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ON THE DAMPING AND POLARIZATION OF HELICON WAVES IN SEMICONDUCTORS AND METALS

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The coefficient for Cerenkov and collisional absorption of helicons in a Fermi gas of particles with a nonisotropic dispersion law is found for an arbitrary ratio of the phase velocity of the wave to the average velocity of the particles.

1. In uncompensated semiconductors and metals, just as in an ordinary plasma, the propagation of helicoidal waves (helicons) is possible—electromagnetic waves with a frequency appreciably smaller than the cyclotron frequency of electrons (holes) and a wavelength considerably greater than the Larmor radius of electrons (holes) with average velocity.

Damping of helicons is due either to collisions or else to the collisionless Landau mechanism, i.e., an exchange of energy between the wave and resonance electrons, having a velocity v_z along the external magnetic field H_0 close to the wave's phase velocity $\omega/k_{\parallel}$ along H_0, and "captured" in an adiabatic trap formed by the field H_0 and the variable magnetic field of the wave, $H_z \approx H^{\sim}\cos(\mathbf{k}\cdot\mathbf{r}-\omega t)$.[1,2] In the collisionless case ($\omega\tau \gg 1$ for $\omega/k \sim \langle v\rangle$, where τ is the relaxation time of the carriers) the Cerenkov damping of helicons may appreciably exceed collisional damping. For particles with a Maxwellian velocity distribution the Cerenkov damping of helicon waves was previously determined in[3] for a homogeneous plasma and in[1] for an inhomogeneous plasma cylinder in the case of arbitrary values for the ratio of the phase velocity to the thermal velocity of the particles. For a degenerate Fermi gas the Cerenkov damping of helicons with phase velocity much smaller than the limiting Fermi velocity was investigated in article[4] by Kaner and Skobov for the case of an isotropic quadratic dispersion law for the carriers, for the case of an isotropic quadratic dispersion law for the carriers, for the case of an ellipsoidal Fermi surface in the article by Walpole and McWhorter,[5] and for an arbitrary carrier dispersion law in article[6] by Kaner and Skobov. Collisional damping of helicons and the influence of collisions on Cerenkov damping of helicons in metals with $\omega/k \ll v_F$ for carriers with an isotropic quadratic dispersion law are investigated in the article by Buchsbaum and Platzman.[2] Cerenkov damping of helicons in metals has recently been experimentally observed.[7-10]

In the present article the coefficient for the damping of helicon waves (taking account of both the collisionless Landau mechanism as well as that due to collisions) is determined for a gas of quasiparticles with an arbitrary dispersion law and a nondegenerate Fermi distribution; the damping is calculated for an arbitrary ratio of the phase velocity to the average velocity of the quasiparticles in the case of closed quasiparticle orbits in momentum space. It is shown that the Cerenkov damping rapidly decreases with increase of the ratio of phase velocity to average velocity of the particles.

The question of the polarization of helicon waves is also discussed. In a plasma under low pressure, when the energy density of the magnetic field $H_0^2/8\pi$ is much larger than the energy density of the particles, the component of the electric field intensity of the helicon wave in the direction of the external magnetic field is much smaller than the transverse components; in a plasma under high pressure the components of the electric field of the helicon wave which are parallel and perpendicular to H_0 are of the same order.

In ordinary metals ($n_0 \sim 10^{22}$ cm^{-3}) $\omega/k \ll v_F$ for $H_0 < 10^6$ Oe. Only in metals containing a small number of carriers or in semiconductors can the helicon phase velocity be of the order of the average carrier velocity for small magnetic field values. The results of the present work may be applied to semiconductors with electron concentrations $n_0 \sim (10^{16}$ to $10^{17})$ cm^{-3}; then for $m^* \sim 0.01$ m and $\omega/k \sim v_F \sim 10^8$ cm/sec we find that $k \sim 4\pi e n_0 v_F/cH_0 \sim 10^3$ cm^{-1} and $\omega \sim 10^{11}$ sec^{-1} for $H_0 \sim (10^2$ to $10^3)$ Oe; here Landau damping of helicons will be comparable with or larger than the damping due to collisions if the mean free path $l \gtrsim 10^{-3}$ cm ($\tau \sim 10^{-11}$ sec). Such mean free paths can be achieved in sufficiently pure semiconductors, for example, in InSb at T = 77°K $l \sim 5\times10^{-3}$ cm ($\tau \sim 5\times10^{-11}$ sec).[11]

2. The electric field intensity $\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0 e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ of the electromagnetic waves inside a dielectric medium with dielectric permittivity ϵ_{ij} is determined from the system of equations

$$\Lambda_{ij}E_j = 0, \tag{1}$$

where $\Lambda_{ij} = N^2(\kappa_i\kappa_j - \delta_{ij}) + \epsilon_{ij}(\omega, \mathbf{k})$, $N = ck/\omega$ is the index of refraction, and $\boldsymbol{\kappa} = \mathbf{k}/k$ is a unit vector in the direction of the wave vector.

The condition for solvability of the system (1), $\det(\Lambda_{ij}) = 0$, gives the dispersion equation

$$AN^4 + BN^2 + C = 0, \tag{2}$$

where

$$A = \varkappa_i\varepsilon_{ij}\varkappa_j, \quad B = {}^1/_2(\varepsilon_{ij}\varepsilon_{ji} - \varepsilon_{ii}\varepsilon_{jj} + e_{ijk}e_{pqr}\varepsilon_{ip}\varepsilon_{jq}\varkappa_k\varkappa_r),$$

$$C = \det(\varepsilon_{ij}),$$

and e_{ijk} is the unit pseudotensor of the third rank.

The dielectric permittivity tensor has the form[4]

$$\varepsilon_{ij} = -\frac{ie^2}{\pi^2\hbar^3\omega}\int_0^\infty d\varepsilon\frac{\partial f_0}{\partial\varepsilon}\int_{-p_{zmax}}^{p_{zmax}} dp_z\frac{m(\varepsilon, p_z)}{\omega_H(\varepsilon, p_z)}\int_0^{2\pi} d\tau v_i(\varepsilon, p_z, \tau)$$
$$\times\exp\left[-\frac{i}{\omega_H(\varepsilon, p_z)}\int^{\tau}(\mathbf{kv}(\varepsilon, p_z, \tau'') - \omega - i\nu)\,d\tau''\right]\int^{\tau} v_j(\varepsilon, p_z, \tau')$$
$$\times\exp\left[\frac{i}{\omega_H(\varepsilon, p_z)}\int^{\tau'}(\mathbf{kv}(\varepsilon, p_z, \tau'') - \omega - i\nu)\,d\tau''\right]d\tau', \quad (3)$$

where $f_0 = [\exp((\epsilon - \epsilon_F)/kT) + 1]^{-1}$, $\mathbf{v} = \partial\epsilon/\partial\mathbf{p}$ is the velocity, $\omega_H = eH_0/m(\epsilon, p_z)c$ is the cyclotron frequency of particles with charge $(-e)$ and dispersion law $\epsilon(\mathbf{p})$, the energy ϵ and the component p_z of a particle's momentum in the direction of the external magnetic field $\mathbf{H}_0$ are integrals of the equations of motion*

$$\frac{d\mathbf{p}}{dt} = -\frac{e}{c}[\mathbf{vH}_0];$$

$m(\epsilon, p_z)$ is the effective mass,

$$m = \frac{1}{2\pi}\frac{\partial S(\varepsilon, p_z)}{\partial\varepsilon} \qquad S = \iint dp_x\,dp_y = \int d\varepsilon\oint\frac{dl}{v^1}$$

is the area of the intersection of the surface $\epsilon(\mathbf{p}) = \epsilon$ by the plane p_z = const, $\tau = \omega_H t$, and t is the time of motion of a particle along an orbit in the magnetic field.

We shall consider the case of closed orbits. In this case $\mathbf{v}$ and $\mathbf{p}$ are periodic (with period 2π) functions of τ which one can represent in the form of Fourier series:

$$v_z = \bar{v}_z(\varepsilon, p_z) + \sum_{n\neq 0} v_z^{(n)}(\varepsilon, p_z)e^{in\tau},$$
$$v_{x,y} = \sum_{n\neq 0} v_{x,y}^{(n)}(\varepsilon, p_z)e^{in\tau}, \quad (4)$$

where $\bar{v}_z(\epsilon, p_z)$ is the average value over a period of the particles' velocities, $\bar{v}_x = \bar{v}_y = 0$.

Performing the integration in (3) with respect to τ and taking (4) into account, we obtain

$$\varepsilon_{ij} = \frac{2e^2}{\pi\hbar^3\omega}\sum_{n=-\infty}^{\infty}\int_0^\infty d\varepsilon\frac{\partial f_0}{\partial\varepsilon}\int_{-p_{zmax}}^{p_{zmax}} dp_z\frac{m(\varepsilon, p_z)A_i^{(n)*}(\varepsilon, p_z, \mathbf{k})A_j^{(n)}(\varepsilon, p_z, \mathbf{k})}{\omega - n\omega_H(\varepsilon, p_z) - k_z\bar{v}_z(\varepsilon, p_z) + i\nu}, \quad (5)$$

where

$$\mathbf{A}^{(n)}(\varepsilon, p_z, \mathbf{k}) = \frac{1}{2\pi}\int_0^{2\pi} d\tau\,\mathbf{v}(\tau)\exp\{-in\tau + i\mathbf{k}\boldsymbol{\rho}(\varepsilon, p_z\tau)\}, \quad (6)$$

$$\boldsymbol{\rho}(\varepsilon, p_z, \tau) = \frac{1}{\omega_H(\varepsilon, p_z)}\sum_{n\neq 0}\frac{\mathbf{v}^{(n)}(\varepsilon, p_z)}{in}e^{in\tau}. \quad (7)$$

In the presence of several kinds of current carriers, the tensor ϵ_{ij} is represented in the form of a sum of expressions (5) for each kind of particle.

3. In the case of helicon waves the following inequalities are valid:

$$\omega \ll \omega_H, \quad k\rho_L \ll 1, \quad \nu \ll \omega_H, \quad (8)$$

where $\rho_L = \langle v\rangle/\omega_H$ denotes the Larmor radius of the particles, and $\langle v\rangle$ is the average (with respect to the Fermi distribution) value of the velocity. Taking inequalities (8) into consideration, from Eq. (5) it is not difficult to find that

$$\varepsilon_{xy} = -\varepsilon_{yx} = -i\varepsilon_2 = i\eta\frac{4\pi n_0 ce}{\omega H_0}, \quad (9)$$

where η is the sign of the charge, and n_0 is the equilibrium density:

$$n_0 = \frac{1}{4\pi^3\hbar^3}\int f_0\,d\mathbf{p}.$$

*$[\mathbf{vH}] \equiv \mathbf{v}\times\mathbf{H}$.

The remaining components of the tensor ϵ_{ij} are given by

$$\varepsilon_{xx} = \frac{\sigma}{\omega}\int d\mathbf{p}\frac{\partial f_0}{\partial\varepsilon}\left[\frac{\overline{(\mathbf{k}\rho v_x)}^2}{\omega + i\nu - k_z\bar{v}_z} + \frac{\overline{(p_y - \bar{p}_y)^2}(k_z\bar{v}_z - \omega - i\nu)}{m^2\omega_H^2}\right],$$
$$\varepsilon_{yy} = \frac{\sigma}{\omega}\int d\mathbf{p}\frac{\partial f_0}{\partial\varepsilon}\left[\frac{\overline{(\mathbf{k}\rho v_y)}^2}{\omega + i\nu - k_z\bar{v}_z} + \frac{\overline{(p_x - \bar{p}_x)^2}(k_z\bar{v}_z - \omega - i\nu)}{m^2\omega_H^2}\right],$$
$$\varepsilon_{zz} = \frac{\sigma}{\omega}\int d\mathbf{p}\frac{\partial f_0}{\partial\varepsilon}\frac{\bar{v}_z^2}{\omega + i\nu - k_z\bar{v}_z}, \quad (10)$$
$$\varepsilon_{xz} = -\varepsilon_{zx} = -i\frac{\sigma}{\omega}\int d\mathbf{p}\frac{\partial f_0}{\partial\varepsilon}\left[\frac{\bar{v}_z\overline{(\mathbf{k}\rho v_x)}}{\omega + i\nu - k_z\bar{v}_z} - \frac{\overline{(v_z - \bar{v}_z)(p_y - \bar{p}_y)}}{m\omega_H}\right]$$
$$\varepsilon_{yz} = -\varepsilon_{zy} = -i\frac{\sigma}{\omega}\int d\mathbf{p}\frac{\partial f_0}{\partial\varepsilon}\left[\frac{\bar{v}_z\overline{(\mathbf{k}\rho v_y)}}{\omega + i\nu - k_z\bar{v}_z} + \frac{\overline{(v_z - \bar{v}_z)(p_x - \bar{p}_x)}}{m\omega_H}\right]$$

where $\sigma = e^2/\pi^2\hbar^3$, and the bar denotes averaging with respect to τ.

Expressions (9) and (10) are given in a coordinate system in which the z axis is parallel to $\mathbf{H}_0$, and the wave vector lies in the (X0Z) plane so that $\mathbf{k} = (k\sin\theta, 0, k\cos\theta)$ where θ is the angle between $\mathbf{k}$ and $\mathbf{H}_0$. In order to obtain expressions (9) and (10) the following change of the variables of integration was used:

$$(\varepsilon, p_z, \tau)\to(p_x, p_y, p_z)\ [4], \quad \int d\mathbf{p}\varphi = \int d\varepsilon\int m\,dp_z\int d\tau\,\varphi.$$

For particles with an isotropic quadratic dispersion law

$$\varepsilon = p^2/2m, \quad \mathbf{v} = (-v_\perp\sin\tau, v_\perp\cos\tau, v_\parallel),$$
$$\boldsymbol{\rho} = [(v_\perp/\omega_H)\cos\tau, \quad (v_\perp/\omega_H)\sin\tau, 0],$$

so that $\epsilon_{xz} = \epsilon_{zx} = 0$. If $\epsilon = p^2/2m$ and the gas is degenerate, then expressions (10) have simple forms:

$$\varepsilon_{xy} = -i\frac{\omega_p^2}{\omega\omega_H} = -i\varepsilon_2, \quad \varepsilon_{xx} = \frac{\omega_p^2(i\nu + \omega)}{\omega\omega_H^2},$$
$$\varepsilon_{yy} = \varepsilon_{xx} - \frac{3\omega_p^2 kv_F\sin^2\theta}{8\omega\omega_H^2\cos\theta}\left[(1 - \zeta^2)^2\ln\frac{1+\zeta}{-1+\zeta} - 2\zeta^3 + \frac{10}{3}\zeta\right],$$
$$\varepsilon_{zz} = \frac{3\omega_p^2}{k^2v_F^2\cos^2\theta}\left[1 - \frac{\zeta}{2}\ln\frac{1+\zeta}{-1+\zeta}\right]\frac{\omega + i\nu}{\omega},$$
$$\varepsilon_{yz} = -i\frac{\omega_p^2\sin\theta}{\omega\omega_H\cos\theta}\left[1 - \frac{3}{2}\zeta^2 - \frac{3}{4}\zeta(1 - \zeta^2)\ln\frac{1+\zeta}{-1+\zeta}\right], \quad (11)$$

where $\omega_p^2 = 4\pi n_0e^2/m$ is the plasma frequency, v_F is the limiting Fermi velocity, $v_F = (3\pi^2\hbar^3n_0/m^3)^{1/3}$, and

$$\zeta = \frac{\omega + i\nu}{kv_F\cos\theta}, \quad |\mathrm{Im}\,\zeta| \ll 1, \quad \mathrm{Re}\,\zeta < 1.$$

In order of magnitude, for $\omega/k\langle v\rangle \lesssim 1$ in a plasma under low pressure ($\beta = 4\pi n_0 m\langle v^2\rangle/H_0^2 \ll 1$) we have

$$\varepsilon_{zz} \sim \omega_p^2/k^2\langle v^2\rangle \gg |\varepsilon_{xy}| \sim |\varepsilon_{zy}| \sim |\varepsilon_{xz}| \sim$$
$$\sim \omega_p^2/\omega\omega_H \gg |\varepsilon_{xx}| \sim |\varepsilon_{yy}| \sim \omega_p^2/\omega_H^2. \quad (12)$$

Taking these inequalities into account, in the zero-order approximation one can set $A = \epsilon_{zz}\cos^2\theta$, $B = 0$, and $C = \epsilon_{zz}\epsilon_{xy}^2$ in the dispersion equation (2). Then from Eq. (2) we find that

$$N^2 = 4\pi n_0 ec/\omega H_0\cos\theta \quad \text{or} \quad \omega = cH_0k^2\cos\theta/4\pi n_0 e. \quad (13)$$

To the next approximation, assuming $\gamma = \mathrm{Im}\,k \ll \mathrm{Re}\,k$, we obtain a final expression for the coefficient of Cerenkov damping of helicon waves

$$\frac{\gamma}{k} = \mathrm{Im}\left[\frac{\varepsilon_{xx} + \varepsilon_{yy}\cos^2\theta}{4\varepsilon_2\cos\theta} + \frac{(\varepsilon_{xy}\sin\theta - \varepsilon_{yz}\cos\theta)^2 + \varepsilon_{xz}^2}{4\varepsilon_2\varepsilon_{zz}\cos\theta}\right], \quad (14)$$

where the quantities ϵ_{ij} are determined by formulas (10)

In order of magnitude, for $k\langle v\rangle \gtrsim \omega \gtrsim \nu$ we obtain from here the result that $\gamma/k \sim k\rho \ll 1$, where $\rho = \langle v\rangle/\omega_H$ is the Larmor radius of the particles.

4. In the general case expression (14) is complicated. It simplifies greatly for a degenerate Fermi gas with an isotropic dispersion law. Substituting expression (11) for ϵ_{ij} into Eq. (14), for $\nu \ll \omega$

$$\frac{\gamma}{k}=\frac{\omega \sin^2\theta}{\omega_H \cos\theta} f_F(\zeta), \qquad (15)$$

where

$$f_F(\zeta)=\frac{3\pi}{128}\left\{\frac{(2\chi)^2}{\zeta}+\frac{\zeta[(2\zeta+\chi\Lambda)^2-(\pi\chi)^2]+4\chi[2\zeta+\chi\Lambda](1-\zeta\Lambda/2)}{(1-\zeta\Lambda/2)^2+(\pi\zeta/2)^2}\right\}$$

$$\chi=(1-\zeta^2), \quad \Lambda=\ln[(1+\zeta)/(1-\zeta)].$$

In the region of small phase velocities ($\zeta \ll 1$), $f_F(\zeta) = 3\pi/32\,\zeta$ and expression (15) goes over into the expression obtained in[4]. For $\zeta \to 1$ the damping γ slowly (logarithmically) tends to zero:

$$f_F(\zeta) = (3\pi/8)[\ln(1-\zeta)]^{-2}.$$

A graph of the monotonically decreasing function $f_F(\zeta)$, characterizing the dependence of the damping on the ratio of the phase velocity to the limiting Fermi velocity is shown in the Figure. For comparison, a graph of the function $f_M(z)$, which determines the damping coefficient for helicon waves in a gas with a Maxwellian velocity distribution for the particles,[3] is also shown on the same Figure:

$$\text{FORMULA} \quad \frac{\gamma}{k}=\frac{\omega}{\omega_H}\frac{\sin^2\theta}{\cos\theta} f_M(z) \qquad (16)$$

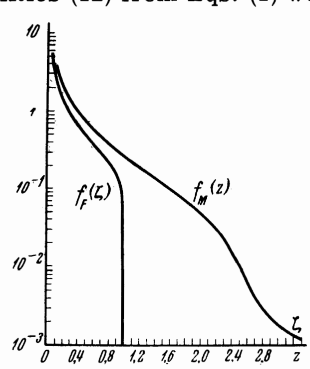
$$f_M(z)=\frac{\sqrt{\pi}}{8z}\cdot\left[1+\frac{1}{(1-\sqrt{\pi}\,zv)^2+(\sqrt{\pi}\,ze^{-z^2})^2}\right]e^{-z^2}$$

where

$$v(z)=\frac{2}{\sqrt{\pi}}e^{-z^2}\int_0^z e^{t^2}\,dt \qquad z=\frac{\omega}{\sqrt{2}\,kv_T\cos\theta} \qquad v_T=\left(\frac{T}{m}\right)^{1/2}$$

Taking collisions into account by using expression (11), it is not difficult to obtain from (14) the expressions of article[2] for the coefficients for collisional and Cerenkov absorption of helicon waves in a degenerate Fermi gas for $\zeta \ll 1$.

5. Now let us consider the polarization of helicon waves. In a plasma under small pressure, upon fulfillment of inequalities (12) from Eqs. (1) we obtain



$$E_y/E_x = i\cos\theta, \quad E_z/E_x = (i\varepsilon_{yz}\cos\theta-\varepsilon_2\sin\theta+\varepsilon_{xz})/\varepsilon_{zz}. \qquad (17)$$

From (17) it follows that the electric field component E_z which is parallel to the magnetic field is considerably smaller than the components E_x and E_y which are perpendicular to the magnetic field. For $\omega/k \lesssim \langle v\rangle$ in the general case of particles with an arbitrary dispersion law $\epsilon_{xz} \sim \epsilon_{yz} \sim \epsilon_2 \sim \omega_p^2/\omega\omega_H$ and

$$E_z/E_x \sim (k^2\langle v^2\rangle/\omega\omega_H) \sim \beta \ll 1.$$

In a gas consisting of particles with an isotropic quadratic dispersion law, $\epsilon_{xz} \approx 0$ and for $|\omega|/k \ll \langle v\rangle$, $\epsilon_{yz} i\sin\theta \approx \epsilon_2\sin\theta$ so that in this case the component E_z turns out to be very small: $E_z/E_x \sim \beta\omega/k\langle v\rangle$.

In the case of a plasma under large pressure ($\beta \gtrsim 1$) the phase velocity of the helicon waves is appreciably smaller than the average velocity of the particles,

$$\omega/k\langle v\rangle \sim (\omega/\omega_H)^{1/2} \sim k\rho_L \ll 1.$$

For $\beta > 1$ the coefficient for the damping of helicon waves is determined as before by formula (14). In this case the component E_z is of the same order as the transverse components E_x and E_y.

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