PLASMA DYNAMICS IN A STRONG HIGH-FREQUENCY FIELD

L. M. GORBUNOV

P. N. Lebedev Physics Institute, U.S.S.R. Academy of Sciences

Submitted December 8, 1968

Zh. Eksp. Teor. Fiz. 56, 1693–1699 (May, 1969)

The quasistatic approximation is used to analyze two nonlinear problems concerning the motion of a transparent plasma layer in a traveling electromagnetic wave of rather large amplitude (expansion of the layer, acceleration of the layer, and compression of the layer).

INTRODUCTION

THE analysis of radiation acceleration of a plasma[1] has stimulated a number of investigations of the motion of a transparent plasma in a strong high-frequency (rf) field. In particular, Kovrizhnykh[2] and Yankov[3] have considered the forces that act on various plasma configurations in an rf field. However, the distribution of plasma density has been assumed to remain constant in [2,3] so that the conclusions reached in that work are qualitative in nature.

It will be evident that one effect of the force associated with the rf field is to cause a modification of the plasma density, which, in turn, leads to a modification of the rf field in the plasma; this field modification then influences the forces that act on the plasma. For this reason, in studying the dynamics of a plasma in an rf field it is necessary to consider the equation of motion and the field equation in the plasma together. In the present work we consider two simple examples in which it is possible to carry through a self-consistent solution for the system of field equations and equations of motion and, in this way, to investigate the dynamics of a transparent plasma in a strong rf field. The analysis is carried out within the framework of single-fluid hydrodynamics under the assumption that the thermal pressure forces can be neglected.

In the first problem the electromagnetic wave propagates along the plasma layer, the direction of the electric field being parallel to the density gradient. It is shown that under these conditions the plasma layer tends to spread; in the course of a definite time density jumps appear at the edges of the layer (discontinuities).

In the second problem we consider the acceleration of a plasma layer when an electromagnetic wave is propagated through it. The density gradient is perpendicular to the electric field in this case. It is shown that in addition to acceleration of a plasma layer as a whole there is also a compression effect. The characteristic time for compression is determined, as are the conditions under which compression occurs.

It is assumed in both problems that the rf wavelength is large compared with the thickness of the plasma layer.

1. INITIAL EQUATIONS

As is well-known, in an rf field the electrons in the plasma are subject, on the average, to a force given by[4]

$$\mathbf{f} = -\frac{e^2}{2m\omega_0^2}\nabla\langle E^2\rangle, \tag{1.1}$$

where ω_0 and $\mathbf{E}$ are the frequency and strength of the rf field in the plasma and the brackets $\langle\ldots\rangle$ denote an average over the high-frequency. If the plasma density is low so that $\omega_0 > \omega_p$, where ω_p is the plasma frequency corresponding to the maximum density, the external field $\mathbf{E}_0$ is only weakly distorted and can be represented in the form

$$\mathbf{E} = \mathbf{E}_0 + \delta\mathbf{E},$$

where the perturbations in the electric and magnetic fields $\delta\mathbf{E}$ and $\delta\mathbf{B}$ arise by virtue of the presence of the plasma, being given by the equations

$$\operatorname{rot}\delta\mathbf{B} = \frac{1}{c}\frac{\partial\delta\mathbf{E}}{\partial t} + \frac{4\pi}{c}\mathbf{j},\quad \operatorname{rot}\delta\mathbf{E} = -\frac{1}{c}\frac{\partial\delta\mathbf{B}}{\partial t},\operatorname{div}\delta\mathbf{E} = 4\pi\rho, \tag{1.2}$$

where the current density $\mathbf{j}$ and the charge density ρ are functions of the external field:

$$\mathbf{j} = en\mathbf{v}_0,\quad \frac{\partial\mathbf{v}_0}{\partial t} = \frac{e}{m}\mathbf{E}_0,\quad \frac{\partial\rho}{\partial t} + \operatorname{div}\mathbf{j} = 0. \tag{1.3}$$

We consider the case in which the force exerted on the plasma by the external field can be neglected compared with the force that arises by virtue of the plasma:

$$|\nabla\langle E_0^2\rangle| \ll 2|\nabla\langle\mathbf{E}_0\delta\mathbf{E}\rangle|. \tag{1.4}$$

It will be evident that the inequality in (1.4) is satisfied if $\mathbf{E}_0$ is the field of a plane traveling wave because in this case $\nabla\langle E_0^2\rangle = 0$. The equation of motion for a plasma that is neutral on the average and the equation of continuity can then be written

$$\frac{\partial\mathbf{v}}{\partial t} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{Ze^2}{mm_i\omega_0^2}\nabla\langle\mathbf{E}_0\delta\mathbf{E}\rangle, \tag{1.5}$$

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{v}) = 0, \tag{1.6}$$

where $Z|e|$ and m_i are the charge and mass of the ions, $\mathbf{v}$ is the plasma velocity and n is the electron density. The perturbation of the electric field in Eq. (1.5) is given by Eqs. (1.2) and (1.3) which can be written in the following manner in view of the high frequency of the external field:

$$\operatorname{rot}\operatorname{rot}\delta\mathbf{E} - \frac{\omega_0^2}{c^2}\delta\mathbf{E} = -\frac{4\pi e^2 n}{mc^2}\mathbf{E}_0,\quad \operatorname{div}\delta\mathbf{E} = \frac{4\pi e^2\mathbf{E}_0}{m\omega_0^2}(\nabla n). \tag{1.7}$$

The system of equations (1.5)–(1.7) can be used to analize various problems concerning the motion of a transparent plasma in a strong rf field when the thermal pressure can be neglected.

The latter situation obtains when

$$\frac{Ze^2}{m\omega_0^2}|\nabla\langle \mathbf{E}_0\delta\mathbf{E}\rangle| \gg \frac{T}{n}|\nabla n|, \qquad (1.8)$$

where T is the plasma temperature.

2. PLASMA EXPANSION

We consider a plasma layer in which the density depends on the single coordinate x and the time. It is assumed that the rf field propagates along the OY axis and that the electric field is in the direction given by the OX axis:

$$\mathbf{E}_0 = \left[E_0 \cos\left(\omega_0 t - \frac{\omega_0}{c} y\right), 0, 0\right].$$

When the thickness of the layer is much smaller than c/ω_0, the solution of Eq. (1.7) is given by

$$\delta E = \frac{4\pi e^2}{m\omega_0^2} n E_0 \cos\left(\omega_0 t - \frac{\omega_0}{c} y\right). \qquad (2.1)$$

Using Eq. (2.1) we now write Eqs. (1.5) and (1.6) in the form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\gamma \frac{\partial n}{\partial x} \qquad (2.2)$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0, \qquad (2.3)$$

where

$$\gamma = \frac{Z}{2}\frac{4\pi e^2}{m_i}\frac{v_E^2}{\omega_0^2}, \quad v_E = \frac{eE_0}{m\omega_0}.$$

The solution of Eqs. (2.2) and (2.3) that corresponds to the initial condition

$$t = 0, \quad v = 0, \quad n = n_0 \operatorname{ch}^{-2}(x/a), \qquad (2.4)$$

is of the form[5]

$$v = \frac{2\gamma t}{a}\operatorname{th}\left(\frac{x - vt}{a}\right) \qquad (2.5)$$

$$n = \left(n_0 - \frac{\gamma t^2 n^2}{a^2}\right)\operatorname{ch}^{-2}\left(\frac{x - vt}{a}\right), \qquad (2.6)$$

where n_0 is the initial density at x = 0 and a is the characteristic initial width of the plasma layer.

Using the dimensionless variables

$$\nu = \frac{n}{n_0}, \quad \xi = \frac{x}{a}, \quad \tau = \frac{1}{\sqrt{2Z}}\frac{v_E}{a\omega_0} t\omega_{L_i}, \quad w = \sqrt{2Z}\frac{\omega_0}{\omega_{L_i}}\frac{v}{v_E},$$

where $\omega_{Li}^2 = 4\pi e^2 Z n_0/m_i$, and introducing Eqs. (2.5) and (2.6), we have

$$\xi = \operatorname{Arch}\sqrt{\frac{1-\nu^2\tau^2}{\nu}} + 2\nu\tau^2\sqrt{1 - \frac{\nu}{1-\nu^2\tau^2}}. \qquad (2.7)$$

From the curve of the function $\nu(\xi, \tau)$ (Fig. 1) plotted from Eq. (2.7) it will be evident that the plasma layer spreads apart as a function of time. Under these conditions the maximum density is maintained at the center of the layer, this density being given by

$$\nu_{max} = \nu(0, \tau) = \frac{1}{2\tau^2}(\sqrt{1 + 4\tau^2} - 1).$$

Starting at some value of τ the density is no longer a single-valued function of coordinates; this indicates the formation of a density discontinuity.[6] Under these conditions it must be true that $\partial\xi/\partial\nu = 0$ and $\partial^2\xi/\partial\nu^2 = 0$, these relations giving the time at which the discontinuity is formed τ_0 as well as the density at the discontinuity point ν_0. Using Eq. (2.7) and assuming that $(\nu_0\tau_0)^2 \ll 1$ we have

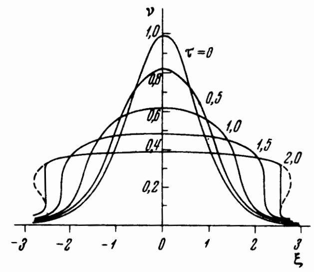

FIG. 1. Variation of plasma density ($\nu = n/n_0$) in expansion of the layer $\left(\xi = x/a, \ \tau = \frac{1}{\sqrt{2Z}}\frac{v_E}{a\omega_0} t\omega_{L_i}\right)$

$$\tau_0 \approx 1{,}6 \quad \left(t \approx 1{,}6\sqrt{2Z}\frac{a\omega_0}{v_E}\frac{1}{\omega_{L_i}}\right), \quad \nu_0 \approx 0{,}2 \quad (n \approx 0{,}2n_0). \qquad (2.8)$$

In accordance with Eqs. (2.5) and (2.6) the plasma velocity is given by

$$w = 2\nu\tau\sqrt{1 - \frac{\nu}{1-\nu^2\tau^2}}. \qquad (2.9)$$

It is evident from the curve of the function $w(\xi, \tau)$ given in Fig. 2 that the velocity is a maximum at a specific point, this point receding from the origin of coordinates in the course of time. At the discontinuity the velocity is

$$w_0 \approx 0{,}95 \quad \left(v \approx 0.95\frac{\omega_{L_i}}{\omega_0}\frac{1}{\sqrt{2Z}}v_E\right). \qquad (2.10)$$

Equation (1.5) does not hold at the discontinuity since (1.1) has been obtained under the assumption that the characteristic scale length for the change in density is much larger than the oscillation amplitude of the electron in the rf field v_E/ω_0.

In concluding this section we indicate that the condition that must be satisfied if the thermal pressure of a plasma can be neglected, (1.8), becomes

$$1 > \nu \gg \frac{2}{Z}\frac{T}{\mathcal{E}_E}\frac{\omega_0^2}{\omega_p^2}, \qquad (2.11)$$

where $\mathcal{E}_E$ is the oscillation energy of the electron in the rf field ($\mathcal{E}_E = mv_E^2/2$), $\omega_p^2 = 4\pi n_0 e^2/m$.

3. COMPRESSION AND ACCELERATION OF THE PLASMA

We now consider a plasma layer in which the density again depends only on the single spatial variable x and

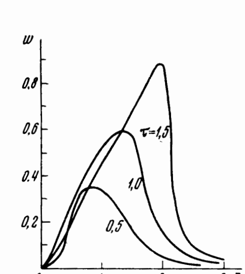

FIG. 2. Variation of plasma velocity $\left(w = \sqrt{2Z}\frac{\omega_0}{\omega_{L_t}}\frac{v}{v_E}\right)$ in expansion of the layer.

on the time. An electromagnetic wave propagates through the plasma and the electric field associated with the wave is perpendicular to the OX axis:

$$\mathbf{E}_0 = \left(0, E_0 \cos\left(\omega_0 t - \frac{\omega_0}{c} x\right), 0\right).$$

From Eq. (1.7) we have

$$\frac{\partial^2 \delta E}{\partial x^2} + \frac{\omega_0^2}{c^2}\delta E = \frac{4\pi e^2}{mc^2} n E_0 \cos\left(\omega_0 t - \frac{\omega_0}{c} x\right). \tag{3.1}$$

The solution of Eq. (3.1) that satisfies the condition for a wave propagating to the right when $x \to +\infty$ and to the left when $x \to -\infty$ is of the form ($k_0 = \omega_0/c$)

$$\delta E(x, t) = \frac{2\pi e^2}{m\omega_0^2} k_0 E_0 \left[\sin(k_0 x - \omega_0 t) \int\limits_{-\infty}^{x} dx' n(x', t) + \int\limits_{x}^{\infty} dx' n(x', t) \sin(2k_0 x' - k_0 x - \omega_0 t)\right]. \tag{3.2}$$

Using Eq. (3.2) we can now transform Eq. (1.5) to the form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\beta \int\limits_{\infty}^{x} dx' n(x', t) \cos 2k_0(x - x'), \tag{3.3}$$

where

$$\beta = \frac{Z}{2}\frac{v_E^2}{c^2}\frac{4\pi e^2}{m_i}.$$

It will be assumed that the wavelength of the external field c/ω_0 is much larger than the thickness of the plasma layer. In this case Eqs. (3.3) and (1.6) can be written in the form

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = -\beta \int\limits_{\infty}^{x} dx' n(x', t); \tag{3.4}$$

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x}(nv) = 0. \tag{3.5}$$

We now introduce the new function

$$A = -\beta \int\limits_{\infty}^{x} dx' n(x', t), \tag{3.6}$$

by means of which we can write the electron density in the form

$$n = -\frac{1}{\beta}\frac{\partial A}{\partial x}. \tag{3.7}$$

We integrate Eq. (3.5) over coordinate from ∞ to x and assume that $n(\infty, t) = 0$. As a result we obtain the following system of equations for determining A and v:

$$\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial x} = A, \tag{3.8}$$

$$\frac{\partial A}{\partial t} + v\frac{\partial A}{\partial t} = 0. \tag{3.9}$$

The general solution of this system of equations can be obtained easily by converting from the variables t and x to the variables A and v. The solution is of the form

$$x = \frac{v^2}{2A} + f_1(A), \quad t = \frac{v}{A} + f_2(A),$$

where f_1 and f_2 are arbitrary functions that are determined from the initial conditions. In particular, if the condition in (2.4) is satisfied at t = 0 then

$$v = tA_0\left[1 - \mathrm{th}\left(\frac{x - vt/2}{a}\right)\right], \tag{3.10}$$

$$A = A_0\left[1 - \mathrm{th}\left(\frac{x - vt/2}{a}\right)\right], \quad A_0 = \frac{a}{2Z}\frac{v_E^2}{c^2}\omega_{L_i}^2. \tag{3.11}$$

Introducing the dimensionless variables

$$\xi = \frac{x}{a}, \quad \nu = \frac{n}{n_0}, \quad \tau = \frac{1}{2\sqrt{Z}}\frac{v_E}{c} t\omega_{L_i}, \quad w = \sqrt{Z}\frac{c}{v_E}\frac{v}{a\omega_{L_i}}, \tag{3.12}$$

and making use of Eqs. (3.7), (3.10), and (3.11) we have

$$\xi = w\tau + \mathrm{Arth}(1 - w/\tau), \tag{3.13}$$

$$\nu = \frac{1}{\mathrm{ch}^2(\xi - w\tau) - \tau^2}. \tag{3.14}$$

It follows from Eqs. (3.13) and (3.14) that the plasma density is a maximum at the point $\xi = \tau^2$ [i.e., $x = (av_E^2/4Zc^2)(t\omega_{Li})^2$], being given by

$$\nu_{max} = 1/(1 - \tau^2). \tag{3.15}$$

The plasma velocity at this point is $w = \tau$ [i.e., $v = (v_E^2/2Zc^2)a\omega_{Li}^2 t$]. In Figs. 3 and 4 we show the functions $\nu(\xi, \tau)$ and $w(\xi, \tau)$ plotted from Eqs. (3.13) and (3.14).

Since the plasma density must remain bounded it follows that $\tau < 1$, ($t < 2\sqrt{Z}c/v_E\omega_{Li}$). Actually, the limitation on τ appears somewhat earlier. This limitation is associated with the violation of the assumption that the plasma is transparent $\nu_{max} \ll \nu_p$, where $\nu_p = \omega_0^2 m/4\pi e^2 n_0$. Hence

$$\tau < \sqrt{1 - 1/\nu_p} \approx 1 - 1/2\nu_p. \tag{3.16}$$

The condition in (1.8) in the present case leads to the following inequality, for which the present analysis holds:

$$1 \lesssim Z\frac{a^2}{(c/\omega_0)^2}\frac{\mathscr{E}_E}{T}\frac{\omega_p^2}{\omega_0^2}. \tag{3.17}$$

CONCLUSION

In conclusion we present several numerical estimates. First consider the expansion of the plasma. We shall consider a hydrogen plasma with a density $n_0 = 3 \times 10^9\ \mathrm{cm}^{-3}$. We take the values $a \approx 0.3$ cm, $a\omega_0/c \approx 1/30$, $\omega_p^2/\omega_0^2 \approx 1/30$, $T \approx 1$ eV. (These data are taken from [7] in which an investigation was made of the expansion of a plasma jet in an rf field. However, in this work the conditions in (1.8) and (1.4) were not satisfied

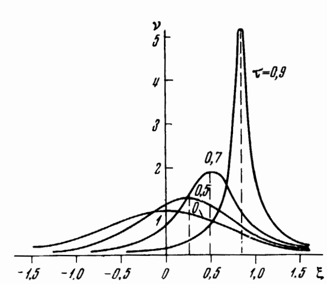

FIG. 3

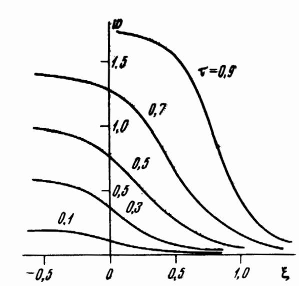

FIG. 4

FIG. 3. Variation of plasma density in acceleration and compression of the layer.

FIG. 4. Variation of plasma velocity in acceleration and compression of the layer.

properly). If we now neglect the thermal pressure in the plasma expansion in accordance with Eq. (2.11), it is necessary that $\mathcal{E}_E > 2T\omega_0^2/\omega_p^2 \approx 60$ eV (i.e., $E_0 > 3$ kV/cm). In accordance with Eq. (2.8) the time required for the formation of the discontinuity with $\mathcal{E}_E \approx 100$ eV is $t = 10^{-6}$ sec; the velocity at the discontinuity is $v \approx 1.5 \times 10^6$ cm/sec.

In accordance with Eq. (3.17), the compression and acceleration of the plasma should appear when $\mathcal{E}_E > 100$ eV if it is assumed that $a\omega_0/c \approx 1/3$, $\omega_p^2/\omega_0^2 \approx 1/10$ and $T \approx 1$ eV. If $\mathcal{E}_E \approx 200$ eV the time during which the present analysis holds is given by $t < 10^{-6}$ sec. Under these conditions the velocity of the plasma layer as a whole is $v \approx 7 \times 10^5$ cm/sec and the distance traversed by the layer is $x \approx 2$ cm.

The author is indebted to Yu. M. Aliev, R. R. Ramazashvili, A. A. Rukhadze and V. P. Silin for discussion.

[1] V. I. Veksler, Atomnaya énergiya (Atomic Energy) 2, 427 (1957).

[2] L. M. Kovrizhnykh, Zh. Eksp. Teor. Fiz. 33, 72 (1957) [Sov. Phys.-JETP 6, 54 (1958)].

[3] V. V. Yankov, Zh. Eksp. Teor. Fiz. 32, 926 (1957). [Sov. Phys. JETP 5, 753 (1957)]; Zh. Tekh. Fiz. 36, 438 (1966) [Sov. Phys.-Tech. Phys. 11, 324 (1966)].

[4] A. V. Gaponov and M. A. Miller, Zh. Eksp. Teor. Fiz. 34, 242 (1958) [Sov. Phys. JETP 7, 168 (1958)]; ibid. 34, 751 (1958) [Sov. Phys. JETP 7, 515 (1958)].

[5] R. Mises, Mathematical Theory of Compressible Flow, (Russian Translation) IIL 1961.

[6] L. D. Landau and E. M. Lifshitz, Fluid Mechanics, Addison Wesley, Reading, Mass., 1959.

[7] G. S. Luk'yanchikov, M. D. Raĭzer, and A. G. Frank, Zh. Tekh. Fiz. 38, 296 (1968) [Sov. Phys.-Tech. Phys. 13, 215 (1968)].

Translated by H. Lashinsky
193