GIANT OSCILLATIONS OF SOUND ABSORPTION BY METALS IN THE CASE OF OPEN TRAJECTORIES

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Absorption of short-wave sound ($qL \gg 1$; q is the sound wave vector, L the electron mean free path) in a quantizing magnetic field is considered for the case when sound absorption is due to electrons on open or closed trajectories closely approaching the Brillouin zone boundaries. The trajectory responsible for absorption can be chosen by varying the direction of propagation of sound; in other words, a transition from closed to open trajectories can be realized by this means. The magnetic field dependence of the sound absorption coefficient exhibits a number of peaks separated by low absorption regions. A periodicity of the peaks with respect to $1/H$ (H is the magnetic field strength) is present; the peak width depends on the type of trajectory responsible for the absorption. In the case of closed trajectories the peaks are in general narrower than the space between them, whereas in the case of open trajectories the gaps between the peaks become narrow, i.e., giant transparency oscillations occur. The results of the work can be applied to uniaxial metals with a Fermi surface of the "corrugated cylinder" type or to cubic metals with a Fermi surface of the copper type for certain directions of the magnetic field.

IN the case of electron trajectories that are closed in p-space, the absorption of sound by metals in a strong magnetic field can experience the so called giant quantum oscillations. This effect, predicted by V. Gurevich, Skobov, and Firsov, was investigated in sufficient detail both theoretically and experimentally in a number of subsequent papers[1-3]. It consists in the fact that the dependence of the sound absorption coefficient on the magnetic field H is a system of narrow peaks separated by broad regions of relatively weak absorption, and the arrangement of the peaks has in the simplest case a periodicity in $1/H$.

In metals with open Fermi surfaces, at definite orientations of the magnetic field, there can occur both closed and open trajectories, and, depending on the direction of the propagation of the sound, the absorption may be due to one type or to the other.

A classical theory of sound absorption by electrons on open trajectories was constructed by Kaner, Peshanskii, and Privorotskii[4]. Our purpose is to construct a theory of quantum oscillations, which can take place upon absorption of sound by electrons on open periodic trajectories[1)], similar to Fig. 1a. More accurately, the effects considered by us can be observed if the absorption of sound is due to trajectories located in the vicinity of the trajectory with self-intersection: closed trajectories coming close to the limit of the reciprocal-lattice cell (Fig. 1b), and open trajectories with a narrow "neck." A study of such giant oscillations can yield valuable information concerning the structure of the Fermi surface. In particular, it is possible to determine directly the diameter of the "neck" (Fig. 1a) joining the parts of the Fermi surface lying in neighboring cells of the reciprocal lattice.

1)Such trajectories arise in uniaxial metals with a Fermi surface of the "corrugated cylinder" type or in cubic metals with Fermi surface of the copper type, if the magnetic field is directed, for example, along [110].

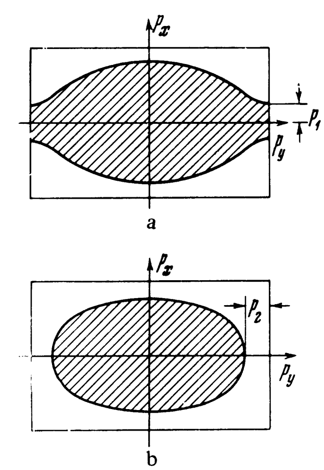

FIG. 1. Trajectories responsible for sound absorption: a – open, b – closed.

1. We consider first qualitatively the question of the origin of the giant oscillations.

At low temperatures, when the electron mean free path is sufficiently large, the absorption of the sound can be regarded as direct absorption of ultrasonic phonons by conduction electrons of the metal. In sufficiently strong magnetic fields, such an absorption occurs without a change in the Landau quantum number n. The sound absorption is then due not to all the electrons, but only to those whose average velocity $\tilde{v}$ in the sound propagation direction is equal to the velocity of the sound w

$$\tilde{v}\cos\theta = w; \tag{1}$$

here θ is the angle between the sound propagation direction and the average electron velocity (in the case of closed trajectories, the average velocity is parallel to the direction of the magnetic field).

Indeed, using the energy and momentum conservation laws in electron-phonon collisions, we can write[2)]

$$\begin{aligned}\varepsilon_n(p_z+\hbar q_z) &\equiv \varepsilon_n(p_z)+\hbar v_z q_z\\ &= \varepsilon_n(p_z)+\hbar\omega_q,\end{aligned} \tag{2}$$

where $v_z = \tilde{v}$, p_z is the projection of the quasimomentum of the electron in the direction of the magnetic field **H**, and $q_z = q \cos\theta$. We see that Eq. (1) follows from Eq. (2). The value of p_z satisfying (1) will be denoted by p_z^0.

On the other hand, electrons situated in the region of the smearing of the Fermi distribution, the width of which is of the order of kT (k—Boltzmann constant), can take part in the absorption. If

$$\hbar\Omega \gg kT, \tag{3}$$

where Ω is the frequency of revolution of the electron in the magnetic field, then intervals of allowed and forbidden values of p_z exist in the region of the smearing of the Fermi distribution.

In order to understand this, let us draw the $\epsilon_n(p_z)$ dependence for several successive values of n that are closest to the Fermi level (Fig. 2). The lines $\epsilon_n(p_z)$ intersect a strip of width kT, located at the level of the chemical potential ζ. The width of the strip is much smaller than the distance between lines, in agreement with condition (3). The intervals of the allowed values of p_z, i.e., of those values corresponding to the region of the smearing of the Fermi distribution, are shown in the figure by thick segments. The position of such segments depends on the values of the magnetic field, since the distance between the lines in the figure changes with variation of the field.

If the value of H is such that the momentum p_z^0 falls in the interval of allowed values of p_z, then the sound is strongly absorbed by the free electrons, i.e., giant oscillations take place. At another value of H, the momentum p_z^0 may fall in the interval of forbidden values of p_z, and then there is no absorption of sound by the free carriers.

In the case of an open Fermi surface, at definite directions of the magnetic field **H** and of the wave vector of the sound **q**, the z projection of the electronic quasimomentum p_z^0, determined from the energy and momentum conservation laws, can correspond to an open trajectory, i.e., electrons on the open trajectory are responsible for the absorption of the sound.

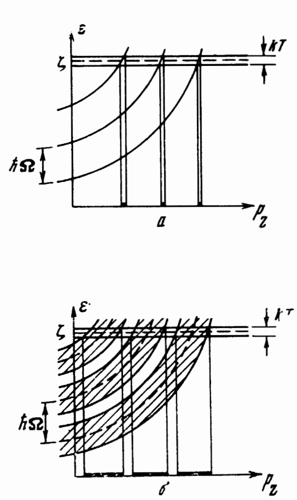

FIG. 2. Dependence of the energy on p_z: a – case closed trajectories, b – case of open trajectories.

[2)]Here q_z is assumed small compared with $p_z/\hbar$ and ω is the frequency of sound.

Since, as we have seen, the oscillation picture is determined by the energy spectrum of the conduction electrons, let us see how the spectrum changes, in the quasiclassical approximation, on going over to open trajectories. If the closed trajectories pass close to the boundaries of the reciprocal-lattice cell, a probability of "tunneling" of the electron from one trajectory to the other appears. When the "tunneling" probability is negligibly small, the given values of n and p_z correspond to one definite energy level, namely the Landau level.

On the other hand, if the "tunneling" probability is not too small, the given values of n and p_z correspond to an entire energy interval: the Landau level broadens into a narrow band. For those trajectories that reach the boundaries of the reciprocal-lattice cell, i.e., become open, the width of these bands becomes large—comparable with the distance between bands. These bands will be represented on the diagram not by lines but by strips of finite width (see Fig. 2b, the shaded strips). The thicker the "neck" joining the open-trajectory sections located in neighboring reciprocal-lattice cells, the wider these strips and the smaller the distance between their edges, i.e., the width of the "forbidden band."

Such a broadening of the Landau levels causes the dimensions of the allowed intervals p_z corresponding to the electronic states in the interval of the temperature smearing of the Fermi distribution to increase. If the width exceeds kT, the magnitude of these intervals is determined only by the form of the energy spectrum, and not by the thermal smearing of the Fermi distribution as in the case of closed trajectories.

Let us consider now the absorption of sound by electrons with such a spectrum. In our qualitative analysis we confine ourselves to the most interesting case, when the transitions occur within the limits of a single magnetic band. We shall consider below the general case.

It is seen from Fig. 2b that if the value of p_z^0 determined by the energy and momentum conservation laws falls in the allowed interval of values of p_z, there is strong absorption. In the opposite case there is no absorption of sound by the free carriers.

Thus, on going over to open trajectories, the peaks of the absorption broaden, and their width practically ceases to depend on the temperature. If the forbidden bands are sufficiently narrow, it is possible to observe giant oscillations of transparency when deep and narrow dips become superimposed on the smooth dependence of the absorption coefficient on the magnetic field.

The oscillation picture in the case of open trajectories also has a periodicity in 1/H, with a period equal to

$$\Delta(1/H) = 2\pi eH/cS, \tag{4}$$

where $S(\zeta, p_z^0)$ is the area bounded by the open trajectory and by the limits of the reciprocal-lattice cell in the case of open trajectories, or the area bounded by the trajectory in the case of closed trajectories (the shaded areas in Figs. 1a and b).

We note one more characteristic feature of the absorption of sound in the case of open trajectories. Unlike the case of closed trajectories, the average electron velocity $\tilde{v}$ turns out to be not parallel to the direction of the magnetic field, namely, a nonzero component of the average velocity along the x axis appears (we assume that in p-space the trajectories are open in the direction of p_y). Thus, we have in place of condition (1), in the case of open trajectories,[3]

$$\tilde{v}_z q_z + \tilde{v}_x q_x = \omega. \qquad (5)$$

It is seen from condition (5) that by choosing the direction of propagation of the sound it is possible to select the trajectories responsible for the absorption of the sound, and to change over from closed to open trajectories. From the change of the oscillation picture occurring in this case it is possible to assess the thickness of the "neck" joining those parts of the Fermi surface which lie in neighboring reciprocal-lattice cells.

2. Let us proceed to calculate the absorption coefficient. In the case of periodic open trajectories (see, for example,[5]), the energy spectrum in the quasi-classical approximation ($n \gg 1$) is determined by the following expression ($\mathbf{A} = (-H_y, 0, 0)$, $\mathbf{H} \parallel z$)[6]:

$$\varepsilon_{n,\sigma}(p_x, p_z) = \overset{0}{\varepsilon}_{n,\sigma}(p_z) + (-1)^{n+1}\frac{\hbar\Omega}{\pi}\arcsin\left(\frac{1}{A}\cos\pi\frac{p_x}{p_0}\right). \qquad (6)$$

We shall not consider here transitions with spin flip.

In the case of open trajectories, $A(\epsilon, p_z)$ is determined by the expression

$$A(\varepsilon, p_z) = \left[1 + \exp\left(-\pi\frac{p_1^2 a_H^2}{\hbar^2}\sqrt{\frac{\rho_p}{p_1}}\right)\right]^{1/2}, \qquad (7)$$

and in the case of closed trajectories by the expression

$$A(\varepsilon, p_z) = \left[1 + \exp\left(\pi\frac{p_2^2 a_H^2}{\hbar^2}\sqrt{\frac{\rho_p}{p_2}}\right)\right]^{1/2} \qquad (8)$$

Here ρ_p is the radius of curvature of the trajectory near the cell boundary (in the case of closed trajectories—at the classical turning point), and the symbols p_1 and p_2 are defined in Figs. 1a and b. The quantum number p_x/p_0 plays the role of the quasimomentum in a periodic field. The quantity

$$p_0 = \hbar a_y / 2a_H^2 \qquad (9)$$

plays the role of the boundary momentum of the magnetic band, a_y is the period of the lattice in the y direction, and $a_H = (c\hbar/eH)^{1/2}$.

It is seen from expression (6) that the spectrum is an aggregate of bands made up of the Landau levels, and the centers of the bands are determined by the condition

$$a_H^2 S(\varepsilon^\pm, p_z) / 2\hbar^2 = \pi(n + \gamma), \quad \varepsilon^\pm = \varepsilon + \sigma\mu H. \qquad (10)$$

Here γ is a constant of the order of unity, $\sigma = \pm 1$, μ is the effective magneton describing the spin splitting of the energy levels.

Inside the n-th band, the value of A depends little on the energy. In formula (6), Ω is the frequency characterizing the periodicity of the motion of electron over the open trajectory, and is equal to[4]

$$\Omega = \frac{eH}{m^* c}, \quad m^* = \frac{1}{2\pi}\left(\frac{\partial S}{\partial \varepsilon}\right)_{p_z}. \qquad (11)$$

Here m* plays the role of the effective mass when the electron moves on an open trajectory, and has a continuous dependence on the energy inside the band.

We note that in the limiting case of a narrow "neck," the quantity in the argument of the exponential in (7) tends to 0, and the forbidden and allowed intervals turn out to be equal in width. We arrive at a similar limit by letting p_z in expression (8) go to 0, i.e., for closely passing closed trajectories. Since the width of the bands changes strongly when the thickness of the "neck" changes (or when p_1 changes), it is most convenient to work in the region near a trajectory with self-intersection.

The absorption coefficient Γ, neglecting the interaction of the electrons with the scatterers, is expressed by the following formula:

$$\Gamma = \frac{\pi}{\rho V_0 u_0^2 w}\sum_{\alpha\alpha'}\frac{\partial F}{\partial \varepsilon}|\langle\alpha'|U|\alpha\rangle|^2\delta\left(\frac{\varepsilon_{\alpha'} - \varepsilon_\alpha}{\hbar} - \omega\right). \qquad (12)$$

Here ρ is the density of the crystal, V_0 is its volume, u_0 is the amplitude of the sound, w is its phase velocity, α is the aggregate of quantum numbers of the electron in the magnetic field, and

$$F(\varepsilon) = \left[1 + \exp\left(\frac{\varepsilon - \zeta}{kT}\right)\right]^{-1}, \quad U = \exp(i\mathbf{qr})\Lambda_{ik}u_{ik}^0, \qquad (13)$$

where $\Lambda_{ik}(\hat{\mathbf{p}} - e\mathbf{A}/c)$ is the tensor of the deformation potential, and $\overset{0}{u}_{ik}$ is the maximum value of the deformation tensor in the sound wave.

Using the procedure of[2], we obtain

$$\Gamma = \frac{eH}{16\pi\rho u_0^2 wkTc\hbar^2}\sum_\sigma\frac{1}{2p_0}\int_{-p_0}^{p_0}dp_x\int dp_z\sum_{n,l}|I_{n+l,n}|^2 \times \mathrm{ch}^{-2}\frac{\zeta - \varepsilon_{n,\sigma}(p_x, p_z)}{2kT}\delta(\omega' - \omega), \qquad (14)$$

where

$$\omega' = \hbar^{-1}[\varepsilon_{n+l}(p_x + \hbar q_x, p_z + \hbar q_z) - \varepsilon_n(p_x, p_z)]; \qquad (15)$$

$I_{n+l,n}$ is shorthand for the matrix element

$$\langle n + l, p_z + \hbar q_z, p_x + \hbar q_x|\Lambda_{ik}u_{ik}^0 e^{i\mathbf{qr}}|n, p_z, p_x\rangle.$$

The expression for this matrix element has the simplest form in two limiting cases: when the width of the Landau band $\Delta\epsilon$ is much smaller than the distance between the centers of the neighboring bands (case of closed trajectories), or when the distance between the edges of the neighboring bands is much smaller than $\hbar\Omega$ (case of open trajectories):

$$|I_{n+l,n}| = \left|\frac{1}{T_{\text{кл}}}\int_0^{T_{\text{кл}}}dt\,\Lambda_{ik}u_{ik}^0\exp\left[i\mathbf{q}\int_0^t(\mathbf{v} - \tilde{\mathbf{v}})\,dt' - il\Omega t\right]\right|. \qquad (16)$$

In this expression, the integral is calculated along a classical trajectory, and $\tilde{v}$ is the average velocity on the trajectory[5]. To analyze the form of the oscillation curves, it is usually unnecessary to know the exact

[3]It should be noted that the condition determines generally speaking, in the case of open trajectories, not the discrete values of p_z, but entire intervals of such values. This is discussed in more detail later.

[4]The frequency Ω is connected with the period of the classical motion of the electron over the open trajectory by the relation $\Omega T_{cl} = \pi$.

[5]It must be indicated that in the case of open trajectories this expression vanishes for odd l. This corresponds to the impossibility of classical motion between the upper and lower sections of the trajectory (see Fig. 1a).

expressions for these matrix elements, since the latter represent smooth functions of the magnetic field. From the point of view of present-day experimental possibilities, greatest interest attaches to the case $\mathbf{q}\cdot\tilde{\mathbf{v}} \ll \Omega$. In this case the expression for the coefficient Γ contains only the matrix elements I_{nn}, which are all of the order of magnitude $\Lambda_{ik}u_{ik}^{0}$.

Let us consider expression (14). Let the width of the forbidden sections of the spectrum be large compared with the temperature smearing. For each individual term in the sum (14) we have after integrating with respect to p_z and taking (15) into account

$$\Gamma_{nl\sigma} \sim \int dp_x \frac{|I_{n+l,\,n}|^2}{|\Delta v_z|}\operatorname{ch}^{-2}\frac{\zeta-\varepsilon_{n,\sigma}(p_z{}^{nl}(p_x),p_x)}{2kT}. \tag{17}$$

Here, just as in[2]

$$\Delta v_z = \hbar q_z(\partial\tilde{v}_z/\partial p_z)_n. \tag{18}$$

We shall assume below that the relation $\hbar q \ll p_0$ is satisfied. The function $p_z^{nl}(p_x)$ is the solution of the equation $\omega' - \omega = 0$.

The integrand in (17) contains the product of the smooth function $|I_{n+l,n}|^2/|\Delta v_z|$ and the function $\cosh^{-2}[(\epsilon-\zeta)/2kT]$, which has a sharp maximum at $\epsilon = \zeta$ when $\zeta \gg \hbar\Omega \gg kT$. The main contribution to the integral (17) is made by values of p_x for which the condition $|\zeta - \epsilon| \lesssim kT$ is satisfied. Such values always exist if the Fermi level lies inside the n-th Landau band. In this case the absorption coefficient is maximal and giant oscillations take place. But if the Fermi level falls in the forbidden energy interval, then the coefficient $\Gamma_{nl\sigma}$ is exponentially small.

When the magnetic field changes, the mutual placement of the Fermi level and the Landau bands changes. However, the absorption remains large, so long as the Fermi level is inside the allowed band. Since the electron energy depends on p_x, the conditions $\omega' - \omega = 0$ and $|\zeta - \epsilon| \lesssim kT$, which determine the giant oscillation, can be satisfied only for electrons with definite p_x (in contrast to the case of closed trajectories, when these conditions are satisfied simultaneously for electrons with all p_x). Taking all the continuous functions outside the integral sign with respect to p_x, we have

$$\Gamma_{l\sigma}^{max} = \frac{eH|I_l|^2}{16\pi\rho u_0{}^2 kTc\hbar w|\Delta v_z|}$$
$$\times\frac{1}{2p_0}\int_{-p_0}^{p_0} dp_x \sum_n \operatorname{ch}^{-2}\frac{\zeta-\varepsilon_{n,\sigma}(p_z^{nl}(p_x),p_x)}{2kT}. \tag{19}$$

The second factor in this formula, which depends little on the magnetic field (if the Fermi level falls in the allowed zone), shows what part of the electronic states contributes to the absorption. It is of the order of $kT/\Delta\epsilon$ if $\Delta\epsilon \gg kT$, and of the order of unity if $\Delta\epsilon \lesssim kT$.

Let us analyze in greater detail the conditions that determine the giant oscillation for the spectrum (6). Let $q_x = 0$, and then the condition $\omega' - \omega = 0$ is rewritten in the form

$$g(p_z) = \frac{eH}{cq_z}\left\{l + \frac{(-1)^n[1-(-1)^l]}{\operatorname{sign} l}\frac{2}{\pi}\arcsin\left(\frac{1}{A}\cos\pi\frac{p_x}{p_0}\right)\right\}, \tag{20}$$

where

$$g(p_z) = \frac{1}{2\pi}\left[\left(\frac{\partial S}{\partial p_z}\right)_\varepsilon + \frac{w}{\cos\theta}\left(\frac{\partial S}{\partial\varepsilon}\right)_{p_z}\right]. \tag{21}$$

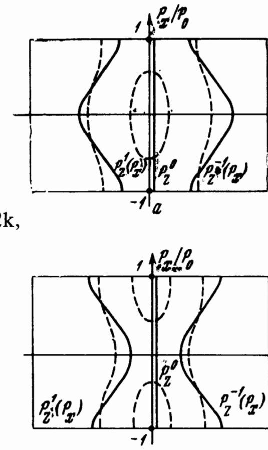

FIG. 3. Plots of $p_z^{nl}(p_x)$: a – n = 2k, b – n = 2k + 1.

Equation (21) determines the family of the functions $p_z^{nl}(p_x)$ (Fig. 3). The dashed lines show the solutions of the equation $\epsilon_n(p_z, p_x) = \zeta$ for different values of the magnetic field. The absorption is determined by the regions near the points of intersection of the solid and dashed curves. If there are no such points, there is no direct absorption of sound by the electrons.

It is easy to write out the conditions that determine the boundaries of the maxima, and also to calculate the dependence of the absorption coefficient on the magnetic field (when $l = 0$, $\hbar\Omega \gg kT$, $A - 1 \ll 1$):

$$\Gamma_{0,\sigma} = \frac{eH|I_0|^2}{16\pi c\hbar\rho u_0{}^2 wkT|\Delta v_z|}\,\frac{\operatorname{sh}(\Delta\varepsilon/2kT)}{\Delta\varepsilon/2kT}$$
$$\times\sum_n \operatorname{ch}^{-1}\left[\frac{\zeta-\overset{0}{\varepsilon}_{n,\sigma}(p_z{}^0)+{}^1/_2\Delta\varepsilon}{2kT}\right]\operatorname{ch}^{-1}\left[\frac{\zeta-\overset{0}{\varepsilon}_{n,\sigma}(p_z{}^0)-{}^1/_2\Delta\varepsilon}{2kT}\right]. \tag{22}$$

It is easy to see that when $\Delta\epsilon \ll kT$ formula (22) goes over into an expression that is valid in the case of closed trajectories[1,2]. Thus, expression (22) is a good interpolation formula for a transition from one case to another.

Let us consider now the case when $q_x \neq 0$. Let $\hbar q_x \ll p_0$, and then we can write ($l = 0$)

$$q_z\tilde{v}_z + q_x\tilde{v}_x = \omega.$$

Here

$$\tilde{v}_x = v_{\text{cl}}\frac{\sin(p_x/p_0)}{[(A^2-1)+\sin^2(p_x/p_0)]^{1/2}}, \tag{23}$$

$v_{cl} = \hbar\Omega/p_0$ is the classical velocity of the electron on the branch of the open trajectory. Thus, we see that by varying the direction of propagation of the sound we can select the trajectory responsible for the absorption.

In the case of more complicated trajectories, for example those considered by Azbel'[7], the energy spectrum has a much more complicated form than (6). Our analysis does not pertain to this case.

An interesting change in the oscillation picture may occur as the result of interband magnetic breakdown, which is possible when the interband energy gap is small. Thus, Pippard[8] has shown that a set of forbidden gaps is superimposed on the set of allowed bands in the energy spectrum. Consequently, dips are superimposed on the oscillation maxima, and their period in 1/H is determined by the area of the closed orbit in the upper band passing near the boundary of the reciprocal-lattice cell.

In conclusion, let us consider the question of the possibility of observing giant oscillations on open trajectories.

It is easy to show that the criteria for the resolvability of the oscillation maxima are similar to those obtained in[1,2] (L—mean free path of the conduction electrons):

$$qL(\Delta\tilde{\varepsilon}/\zeta)^{1/2} \gg 1, \quad l=0,$$
$$qL(\Delta\tilde{\varepsilon}/\zeta) \gg 1, \quad l\neq 0. \tag{24}$$

Here $\Delta\tilde{\epsilon}$ is the width of the forbidden section in the energy spectrum (if the absorption of the sound is due to a trajectory located near the trajectory with self-intersection, then $\Delta\tilde{\epsilon} \sim \hbar\Omega$).

It should be noted, however, that giant absorption oscillations on closed trajectories are usually observed for anomalously small electron groups (or else for normal groups at very small effective mass). It is much more difficult to observe giant oscillations of absorption on normal electron groups, although this is possible.

Similar difficulties should arise also in the case of open trajectories, which appear only at sufficiently large electron concentrations, when the limiting Fermi momentum p_F approaches $\hbar/a$, (a—lattice constant). As a result, the distance between the maximum decreases strongly, making their resolution quite difficult. We can obtain for the period of the oscillations the following estimate:

$$\Delta H/H \sim (a/a_H)^2\beta, \tag{25}$$

where β is the ratio of the area of the reciprocal-lattice cell to the area of the intersection of the part of the Fermi surface contained in one reciprocal-lattice cell and the plane p_z = const (see Fig. 1a). When H $\sim 10^5$ Oe, we have $\Delta H \sim 2\times 10^3$ Oe.

Thus, although the conditions for observing giant oscillations on open trajectories are less favorable than in the case of closed trajectories, at the present level of the experimental technique their observation is perfectly feasible.

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