PHOTOMAGNETIC EFFECT IN A QUANTIZED MAGNETIC FIELD FOR ELECTRONS HEATED BY LIGHT

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It is shown that in a number of semiconductors the photomagnetic effect at helium temperatures oscillates on variation of the magnetic field, similar to the Nernst effect. A model is proposed to explain oscillations of the Gurevich-Firsov type which were observed in the photomagnetic effect in n-InSb at 4°K.

IN the investigation of the photomagnetic effect (PME) in degenerate n-InSb at helium temperatures, quantum oscillations of two types were observed: Shubnikov–de Haas (SH) and Gurevich–Firsov (GF)[1,2]. Analogous phenomena were observed in a number of other semiconductors: n-InAs, n-HgSe, n-GaSb[2,3]. The results of these experiments cannot be interpreted on the basis of the usual diffusion theory of the PME in a quantizing magnetic field[4], and to understand them it is necessary to take into account the heating of electrons by light, which is appreciable at helium temperatures[5-8].

In the present paper we consider the PME effect in a quantizing magnetic field with allowance for the heating of the electrons by light. Some of the results of this analysis were published by us earlier in the form of a brief communication[9].

1. FORMULATION OF PROBLEM

Estimates show[8] that in the case of weak magnetic fields in a degenerate semiconductor at helium temperatures, the energy relaxation time τ_{ee} of the electron in inter-electron interactions turns out to be smaller by 3–4 orders of magnitude than the energy relaxation time τ_{ac} in interactions with acoustic phonons or the electron lifetime τ_n, and the inequalities $\tau_{ee} \ll \tau_{ac}$ and $\tau_{ee} \ll \tau_n$ are satisfied with ample margin. This allows us to assume that on going over to a quantizing magnetic field the interelectron interaction remains predominant and electrons with energy $\epsilon < \hbar\omega_0 + \zeta_0$ (or $\epsilon < \hbar\omega_0$ if the magnetic field lifts the degeneracy) will have a Fermi distribution with temperature $T(x) = T_0 + T_1(x)$ and a chemical potential $\zeta(x) = \zeta_0 + \zeta_1(x)$. Here T_0 is the lattice temperature, ζ_0 is the equilibrium chemical potential in the magnetic field, and $\hbar\omega_0$ is the energy of the longitudinal optical phonon. We consider the case $\hbar\omega_0 > \zeta_0$, which corresponds to the conditions of the experiment of[1,2].

No general opinion can be advanced with respect to the distribution function at energy $\epsilon > \hbar\omega_0 + \zeta_0$, since the relation between τ_{ee} and τ_{opt}—the lifetime of emission of the optical phonon—can be arbitrary. The small corrections T_1 and ζ_1 are determined by the particle and energy balance equations. The form of the distribution in the region $\epsilon > \hbar\omega_0 + \zeta_0$ is important only for the calculation of the energy lost by the electrons as a result of emission of optical phonons. The contribution of such electrons to the macroscopic fluxes is negligibly small in view of their small number.

In a quantizing magnetic field, the macroscopic heat and particle fluxes can be represented, as in a classical field, in the form

$$\begin{aligned} \mathbf{q}_n &= -\frac{\hat{\sigma}}{e}\left(\mathbf{E} + \frac{1}{e}\nabla\zeta_1\right) - \frac{\hat{\beta}}{e}\nabla T_1, \\ \mathbf{q}_p &= \frac{\hat{\sigma}_p}{e}\left(\mathbf{E} - \frac{1}{e}\frac{T_0}{p_0}\nabla p_1\right), \\ \mathbf{Q}_n &= -\hat{\chi}\left(\mathbf{E} + \frac{1}{e}\nabla\zeta_1\right) - \hat{\varkappa}\nabla T_1. \end{aligned} \tag{1}$$

Here $\mathbf{q}_n$ and $\mathbf{Q}_n$ are the electronic particle and heat fluxes, $\mathbf{q}_p$ is the hole flux, e is the absolute value of the elementary charge, and p_0 and p_1 are the equilibrium concentration of the hole and the non-equilibrium addition to it. The quantities $\hat{\sigma}$, $\hat{\beta}$, $\hat{\chi}$, $\hat{\kappa}$ are tensor kinetic coefficients for the electrons in the quantizing magnetic field. These coefficients are experimentally measured quantities. They were obtained in[10-12] for individual scattering mechanisms. As is well known, in a quantizing magnetic field these coefficients oscillate when the magnetic field is varied. At helium temperatures only the SH oscillations appear, and there are no GF oscillations[2].

The purpose of the present work was to obtain a general expression for the photomagnetic short circuit current in terms of the coefficients $\hat{\sigma}$, $\hat{\beta}$, $\hat{\chi}$, $\hat{\kappa}$, $\hat{\sigma}_p$, without specifying the concrete form of these quantities.

2. PARTICLE AND ENERGY BALANCE EQUATIONS

Let the magnetic field be directed along the z axis, and let the light be incident along the x axis. We denote by J the density of the photon flux entering the sample, by α the light absorption coefficient (we confine ourselves to the simplest case of monochromatic light). Then the system of initial equations for the determination of T_1 and ζ_1 and the boundary conditions for them in the absence of surface recombination can be written in the form

$$\begin{aligned} \frac{\partial q_n{}^x}{\partial x} &= \alpha J e^{-\alpha x} - \left(\frac{\delta n}{\delta t}\right)_{ri}, \quad q_n{}^x = q_p{}^x, \\ \frac{\partial Q_n{}^x}{\partial x} &= \alpha J e^{-\alpha x}(\varepsilon_0 - \zeta_0) - \left(\frac{\delta \varepsilon}{\delta t}\right)_{ri} - P_{ac} - P_{opt}, \end{aligned} \tag{2}$$

$$\zeta_1 = \frac{\partial \zeta_0}{\partial T_0} T_1 + \frac{\partial \zeta_0}{\partial n_0} n_1,$$

$$q_n{}^x|_{x=0} = 0, \quad Q_n{}^x|_{x=0} = 0.$$

Here n_1 is the non-equilibrium addition to the electron concentration; $(\delta n/\delta t)_{ri}$ and $(\delta \epsilon/\delta t)_{ri}$ is the rate of change of the number of electrons and of their thermal energy per unit volume as a result of the recombination and ionization processes; P_{ac} is the power given up by the electrons to the lattice per unit volume in scattering by the deformation and the piezoelectric potentials of the acoustic oscillations; P_{opt} is the power lost by the electrons per unit volume in interactions with longitudinal optical phonons.

The value of P_{ac} in a quantizing magnetic field is obtained by us in the Appendix. It is important to note here that these losses can be represented in the form

$$P_{ac} = n_0 T_1 / \tau_{ac} \tag{3}$$

If we assume for the description of the ionization and recombination processes the Schockley-Read model, then the derivation of the expressions for $(\delta n/\delta t)_{ri}$ in $(\delta \epsilon/\delta t)_{ri}$ is carried out in exactly the same manner as for a classical magnetic field[8]. We then obtain

$$(\delta n/\delta t)_{ri} \cong n_1/\tau_n, \quad (\delta \varepsilon/\delta t)_{ri} \cong \frac{\gamma_{n1}}{\gamma_n} \frac{\zeta_0 n_1}{\tau_n},$$

$$\frac{1}{\tau_n} = \frac{\gamma_n \gamma_p M n_0}{\gamma_n n_0 + \gamma_p M}, \quad n_0 \gamma_n = \int_{n\Omega/2}^{\infty} d\varepsilon g(\varepsilon) W_r(\varepsilon) f_{00}(\varepsilon), \tag{4}$$

$$n_0 \gamma_{n1} = \int_{n\Omega/2}^{\infty} d\varepsilon g(\varepsilon) W_r(\varepsilon) \frac{\varepsilon - \zeta_0}{\zeta_0} f_{00}(\varepsilon).$$

In formulas (4) γ_n and γ_p are the coefficients of capture of the electrons and holes in the traps, M is the trap concentration, $W_r(\epsilon)$ is the electron recombination probability, $f_{00}(\epsilon)$ is the electron equilibrium distribution function, $g(\epsilon)$ is the density of states of the electron in the magnetic field:

$$g(\varepsilon) = \frac{V}{(2\pi)^2} \frac{\hbar\Omega}{2} \left(\frac{2m}{\hbar^2}\right)^{3/2} \sum_{n=0}^{n_{max}} \frac{1}{\sqrt{\varepsilon - (n+1/2)\hbar\Omega}},$$

V is the volume of the crystal, $\Omega = eH/mc$ is the Larmor frequency of the electron, and m is the effective mass of the electron. We shall henceforth be interested also in the connection between n_1 and p_1. In the Schockley-Read model, just as in[8], we obtain

$$p_1 = \gamma_n n_0 n_1 / (\gamma_n n_0 + \gamma_p M). \tag{5}$$

3. ENERGY LOSS P_{opt} DUE TO THE EMISSION OF OPTICAL OSCILLATIONS

We proceed to discuss the form of P_{opt}. If we neglect the small dispersion of the longitudinal optical oscillations, then the general expression for P_{opt} can be represented in the form

$$P_{opt} = \frac{\hbar\omega_0}{V} \sum_{\nu\nu' q} \frac{4\pi}{\hbar} W_{\nu\nu'}(q) [f_\nu (1 - f_{\nu'}) (N_q + 1) - f_{\nu'}(1 - f_\nu) N_q]. \tag{6}$$

Here

$$W_{\nu\nu'}(q) = |I_{nn'}|^2 |c_q|^2 \delta_{p_y, p_y' - q_y} \delta_{p_z, p_z' - q_z} \delta(\varepsilon_\nu - \varepsilon_{\nu'} - \hbar\omega_0) \tag{7}$$

is the probability of the transition of the electron from the state ν to the state ν' with emission of an optical phonon with momentum q, f_ν is the electron distribution function, N_q is the number of optical phonons with momentum q, and

$$I_{nn'} = \int_{-\infty}^{\infty} \exp\left\{\frac{i}{\hbar} q_x x\right\} \varphi_n(x - x_0) \varphi_{n'}(x - x_0') dx,$$

where $\varphi_n(x - x_0)$ is the oscillator wave function. For scattering by optical phonons, we have

$$|c_q|^2 = \frac{2\pi\hbar\omega_0 e^2}{q^2 V} \left(\frac{1}{k_\infty} - \frac{1}{k_0}\right),$$

where k_0 and k_∞ are the static and dynamic dielectric constants.

In formula (6), the summation over ν and ν' contains summation over the quantum numbers n and n' and integration over p_y, p_y' and p_z, p_z'. Integrating with respect to p_y and introducing the new variables

$$\varepsilon = p_z^2/2m + (n + 1/2)\hbar\Omega, \quad \varepsilon' = p_z^2/2m + (n' + 1/2)\hbar\Omega,$$

we can reduce the expression for P_{opt} to the form

$$P_{opt} = \int d\varepsilon \sum_{nn'} \frac{G^{(1)}_{nn'}(\varepsilon) f_{n\varepsilon}(1 - f_{n'\varepsilon'}) - G^{(2)}_{nn'}(\varepsilon) f_{n'\varepsilon'}(1 - f_{n\varepsilon})}{[(\varepsilon - (n + 1/2)\hbar\Omega)(\varepsilon + \hbar\omega_0 - (n' + 1/2)\hbar\Omega]^{1/2}}. \tag{8}$$

Here

$$G^{(1)}_{nn'}(\varepsilon) = \frac{\omega_0 m \Omega}{\pi\hbar^2} \sum_q |I_{nn'}|^2 |c_q|^2 N_q \sum_{\text{зн}} \delta(\pm\sqrt{2m(\varepsilon - (n + 1/2)\hbar\Omega)} \pm$$

$$\pm \sqrt{2m(\varepsilon + \hbar\omega_0 - \hbar\Omega(n' + 1/2))};$$

$G^{(2)}_{nn'}(\epsilon)$ is obtained from $G^{(1)}_{nn'}(\epsilon)$ by replacing N_q with $N_q + 1$. The symbol Σ denotes summation over different combinations of the signs in the argument of the δ function.

A feature of (8) is that at values of magnetic field intensities satisfying the condition $\omega_0 = \Omega N$, where N is an integer, both roots in the denominator can vanish simultaneously, and the integral with respect to energy diverges logarithmically (provided the numerator does not vanish in this case). As is well known[13], a divergence of this type leads to oscillations of the electric conductivity due to scattering by optical phonons, with a period $\Delta(1/H) = e/mc\,\omega_0$—GF oscillations. Thus, in the general case P_{opt} should experience GF oscillations.

The explicit form of P_{opt} can be established directly in two limiting cases: a) very strong interelectron interaction, when $f_{n\epsilon}$ in (8) can be replaced by a Fermi function with temperature $T(x) = T_0 + T_1(x)$, $T_1 \ll T_0$; b) negligibly small interelectron interaction (compared with the interaction with the optical phonons). In case a) the losses P_{opt} oscillate and are proportional to the factor $\exp(-\hbar\omega_0/T_0)$. In n-InSb at helium temperatures we have $\hbar\omega_0/T_0 \approx 70$, and therefore P_{opt} is vanishingly small and it can be assumed that there are no losses on optical phonons. In case b), the distribution function of the fast electrons ($\epsilon > \hbar\omega_0 + \zeta$) can be obtained from the kinetic equation, in which it is sufficient to take into account only the production of photoelectrons and the spontaneous emission of optical phonons. Stimulated emission and absorption can be neglected, since $N_q \ll 1$, and the distribution function is essentially non-equilibrium. The kinetic equation can in this case be written in the form

$$\sum_{\nu'} (W_{\nu\nu'} f_\nu - W_{\nu\nu'} f_{\nu'}) = \alpha J e^{-\alpha x} \frac{\delta(\varepsilon_\nu - \varepsilon_0)}{g(\varepsilon_0)}. \tag{9}$$

Here

$$W_{\nu\nu'} = \sum_q \frac{4\pi}{\hbar} W_{\nu\nu'}(q)$$

and account is taken of the fact that the produced photoelectrons are uniformly distributed over the quantum states with energy ϵ_0.

We break up the energy interval from ϵ_0 to $\hbar\omega_0 + \zeta$ into bands of width $\Delta\epsilon = \hbar\omega_0$ (the last band can also be narrower). The number of such bands is $l = E((\epsilon_0 - \zeta_0)/\hbar\omega_0)$. The solution of (9) can be represented in the form

$$f_\nu^{(1)} = \alpha J e^{-\alpha x} \frac{\delta(\varepsilon_\nu - \varepsilon_0)}{g(\varepsilon_\nu) \sum_{\nu'} W_{\nu\nu'}}, \quad f_\nu^{(k)} = \frac{\sum_{\nu'} W_{\nu'\nu} f_{\nu'}}{\sum_{\nu'} W_{\nu\nu'}}, \tag{10}$$

where $f_\nu^{(k)}$ is the distribution function in the k-th band. We obtain for P_{opt}

$$P_{opt} = \hbar\omega_0 \sum_{k=1}^{l} \sum_{\nu\nu'} W_{\nu\nu'} f_\nu^{(k)} = l\hbar\omega_0 \alpha J e^{-\alpha x}. \tag{11}$$

We see that in this limiting case the energy loss on optical phonons is large and does not depend on the magnetic field at all. From the formal point of view, the absence of oscillations is due here to the fact that the numerator in (8) vanishes simultaneously with the denominator, since the distribution function is inversely proportional to $g(\epsilon_\nu)\sum_{\nu'} W_{\nu\nu'}$. From the physical point of view, the absence of oscillations in the case b) is also perfectly understandable, for in the stationary state the velocity of the energy loss by fast electrons should not depend on $W_{\nu\nu'}$.

In the intermediate case, when the interelectron interaction and the interaction with the optical phonons are comparable and competition takes place for the fast electrons between the two channels of energy loss, the loss P_{opt} turns out to be appreciable and oscillates with variation of the magnetic field, with a period $\Delta(1/H) = e/mc\,\omega_0$.

4. CALCULATION OF THE PHOTOMAGNETIC CURRENT

Using the expressions for the fluxes (1), the system of equations (2), and formulas (3), (4), and (5), we can eliminate the electric field with the aid of the neutrality condition, and obtain a system of equations for T_1 and n_1, as well as a general expression for the photomagnetic short-circuit current ($E_y = 0$) in terms of T_1 and n_1:

$$\begin{gathered} L_p^2 \frac{d^2 n_1}{dx^2} - n_1 = -\alpha J e^{-\alpha x} \tau_n, \\ n_0 L^2 \frac{d^2 T_1}{dx^2} + \frac{\tau_{ac}}{\tau_n} \frac{T_0^2}{\zeta_0} L_1^2 \frac{d^2 n_1}{dx^2} - \frac{\gamma_{n1}}{\gamma_n} \frac{\tau_{ac}}{\tau_n} \zeta_0 n_1 - n_0 T_1 \\ = [P_{opt}(T_1, x) - \varepsilon_0 \alpha J e^{-\alpha x}] \tau_{ac}, \\ \left.\frac{dn_1}{dx}\right|_{x=0} = 0, \quad \left.\frac{dT_1}{dx}\right|_{x=0} = 0, \end{gathered} \tag{12}$$

$$i_{pm} = eh \int_0^d (q_p{}^y - q_n{}^y)\, dx = h \frac{\sigma_{yx}\beta_{xx} - \beta_{yx}\sigma_{xx}}{\sigma_{xx}} T_1(0)$$

$$+ eh\left(\frac{\sigma_{yx}{}^p}{\sigma_{xx}{}^p} - \frac{\sigma_{yx}}{\sigma_{xx}}\right) \frac{L_p^2}{\tau_n} n_1(0). \tag{13}$$

In (12) and (13) we have

$$\begin{gathered} L_p{}^2 = \frac{\sigma_{xx}{}^p T_0}{e^2 p_0} \tau_p, \quad L_1{}^2 = \frac{\sigma_{xx}{}^p \chi_{xx}}{e p_0 \sigma_{xx}} \frac{\zeta_0}{T_0} \tau_p, \\ L^2 = \frac{\varkappa_{xx}\sigma_{xx} - \beta_{xx}\chi_{xx}}{\sigma_{xx} n_0} \tau_{ac}, \end{gathered} \tag{14}$$

$\tau_p = (\gamma_p M)^{-1}$ is the lifetime of the holes, h and d are the dimensions of the sample along the z and x axes. The expression $(\varkappa_{xx}\sigma_{xx} - \beta_{xx}\chi_{xx})/\sigma_{xx}$ is always positive, since it is the coefficient of thermal conductivity under the condition $q_n^x = 0$, $E_y = 0$.

Substituting in (12) the expressions for P_{opt} in the limiting cases a) and b), we can solve the system (12) and find $T_1(x)$ and $n_1(x)$. These derivations and the forms of $T_1(x)$ and $n_1(x)$ are perfectly analogous to those in[8], and we present only the values of $T_1(0)$ and $n_1(0)$:

$$n_1(0) = \frac{\alpha J \tau_n}{1 + \alpha L_p},$$

$$T_1(0) = \frac{\alpha J \tau_{ac}}{n_0} \frac{\zeta_0}{1 + \alpha L} \left\{ \frac{\varepsilon_{eff}}{\zeta_0} - \frac{(L_p + L + \alpha L L_p)\gamma_{n1}/\gamma_n + \alpha L_1{}^2 T_0{}^2/\zeta_0{}^2}{(1 + \alpha L_p)(L_p + L)} \right\}. \tag{15}$$

Here $\epsilon_{eff} = \epsilon_0 - \zeta_0$ in the case a) and $\epsilon_{eff} = \epsilon_0 - \zeta_0 - l\hbar\omega_0$ in case b).

Formulas (13) and (15) determine the value of i_{pm} in the limiting cases a) and b).

In the intermediate case, when the interelectron interaction and the interaction with the optical phonons are comparable, we do not have the explicit form of P_{opt} and we cannot, strictly speaking, find the explicit form of T_1. It can be stated, however, that T_1, and with it also the photomagnetic current, will experience GF oscillations, since the equation for T_1 contains essentially the oscillating term of P_{opt}.

We can assume qualitatively that $T_1(0)$ is determined in this case also by formula (15) with

$$\varepsilon_{eff} = \varepsilon_0 - \zeta_0 - \langle P_{opt} / \alpha J e^{-\alpha x} \rangle,$$

where $\langle P_{opt}/\alpha J e^{-\alpha x}\rangle$ is a certain mean value; in this case ϵ_{eff} experiences GF oscillations. Since $\epsilon_{eff} \sim \epsilon_0$ in this case, we obtain when $\epsilon_0 \gg \zeta_0$ the following simple expression for i_{pm}:

$$i_{pm} \approx h \frac{\sigma_{yx}\beta_{xx} - \beta_{yx}\sigma_{xx}}{\sigma_{xx}} \frac{\alpha J \tau_{ac}}{n_0(1 + \alpha L)} \varepsilon_{eff} + eh\left(\frac{\sigma_{yx}{}^p}{\sigma_{xx}{}^p} - \frac{\sigma_{yx}}{\sigma_{xx}}\right) \frac{\alpha J L_p{}^2}{1 + \alpha L_p}. \tag{16}$$

5. DISCUSSION OF RESULTS

The expression obtained by us for the photomagnetic current (16) contains two terms. The second term is the result of the diffusion theory without heating, while the first term takes the heating into account. Since the first term is proportional to the electronic electric conductivity and the second to the hole conductivity, at low temperatures, when the ratio τ_{ac}/τ_p is not too small, the first term prevails. If we discard the second term of (16), then we obtain for the case $\alpha L \ll 1$ the expression given in[9] for i_{pm}. Similar estimates for the region of classical magnetic fields are given in[8].

The PME under these conditions is essentially sim-

ply the Nernst effect, due to the gradient of the electronic temperature. The photomagnetic current turns out to be proportional to the difference $\sigma_{yx}\beta_{xx} - \beta_{yx}\sigma_{xx}$, and consequently, it is possible in principle to change the sign of the effect when the magnetic field is varied, as was observed in[1-3]. The strong increase of the amplitude of the SH oscillations for the PME compared with the conductivity oscillations, noted in[2], is connected with the partial cancellation of the "constant" components in the difference $\sigma_{yx}\beta_{xx} - \beta_{yx}\sigma_{xx}$. Since the coefficient of the Nernst emf also contains the quantity $\sigma_{yx}\beta_{xx} - \beta_{yx}\sigma_{xx}$, it follows from our result that the PME and the Nernst effect should reverse sign at close values of the magnetic field (a small difference may be introduced by the second term of (16)). Consequently, a comparative investigation of the Nernst effect and of the PME on the same sample can serve as a qualitative verification of the theory. Such an investigation was undertaken in[9] and its results confirm the theory.

The appearance of GF oscillations, which was observed in[2], can be explained, as already noted, by oscillations of the energy loss on the optical phonons. It is important to notice that in our theory the maxima of the electric conductivity correspond to minima of the PME, as was observed also in[2,9].

The vanishing of the GF oscillations with increasing and decreasing electron concentration, and the sharp decrease of the effect at small concentrations, can be explained as being due to the transition to the limiting cases a) and b). The GF oscillations were observed in n-InSb at electron concentrations $n_0 = 2 \times 10^{14} - 5 \times 10^{15}$ cm^{-3}[9], whereas for the case of classic magnetic fields $\tau_{ee} \approx \tau_{opt}$ at $n_0 = 7 \times 10^{15}$ cm^{-3}[8].

APPENDIX

A general expression for the energy loss in the scattering of an electron by the deformation or piezoelectric potentials of acoustic oscillations can be represented in the case when $T_1 \ll T_0$ in the form

$$\left(\frac{\delta\varepsilon}{\delta t}\right)_{ac} = \frac{4\pi}{\hbar} T_1 \frac{m\Omega}{(2\pi\hbar)^2} \sum_{qnn'} \int dp_z \int dp_z' \left(\frac{\hbar\omega_q}{T_0}\right)^2 \delta(p_z - p_z' + q_z)$$
$$\times |c_q|^2 N_q f_{00}(\varepsilon_{np_z})[1 - f_{00}(\varepsilon_{n'p_z'})]|I_{nn'}|^2 \delta(\varepsilon_{np_z} - \varepsilon_{n'p_z'} + \hbar\omega_q). \quad (A.1)$$

In the case of scattering by a deformation potential

$$|c_q|^2 = E_c^2 q^2 / V\rho\hbar\omega_q, \quad \hbar\omega_q = sq,$$

where s is the speed of sound, E_c is a constant of the deformation potential, and ρ is the density of the crystal. For scattering by a piezoelectric potential, we assume for $|c_q|^2$ the simplified expression[14]:

$$|c_q|^2 = \hbar^2 P^2 / 2\rho sqV,$$

where P is the coupling constant averaged over the angles and polarizations. It is easy to establish its connection with the modulus of the piezoelectric elasticity e_{14}[15].

An analysis of (A.1) shows that the main contribution in integration with respect to q is made by the momenta $q \approx \sqrt{8m\zeta_0}$. An estimate shows that in our case $\hbar\omega_q \ll \hbar\Omega$ and P_{ac} experiences SH oscillations similar to those of the transverse resistance, more accurately, similar to those of the transverse-resistance due to scattering by phonons. Explicit expressions for P_{ac} can be obtained only in limiting cases. When $\hbar\Omega \ll \zeta_0$, to calculate the monotonic part, the summation over n and n′ in (A.1) can be replaced by integration. We then obtain the classical result given in[8,15].

In the limiting quantum case $\hbar\Omega > \zeta_0$, it is possible to retain in the sum over n only the terms with n = n′ = 0.

1) $T_0^2 \gg 8ms^2\hbar\Omega$, $\zeta_0 - \hbar\Omega/2 \gg T_0$:

$$P_{def} = n_0 T_1 \frac{27E_c^2 T_0^3 \Omega^4 \sqrt{m}}{8\sqrt{2}\rho s^4 \zeta_{00}^{3/2}}, \quad (A.2)$$

$$P_{piezo} = n_0 T_1 \frac{27\hbar^2 P^2 T_0 \Omega^4 \sqrt{m}}{16\sqrt{2}\pi\rho s^2 \zeta_{00}^{5/2}}. \quad (A.3)$$

2) $T_0^2 \gg 8ms^2\hbar\Omega$, $\zeta_0 - \hbar\Omega/2 \ll -T_0$ (the degeneracy in the magnetic field is lifted):

$$P_{def} = n_0 T_1 \frac{E_c^2 m^{1/2} \Omega T_0^{3/2}}{\pi^{3/2}\sqrt{2}\rho s^4 \hbar^3} \ln\frac{1{,}7T_0^2}{ms^2\hbar\Omega}, \quad (A.4)$$

$$P_{piezo} = n_0 T_1 \frac{P^2\Omega\sqrt{m}}{(2\pi)^{3/2}\hbar\rho s^2\sqrt{T_0}} \ln\frac{4{,}4T_0^2}{ms^2\hbar\Omega}. \quad (A.5)$$

3) $T_0^2 \ll 2ms^2\hbar\Omega$, $\zeta_0 - \hbar\Omega/2 \gg T_0$:

$$P_{def} = n_0 T_1 \frac{27E_c^2 T_0^5 \Omega^3}{8\sqrt{2}\pi\hbar\rho s^6 \zeta_{00}^4 \sqrt{m\zeta_{00}}} \left(5!\,\zeta(5) - \frac{T_0^2}{2ms^2\hbar\Omega} 7!\,\zeta(7)\right), \quad (A.6)$$

$$P_{piezo} = n_0 T_1 \frac{81\hbar P^2 \Omega^3 T_0^3}{8\sqrt{2}\pi\rho s^4 \zeta_{00}^4 \sqrt{m\zeta_{00}}} \left(\zeta(3) - \frac{10T_0^2}{ms^2\hbar\Omega}\zeta(5)\right). \quad (A.7)$$

When $\zeta_0 - \hbar\Omega/2 > T_0^2/8ms^2$, the result in (A.6) and (A.7) should be divided by two.

4) $T_0^2 \ll 2ms^2\hbar\Omega$, $\zeta_0 - \hbar\Omega/2 \ll -T_0$:

$$P_{def} = n_0 T_1 \frac{\sqrt{2}E_c^2 T_0^{7/2}}{\pi\hbar^4\rho s^6\sqrt{m}} \left[\Gamma\left(\frac{9}{2}\right)\zeta\left(\frac{9}{2}\right) - \frac{T_0^2}{2ms^2\hbar\Omega}\Gamma\left(\frac{13}{2}\right)\zeta\left(\frac{13}{2}\right)\right], \quad (A.8)$$

$$P_{piezo} = n_0 T_1 \frac{P^2 T_0^{3/2}}{\sqrt{2}\pi\hbar^2\rho s^4\sqrt{m}} \left[\Gamma\left(\frac{5}{2}\right)\zeta\left(\frac{5}{2}\right) - \frac{T_0^2}{2ms^2\hbar\Omega}\Gamma\left(\frac{9}{2}\right)\zeta\left(\frac{9}{2}\right)\right]. \quad (A.9)$$

Here $\zeta_{00} = (3\pi^2\hbar^3 n_0)^{2/3}/2m$ is the equilibrium chemical potential in the absence of a magnetic field, and $\zeta(n)$ is the Riemann zeta function.

P_{piezo} and P_{def} were calculated by the method developed in[14] for the calculation of the conductivity.

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