. Indeed, let us consider the simplest of the unaccounted for diagrams. It is shown in Fig. 2b. Integrations with respect to p_{Z1} and p_{Z2} are independent. Since the Green's functions are even in p_z , and the vertices are odd, the diagram b of Fig. 2 vanishes. For the same reason, all the diagrams with parallel dashed lines for σ_{ZZ} , and in general all diagrams for the conductivity tensor components σ_{ZX} and σ_{ZY} vanish (in one vertex p_z , and in the other p_y or $\partial/\partial x$). To be sure, this argument cannot be used to eliminate diagrams for σ_{ZZ} with meshing lines. The simplest diagrams of this type are shown in Figs. 2c, d, and e. Although each of them is of the same order as that of Fig. 2a, the main term cancels out in the sum, and the result is of order $m/p_0^2 \tau$. We do not present here the corresponding derivations. They are not complicated but quite long.

We now consider the component σ_{yy} . For the diagram of type a on Fig. 2 we obtain

$$j_{0y} = \frac{ie^2}{m_1^2 c} \int \left(p_y - \frac{eH}{c} x \right) \left(p_y - \frac{eH}{c} x \right) \times \sum_n \frac{\psi_n(x - cp_y/eH) \psi_n(x_1 - cp_y/eH)}{\omega - \xi_n + (i/2\tau) \text{sign } \omega} \times \sum_{n_1} \frac{\psi_{n_1}(x_1 - cp_y/eH) \psi_{n_1}(x - cp_y/eH)}{\omega + \omega_0 - \xi_{n_1} + (i/2\tau) \text{sign}(\omega + \omega_0)} \times A_1^y(\omega_0) dx_1 dp_y dp_z d\omega / (2\pi)^3 - \frac{e^2 n_e}{m_1 c} A_1^y(\omega_0).$$

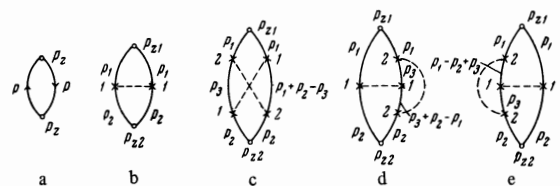


FIG. 2

The integral with respect to x_1 is in fact the matrix element of the oscillator coordinate. After obtaining this integral, a similar role is played by the integral with respect to p_y . In this case

$$x_{n, n+1} = x_{n+1, n} = \sqrt{(n+1)c/eH}.$$

We then obtain

$$\begin{aligned} j_{0y} = & \frac{ie^2}{m_1^2 c} \frac{1}{4\pi} \left(\frac{eH}{c} \right)^2 \int \frac{dp_z d\omega}{2\pi} \sum_n (n+1) \left\{ \left[\left(\omega - \xi_{n+1} + \frac{i}{2\tau} \text{sign } \omega \right) \right. \right. \\ & \times \left. \left(\omega + \omega_0 - \xi_n + \frac{i}{2\tau} \text{sign}(\omega + \omega_0) \right) \right]^{-1} \\ & + \left. \left[\left(\omega - \xi_n + \frac{i}{2\tau} \text{sign } \omega \right) \left(\omega + \omega_0 - \xi_{n+1} + \frac{i}{2\tau} \text{sign}(\omega + \omega_0) \right) \right]^{-1} \right\} \\ & \times A_1^y(\omega_0) - \frac{e^2 n_e}{m_1 c} A_1^y(\omega_0). \end{aligned} \quad (14)$$

This integral is easiest to calculate by subtracting and adding an analogous expression with $\tau \rightarrow \infty$. In the "free" term we can replace all the sign ω by sign ξ , where ξ corresponds to the given denominator. Then, when integrating with respect to ω , we should have $\xi_{n+1} > 0$ and $\xi_n < 0$ (the opposite case is impossible). We then obtain for the free term

$$\begin{aligned} & \frac{e^2}{m_1^2 c} \frac{1}{4\pi} \left(\frac{eH}{c} \right)^2 \sum_n (n+1) \int_{\xi_{n+1} > 0, \xi_n < 0} \frac{dp_z}{2\pi} \\ & \times \left(\frac{1}{\omega_0 + \xi_{n+1} - \xi_n} - \frac{1}{\omega_0 - \xi_{n+1} + \xi_n} \right) = \frac{e^2}{m_1^2 c} \frac{1}{2\pi^2} \left(\frac{eH}{c} \right)^2 \\ & \times \frac{eH/m_1 c}{(eH/m_1 c)^2 - \omega_0^2} \sum_n (n+1) (p_{0n} - p_{0n+1}) A_1^y(\omega_0) \\ & \approx \frac{e^2}{m_1 c} \frac{eH}{2\pi^2 c} \sum_n p_{0n} A_1^y(\omega_0) = \frac{e^2 n_e}{m_1 c} A_1^y(\omega_0). \end{aligned}$$

(We have neglected additions of order $\omega_0^2/(eH/m_1 c)^2$). Thus, this term cancels the last term in the current.

It remains to calculate the difference between the integral with τ and with the free term. Since the integral converges, we shall first integrate with respect to p_z . Assuming $\omega_0 \ll 1/\tau \ll eH/m_1 c$, we obtain

$$\sigma_{yy} = \frac{e^2 m_1}{2\pi^2} \left(\frac{1}{\tau_+} \sum_n \frac{n+1/2}{p_{0n+}} + \frac{1}{\tau_-} \sum_n \frac{n+1/2}{p_{0n-}} \right). \quad (15)$$

We have again omitted the index zero, since the next diagrams make no contribution. Indeed, let us consider the diagram of type b on Fig. 2. In each vertex there is contained in lieu of p_z an operator having matrix elements with $n' = n \pm 1$. Integrating with respect to the x corresponding to the vertex, we obtain the product $\psi_n(x_1 - cp_y/eH) \psi_{n\pm 1}(x_1 - cp_y/eH)$, which then vanishes upon integration with respect to p_y . But it is necessary to be cautious with the next diagrams, for according to (15) the main expression has been decreased compared with σ_{zz} by a factor $1/(\Omega\tau)^2$, where $\Omega = eH/m_1 c$. Calculation of the diagrams of types c, d, and e of Fig. 2 show that terms of the required order vanish as a result of integration with respect to p_y . We succeeded in proving this by direct calculation only in the limiting case of a very strong field, when all the levels with $n = 0$ and 1 take part. However, in all probability, this is valid also in the general case.

The calculation of σ_{xx} and σ_{yy} is analogous. We use here the matrix elements

$$\left(\frac{\partial}{\partial x} \right)_{n, n+1} = - \left(\frac{\partial}{\partial x} \right)_{n+1, n} = \sqrt{\frac{(n+1)eH}{2c}}.$$

From the symmetry of the problem it is obvious that $\sigma_{xx} = \sigma_{yy}$. To calculate σ_{xy} we determine j_{0x} under the influence of the potential A_1^y . We then obtain in lieu of (14)

$$\begin{aligned} j_{x0} = & - \frac{e^2}{m_1^2 c} \frac{1}{4\pi} \left(\frac{eH}{c} \right)^2 \int \frac{dp_z d\omega}{2\pi} \sum_n (1+n) \\ & \times \{ [(\omega - \xi_n + (i/2\tau) \text{sign } \omega) (\omega + \omega_0 - \xi_{n+1} + (i/2\tau) \text{sign}(\omega + \omega_0))]^{-1} \\ & - [(\omega - \xi_{n+1} + (i/2\tau) \text{sign } \omega) (\omega + \omega_0 - \xi_n \\ & + (i/2\tau) \text{sign}(\omega + \omega_0))]^{-1} \} A_1^y(\omega_0). \end{aligned} \quad (16)$$

In this case the answer does not depend on τ (since the terms of zeroth order in $1/\tau$ do not cancel out). This means that we can put $1/\tau \rightarrow 0$ and replace sign ω by sign ξ of the corresponding ξ in the denominator. The subsequent integration is then analogous to the preceding one, and we obtain

$$\sigma_{xy} = n_e e c / H, \quad (17)$$

i.e., the usual Hall conductivity. Obviously, $\sigma_{yx} = -\sigma_{xy}$.

We now analyze the derived formulas. We start with the limiting quantum case $eH/mc \gg p_0^2/m$, when all the electrons are concentrated on one Landau level. In this case

$$p_0 = \frac{2\pi^2 c}{eH} n_e, \quad (18)$$

with $n_e \ll (eH/c)^{3/2}$. Since $n \sim p_{00}^3$, where p_{00} is the Fermi momentum in the absence of a field, it follows that $p_0 \ll p_{00}$. If we recognize that the interaction with the impurity atom is of the order of $p_{00}^2/2m$, we get $U_0 \sim p_{00}^2/mn$. It is then easily seen that

$$U_0 \frac{eH m_1}{c p_0} \sim \frac{p_{00}^2}{p_0^2} \gg 1.$$

According to (13), we obtain

$$\tau = \frac{eH m_1}{4\pi c p_0 N_i} = \frac{1}{(2\pi)^3} \left(\frac{eH}{c} \right)^2 \frac{m_1}{n N_i}, \quad (19)$$

i.e., τ depends only on the impurity density, but not on the concrete form of the interaction. The condition $1/\tau \ll p_0^2/m$ denotes in this case that $N_i \ll n$, i.e., a low impurity density. From (12) and (15), we get

$$\sigma_{zz} = \frac{1}{(2\pi)^3} \left(\frac{e^2 H}{c} \right)^2 \frac{1}{N_i}, \quad (20)$$

$$\sigma_{xx} = \sigma_{yy} = \frac{e c N_i}{\pi H}. \quad (21)$$

The qualitative form of these formulas has a natural physical interpretation. Since each electron moves on a helix with axis along z , the impurities interfere with the conductivity along this axis, and conversely contributes to conductivity in the perpendicular direction. The decrease of the radius of the helix with increasing field decreases also the collision probability, thus leading to an increase of σ_{zz} and to a decrease of σ_{xx} and σ_{yy} . From (17), (20), and (21) we obtain the resistivity ten-

or $\rho_{ik} = \sigma_{ik}^{-1}$:

$$\rho_{zz} = (2\pi)^3 N_i \left(\frac{c}{e^2 H} \right)^2, \\ \rho_{xx} = \rho_{yy} = \frac{N_i H}{\pi n^2 e c}, \quad \rho_{xy} = \frac{H}{n e c}. \quad (22)$$

We now consider the opposite limit $eH/mc \ll p_0^2/2m$. This is the quasiclassical case, in which it is possible to change from summation with respect to n to integration. The small addition gives the so-called "quantum oscillations."

Using the Poisson summation formula

$$\sum_0^\infty \varphi(n) = \int_{-1/2}^\infty \varphi(x) dx + 2 \operatorname{Re} \sum_{h=1}^\infty \int_{-1/2}^\infty \varphi(x) e^{2\pi i h x} dx,$$

we obtain

$$\sum_n \frac{1}{p_{0n\pm}} = \frac{1}{\sqrt{2m_1}} \int_0^{\xi_{\pm}} \frac{dx}{\sqrt{\xi_{\pm} - \Omega x}} + \frac{2}{\sqrt{2m_1}} \operatorname{Re} \sum_{h=1}^\infty \int_0^{\xi_{\pm}} \frac{e^{2\pi i h x} dx}{\sqrt{\xi_{\pm} - \Omega x}}$$

where

$$\xi_{\pm} = \zeta_{\pm} \pm eH/2mc, \quad \Omega = eH/m_1 c.$$

Changing over to the variable $y = \xi_{\pm} - \Omega x$ and continuing in the second integral the limit of integration with respect to y to ∞ , we obtain

$$\sum_n \frac{1}{p_{0n\pm}} \approx \frac{2}{\Omega} \sqrt{\frac{\xi_{\pm}}{2m_1}} + \frac{1}{\sqrt{m_1} \Omega} \sum_h \frac{1}{\sqrt{h}} \cos\left(2\pi k \frac{\xi_{\pm}}{\Omega} - \frac{\pi}{4}\right).$$

In the first term we can put $\xi_{\pm} \approx p_0^2/2m_1$. This cannot be done in the second term, in view of the fact that this term oscillates.²⁾ After substitution, we obtain

$$\sum_n \frac{1}{p_{0n\pm}} = \frac{c p_{00}}{eH} \left[1 + \sqrt{\frac{eH}{c p_{00}^2}} \sum_h \frac{1}{\sqrt{h}} \cos\left(2\pi k \frac{\xi_{\pm}}{\Omega} - \frac{\pi}{4}\right) \right]. \quad (23)$$

The second term is of relative order $(\Omega/\xi)^{1/2}$.

Therefore, upon substitution in (12) for τ we can expand with respect to this term. It is convenient here to go over from the Born amplitude to the true amplitude in the absence of the field. Considering Fig. 1c for an individual electron interacting with an impurity, we obtain

$$A = \frac{U_0}{1 + iU_0 p_{00} m_1 / 2\pi}.$$

Expressing U_0 in terms of A_1 we get

$$\tau_{\pm} = (N_i |A|^2 p_{00} m / \pi)^{-1} \left\{ 1 + [2|A|^2 (p_{00} m / 2\pi)^2 - 1] \sqrt{\frac{eH}{c p_{00}^2}} \sum_h \frac{1}{\sqrt{h}} \cos\left(\frac{2\pi k \xi_{\pm}}{\Omega} - \frac{\pi}{4}\right) \right\}. \quad (24)$$

When this expression is substituted in (12), we can assume that $n_+ \approx n_-$. After this, addition of terms with indices \pm denotes

$$\cos(2\pi k \xi_+ / \Omega - \pi/4) + \cos(2\pi k \xi_- / \Omega - \pi/4) \\ \approx \cos(2\pi k \xi / \Omega - \pi/4) \cos(\pi k m / m_1).$$

²⁾Strictly speaking, ξ also has an oscillating addition. It can be obtained with the aid of formulas (2) and (3). However, when we sum Σp_{0n} in analogy with the summation in (23), we find that the relative order of the addition is $(\Omega/\xi)^{3/2}$, whereas other sums make additions of order $\sqrt{\Omega/\xi}$ to the conductivity. For this reason, we can assume that $n_+ = n_-$.

Thus, as a result we get

$$\sigma_{zz} = \sigma_0 \left\{ 1 + [2|A|^2 (p_{00} m_1 / 2\pi)^2 - 1] \sqrt{\frac{eH}{c p_{00}^2}} \times \sum_h \frac{1}{\sqrt{h}} \cos\left(\frac{\pi k c p_{00}^2}{eH} - \frac{\pi}{4}\right) \cos\left(\pi k \frac{m}{m_1}\right) \right\}, \quad (25)$$

where

$$\sigma_0 = n e^2 \tau_0 / m_1, \quad 1/\tau_0 = N_i |A|^2 p_{00} m_1 / \pi.$$

In analogy with (23), we get

$$\sum_n \frac{n + 1/2}{p_{0n\pm}} \approx \frac{p_{00}^2 c^2}{3(eH)^2} \left[1 + \frac{3}{2} \sqrt{\frac{eH}{c p_{00}^2}} \sum_h \frac{1}{\sqrt{h}} \cos\left(2\pi k \frac{\xi_{\pm}}{\Omega} - \frac{\pi}{4}\right) \right]. \quad (26)$$

Substituting in (15), we get

$$\sigma_{yy} = \sigma_{xx} = \frac{n e^2 m_1}{\tau_0} \left(\frac{c}{eH} \right)^2 \left\{ 1 + \left[\frac{5}{2} - 2|A|^2 \left(\frac{p_{00} m_1}{2\pi} \right)^2 \right] \sqrt{\frac{eH}{c p_{00}^2}} \times \sum_h \frac{1}{\sqrt{h}} \cos\left(\frac{\pi k c p_{00}^2}{eH} - \frac{\pi}{4}\right) \cos\left(\pi k \frac{m}{m_1}\right) \right\}. \quad (27)$$

We recall, incidentally, that we have assumed throughout that $\Omega \gg 1/\tau_0$.

Finally, let us consider the case $\Omega \sim \xi$ and ascertain what occurs when the chemical potential passes through one of the Landau levels, say n_0 . We assume that this is a level with spin orientation along the field. We then can easily obtain from (23) the corresponding value of the field

$$\left(\frac{eH n_0}{c} \right)^{3/2} = \sqrt{2} \pi^2 n_e \left[\sum_{n=1}^{n_0} \sqrt{n} + \sum_n \sqrt{n_0 - n - \frac{m_1}{m}} \right]^{-1}.$$

Let us examine τ in the vicinity of this point. So long as the field is somewhat larger than H_{n_0} , according to (13), τ behaves like a smooth function of the field and is of the order of

$$\tau \sim \frac{eH n_0 m_1}{c N_i p_{0n}} \sim \sqrt{\frac{eH n_0}{c}} \frac{m_1}{N_i} \sim n_e^{1/2} \frac{m_1}{N_i}.$$

Accordingly, the components of the conductivity tensor are of the order of

$$\sigma_{zz}(H_{n_0} + 0) \sim e^2 n_e^{1/2} \frac{n_e}{N_i},$$

$$\sigma_{xx} = \sigma_{yy}(H_{n_0} + 0) \sim e^2 n_e^{1/2} \frac{N_i}{n_e}. \quad (28)$$

However, if we approach this point from below, then the momentum p_{n_0} becomes small near H_{n_0} , and this immediately affects the conductivity. From (3) we have

$$p_{0n_0} = 2\pi^2 n_e \frac{c}{H_{n_0} e} \frac{H_{n_0} - H}{H_{n_0}}.$$

This yields directly

$$\tau \sim n_e^{1/2} \frac{m_1}{N_i} \frac{H_{n_0}}{H_{n_0} - H}, \\ \sigma_{zz}(H \rightarrow H_{n_0} - 0) \sim e^2 n_e^{1/2} \frac{n_e}{N_i} \frac{H_{n_0}}{H_{n_0} - H}, \\ \sigma_{xx} = \sigma_{yy}(H \rightarrow H_{n_0} - 0) \sim e^2 n_e^{1/2} \frac{N_i}{n_e}. \quad (29)$$

Thus, the component σ_{zz} increases when H approaches H_{n_0} from below. As to σ_{xx} and σ_{yy} , they are of the same order as when $H = H_{n_0} + 0$, but it can be shown that the coefficient will be different in this case. Consequently, these components experience a discontinuity.

Actually these singularities become smoothed out somewhat. The point is that the condition for the applicability of the formulas obtained above is, in particular, $p_{0n_0}^2/m_1 \gg 1/\tau$. This imposes the limitation $(H_{n_0} - H)/H_{n_0} \gg N_i/n_e$. Thus, the singularity of σ_{ik} should be smoothed out over this region. In particular, it follows therefore that the maximum reached by σ_{ZZ} is of the order of

$$\sigma_{ZZ \max} \sim e^2 n_e^{1/2} (n_e/N_i)^2. \quad (30)$$

2. MODEL OF A SEMIMETAL

We now consider a model closer to the real situation. We assume that near $\mathbf{p} = 0$ we have isotropic holes with small effective mass, and near $\mathbf{p} = \mathbf{P}$ we have "electrons" that are isotropic with respect to $\mathbf{p} - \mathbf{P}$ and also have a small mass (Fig. 3a). Then, in principle, three different cases are possible.

1. The valence and the conduction bands intersect at $H = 0$, and this intersection remains for all fields (Fig. 3b).

2. The bands do not intersect at $H = 0$, but the intersection occurs at some field value (Fig. 3c).

The fields have intersected at $H = 0$, but the intersection was lifted under the influence of the field (Fig. 3d).

We have considered only the limits of the strongest fields, when only one of the Landau levels is filled. The energies of the electrons and holes are given by

$$\begin{aligned} \varepsilon_e(H, p_z) &= \varepsilon_e + \frac{eH}{2m_{e1}c} - \frac{eH}{2m_{e2}c} + \frac{(\mathbf{p} - \mathbf{P})_z^2}{2m_{e1}}, \\ \varepsilon_h(H, p_z) &= \varepsilon_h - \frac{eH}{2m_{h1}c} + \frac{eH}{2m_{h2}c} - \frac{p_z^2}{2m_{h1}}. \end{aligned} \quad (31)$$

The effective masses m_e and m_h for the spin splitting are also assumed to be small, of the order of m_{e1} and m_{h1} . This corresponds to the real situation in semimetals where, as a result of the spin-orbit interaction, the spin splitting of the levels is of the order of the orbital splitting. We put

$$\Delta = \varepsilon_h - \varepsilon_e, \quad \frac{1}{m^*} = \frac{1}{m_h} - \frac{1}{m_{h1}} + \frac{1}{m_e} - \frac{1}{m_{e1}}. \quad (32)$$

Then, obviously, the aforementioned cases correspond to:

- 1) $\Delta > 0, m^* > 0$,
- 2) $\Delta < 0, m^* > 0$,
- 3) $\Delta > 0, m^* < 0$.

The number of electrons (which equals the number of holes) we have

$$n_h = n_e = \frac{eH}{2\pi^2 c} p_0, \quad (33)$$

where p_0 is determined from the condition that the energies of the electrons and holes be equal to the chemical potential, i.e.,

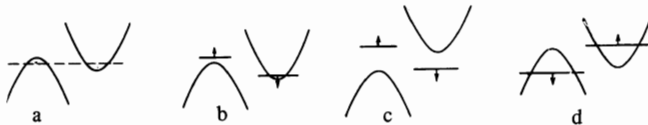


FIG. 3

$$p_0 = \sqrt{2m^{**} \left(\Delta + \frac{eH}{2m^*c} \right)}, \quad (34)$$

where $(m^{**})^{-1} = m_{h1}^{-1} + m_{e1}^{-1}$. With the aid of (34) we easily obtain the conditions under which only the lower level takes part in the spectrum. To this end, obviously, the following inequality should be satisfied:

$$\frac{cp_0^2}{2eH} < 1, \quad \frac{m_{e1}}{m_e}, \quad \frac{m_{h1}}{m_h}.$$

We shall not write out explicitly the relations obtained in this case for the masses and the field H . It is important that these conditions can exist.

We note first that the possible existence of a Fermi spectrum in a semimetal is not obvious in the extreme quantum case. The motion becomes one-dimensional. As is well known, in the one-dimensional case the attraction of the particles must lead to the formation of bound states.³⁾

The situation becomes more complicated by the fact that this question is solved differently for two isolated particles and for two Fermi distributions of the electrons and the holes. It is more or less obvious that the transition from one of these variants to the other occurs when the Fermi energy becomes comparable with the binding energy of the pair (exciton). The latter can be easily estimated. It turns out to be of the order of $m_1 e^4$ if $e^2 m_1 / \sqrt{eH/c} \lesssim 1$.

For semimetals without a magnetic field, the velocity on the Fermi surface is $p_{00}/m_1 \sim 10^8$ cm/sec, i.e., $e^2 m_1 / p_{00} \sim 1$. This means that in strong fields, where $p_0 \sim \sqrt{eH/c} \gg p_{00}$,⁴⁾ the ratio of the exciton binding energy to the Fermi energy is of the order of $(e^2 m_1 / p_0)^2 \ll 1$. It follows therefore that in this case a consideration of individual electrons and holes will not do.

We do not present the details of the estimate in which allowance is made for the Fermi distributions. It is obtained in the same manner as for superconductivity. The binding energy has in this case the form $a(p_0^2/m_1) \times \exp(-bp_0/m_1 e^2)$, where $a \sim 1$ and $b \sim 1$. This energy is much smaller than p_0^2/m_1 . Consequently, there exists an interval of temperatures much lower than the Fermi energy and much higher than the binding energy, in which temperature effects can be neglected and at the same time it is possible to disregard exciton formation. All the following will pertain precisely to this case (see also footnote 4).

Let us consider Fig. 1a. Averaging over the random arrangement of the impurities gave us in the preceding model the product $\psi_n(\mathbf{x} - c\mathbf{p}_y/eH)\psi_n(\mathbf{x}' - c\mathbf{p}_y/eH)$. This was due to the fact that the functions ψ_n are orthonormal. In the present case we may encounter a situation wherein the Green's function on one side of the cross corresponds to holes, and on the other side to electrons. In this case we have the integral

$$\begin{aligned} & \int \exp[i(p_z - p_z' + P_z)z_1 + i(p_y - p_y' + P_y)y_1 + iP_x x_1] \\ & \times \psi_n \left(x_1 - \frac{cP_y}{eH} \right) \psi_{n1} \left(x_1 - \frac{cP_y'}{eH} \right) d^3 r_1 \end{aligned}$$

³⁾The author is grateful to L. P. Gor'kov for suggesting this problem.

⁴⁾Of course, according to (34), it is necessary to have for this purpose $m^* > 0$, i.e., all this pertains only to cases 1) and 2).

$$\times [\omega - \xi_{nh}(p_z) + i\delta \text{sign } \omega]^{-1} [\omega - \xi_{ne}(p_z') + i\delta \text{sign } \omega]^{-1},$$

where

$$\xi_{nh}(p_z) = (p_0^2 - p_z^2) / 2m_{h1}, \quad \xi_{ne} = (p_z^2 - p_0^2) / 2m_{e1}.$$

Although the integral written out above does not vanish exactly, it is very small. Indeed, by definition, at least one of the nonvanishing components of the vector \mathbf{P} is of the order of $1/a$, where a is an atomic dimension. The functions $\psi_n(x - cp_y/eH)$ vanish at a distance $\sqrt{c/eH}$ along x or $\sqrt{eh/c}$ along p_y . We are considering a case in which $\sqrt{eH/c} \sim p_0 \ll P$. Consequently, if $P_y \neq 0$, then the product $\psi_n(x_1 - cp_y/eH)\psi_{n_1}(x - c(p_y - P_y)/eH)$, which appears after integration with respect to y_1 , should be small. If $P_x \neq 0$, then the smallness appears in the result of integration with respect to x_1 owing to the rapidly oscillating factor $\exp[iP_x x_1]$ in the integral. Finally, if $P_z \neq 0$, then P_z appears in one of the denominators, making it large.

Thus, the "diagonality" of G in averaging over the positions of the impurities is retained in this case, too. This, of course, does not exclude summations over either the electron or the hole states in the intervals between the crosses in the diagrams of types b and c on Fig. 1. It is easy to verify that both contributions have the same form, i.e., $m_{e1} + m_{h1}$ appears in (6) in lieu of m_1 .

We now estimate the order of magnitude of the correction to the Born approximation in our case. E_0 is of the order of $P^2/m^{(0)}P^3 \sim 1/m^{(0)}P$, where $m^{(0)}$ is the mass of the free electron and P is of the order of the reciprocal atomic distances. Thus, the relative order of the correction is

$$(eHm_1 / cp_0) / m^{(0)}P.$$

In the quantum region $p_0 \sim \sqrt{eH/c}$, and the correction is of the order of $p_0 m_1 / m^{(0)}P$. In this case p_0 is of the order of the limiting Fermi momentum without a field, or larger. But in semimetals the velocity on the Fermi boundary is of the same order as in ordinary metals, i.e., without a field we have $p_0/m_1 \sim P/m^{(0)}$ (p_0 —Fermi momentum of the semimetal without a magnetic field). It follows therefore that the correction at not too large fields is of the order of $(p_0/P)^2 \sim 10^{-3}$. This means that it can be neglected. Of course, this smallness can be overcome by increasing the magnetic field. But whereas on the limit of the ultraquantum region the field should be of the order of 10^5 Oe, in order to make the correction large the field must be increased by six orders of magnitude, which is not realistic. Thus, we can always use the value of τ^{-1} in the Born approximation, i.e.,

$$\frac{1}{\tau} = N_i U_0^2 \frac{eH}{\pi c} (m_{e1} + m_{h1}) \frac{1}{p_0}. \quad (35)$$

In general, the condition $1/\tau \ll p_0^2/m_1$ does not impose any limitations on the concentration of the impurity in the ultraquantum region.

We proceed to the conductivity tensor. It is easy to see, first, that the entire loop on Fig. 2 should pertain either to electrons or to holes, i.e., the current consists of two independent parts. For the diagonal components we obtain (only the diagrams of type a in Fig. 2 are significant)

$$\sigma_{zz} = e^2 n \tau \left(\frac{1}{m_{e1}} + \frac{1}{m_{h1}} \right), \quad (36)$$

$$\sigma_{yy} = \sigma_{xx} = \frac{e^2}{4\pi^2 p_0 \tau} (m_{e1} + m_{h1}). \quad (37)$$

The Hall component σ_{xy} in the approximation of formula (17) vanishes in this case. In the next approximation, contributions are made by diagrams of types c, d, and e in Fig. 2. We can therefore determine σ_{xy} only in order of magnitude:

$$\sigma_{xy} = -\sigma_{yx} \sim \frac{e^2 m_1^2}{p_0 \tau^2} \frac{c}{eH}. \quad (38)$$

As already mentioned, the obtained formulas, strictly speaking, are valid only for the case of strong fields $eH/m_1 c \gg \Delta$, $\sqrt{eH/c} \gg p_0$. Then, according to (33)–(38)

$$p_0 = \sqrt{\frac{m^{**} eH}{m^* c}}, \quad (39)$$

$$n_h = n_e = \frac{1}{2\pi^2} \sqrt{\frac{m^{**}}{m^*}} \left(\frac{eH}{c} \right)^{3/2}, \quad (40)$$

$$\frac{1}{\tau} = \frac{1}{\pi} N_i U_0^2 (m_{e1} + m_{h1}) \sqrt{\frac{m^*}{m^{**}}} \sqrt{\frac{eH}{c}}, \quad (41)$$

$$\sigma_{zz} = \frac{e^2 n \tau}{m^{**}} = \frac{1}{2\pi} \frac{e^2}{m^* (m_{e1} + m_{h1})} \frac{eH}{c} \frac{1}{N_i U_0^2}, \quad (42)$$

$$\sigma_{xx} = \sigma_{yy} = \frac{e^2}{4\pi^3} \frac{m^*}{m^{**}} N_i U_0^2 (m_{e1} + m_{h1})^2, \quad (43)$$

$$\sigma_{xy} = -\sigma_{yx} \sim e^2 m_1^4 N_i^2 U_0^2 \sqrt{c/eH}. \quad (44)$$

In view of the fact that in this case $\sigma_{xy} \ll \sigma_{xx}$, the diagonal components of the tensor ρ_{ik} are simply the reciprocals of the corresponding σ_{ik} . As to the component ρ_{xy} , we obtain for it

$$\rho_{xy} = -\rho_{yx} \approx \frac{1}{e^2} \sqrt{\frac{c}{eH}}. \quad (45)$$

Thus, the resistance along the field decreases with increasing H , the transverse resistance reaches saturation, and ρ_{yx} does not depend on the impurity density. The behavior of ρ_{xx} differs from the non-quantum case $p_0^2/m_1 \gg \Omega \gg 1/\tau$, where $\rho_{xx} \propto H^2$.

4. CONCLUSION

The described method can be generalized and applied to a large group of various quantum phenomena in metals.

a) **Quasiclassical limit. Quantum oscillations.** The obtained formulas can be usually generalized to the case of a finite temperature. Then the expressions of type (11), (14), and (16) can be used for an arbitrary energy spectrum. After summation over ω , there remains summation over n and p_z , which is carried out by the same method as in the paper of I. Lifshitz and A. Kosevich.^[4] Unlike the previously employed complicated and inconvenient methods for calculating the quantum oscillations in kinetic phenomena^[1, 2, 5, 6], this method has the advantage of being highly automatic and providing for the elimination of errors. Of course, in order to consider high-frequency phenomena it is

necessary to introduce further modifications, in which account is taken of the inhomogeneity of the field at the surface, as well as boundary effects.

b) Quantum limit. In addition to calculating other characteristics of the two models considered in the present paper, we can study also more concrete spectra in the quantum limits. To this end it is necessary to know the entire function $\epsilon(\mathbf{p})$, and not only the vicinity of the Fermi surface. The problem is complicated by the fact that usually, the different branches of the spectrum are close to each other for a large number of values of \mathbf{p} , and cannot be regarded separately upon quantization. In the most important case, however, that of real semimetals, the spectrum is known^[7] and in principle we know how to solve the quantum problem, although this is a complicated matter and was completed only for holes in a field parallel to the principal axis.^{[8] 5)}

Recently a new phenomenon was discovered,^[10, 11] namely, the quantization of the electron motion over jumping trajectories along the surface. In this case the lower quantum levels are important. Although this problem has its own specific features, its solution can apparently likewise be simplified and refined by using

⁵⁾The energy spectrum in very strong fields may be altered by striction effects (the author thanks M. Ya. Azbel' for pointing out this circumstance). Obviously, this occurs when the splitting of the magnetic levels becomes of the same order as the energies responsible for the structure of the spectrum. For semimetals (see [9]), this energy (γ) is of the order of several tenths of an electron volt, whereas the Fermi energy (reckoned from the bottom of the band) is of the order of several hundredths of an electron volt. Consequently, there is an interval of strong fields where the change of the parameters of the spectrum (such as the effective mass) can still be disregarded.

Green's functions expanded in terms of the eigenfunctions of the problem.

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