CONTRIBUTION TO THE THEORY OF DOMAIN STRUCTURES

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General properties of domain structures in magnetic and ferroelectric materials are discussed. Conditions for the coexistence of phases are found which are less restrictive than those usually assumed. It is explained how the problem of the exit of domains to the surface can be correctly stated. It is shown that near the surface of the specimen, the boundaries of ferromagnetic and ferroelectric domains may in general be curved. The distribution of magnetic field near the surface is found for a uniaxial ferromagnet with closure domains.

1. INTRODUCTION

IT is well known that in ferromagnets a domain structure can be formed: that is, a stratification of the specimen into regions of coexisting phases with different values of the magnetic moment **M** (see, for example, [1]). A similar phenomenon occurs in ferroelectrics, and also in superconductors (the intermediate state). Recently the existence of domain structure has been detected in nonferromagnetic metals under the conditions of the de Haas–van Alphen effect.[2] The simplest domain structures are those in ellipsoidal bodies, in thin plane-parallel plates, and in cylindrical specimens of elliptic cross section. If the external field is uniform, the domains in specimens of the shape mentioned form a system of periodically alternating, plane-parallel layers, whose thickness is small in comparison with the dimensions of the specimen. After averaging over the domain structure, the values of the magnetic field **H** and induction **B** (or of the electric field **E** and induction **D**) inside such a specimen are uniform. These quantities are connected with the external field $\mathbf{H}_0$ (or $\mathbf{E}_0$) by the relations

$$H_{0i} = (\delta_{ik} - n_{ik})\langle H_k\rangle + n_{ik}\langle B_k\rangle, \quad (1)$$
$$E_{0i} = (\delta_{ik} - n_{ik})\langle E_k\rangle + n_{ik}\langle D_k\rangle. \quad (1')$$

Here the angular brackets denote an average over the domain structure, and n_{ik} is the tensor of demagnetizing (depolarizing) coefficients. The stratification into domains leads to a decrease of the thermodynamic potential; the decrease is proportional to the volume of the specimen.

The problem of the domain shape for thermodynamic equilibrium was first considered in the well-known work of Landau and Lifshitz.[3] They showed, in particular, that the dimensions of the plane-parallel layers are determined by the condition that the sum of two energies be a minimum: the energy of surface tension on the boundaries separating the phases, and the energy of exit of the domains to the surface. The latter arises because of distortion of the domain structure near the surface of the specimen, at distances of the order of the domain width. In the case of a plane-parallel plate of thickness l, the energy of exit of the domains to the surface, per unit area of the plate, is proportional to the domain width a and independent of l, whereas the surface-tension energy is proportional to l/a. Therefore the domain width is proportional to $\sqrt{l}$.

In the following sections, the general properties of domain structures are discussed, and in particular the conditions for coexistence of phases.

At a boundary between phases there must be continuity of the tangential component of the magnetic field $\mathbf{H}_t$, of the normal component of the magnetic induction B_n, and of the thermodynamic potential (see Sec. 2)

$$\Phi'(\mathbf{H}_t, B_n) = -\frac{1}{4\pi}\int_0^{\mathbf{H}} \mathbf{B}\,d\mathbf{H} + \frac{1}{4\pi} H_n B_n.$$

It has hitherto been supposed that at the boundaries separating phases, there was continuity not only of $\mathbf{H}_t$ but also of H_n.[1)] Actually, the complete set of boundary conditions ($\mathbf{H}_t$ = const, B_n = const, $\Phi'(\mathbf{H}_t, B_n)$ = const) permits a discontinuity of H_n at the surface of separation.

Very important is the fact that far within a specimen of ellipsoidal form, in an arbitrary uniform external field $\mathbf{H}_0$ (and for $\overline{\mathbf{H}}_0 = 0$ even in a specimen of arbitrary form), there is continuity not only of $\mathbf{H}_t$ and of B_n, but also of H_n, so that coexisting phases have equal thermodynamic potentials

$$\tilde{\Phi} = -\frac{1}{4\pi}\int_0^{\mathbf{H}} \mathbf{B}\,d\mathbf{H}.$$

This result is not new, but it has been obtained with allowance for the possibility of a discontinuity of H_n, from the condition of minimization of the total thermodynamic potential

$$\tilde{\Phi} = -\frac{1}{4\pi}\int d^3x \int_0^{\mathbf{H}(\mathbf{x})} \mathbf{B}\,d\mathbf{H}$$

(see Sec. 3). A similar situation occurs in ferroelectrics.

The possibility of a discontinuity of H_n is important in the problem of exit of domains to the surface and, in particular, in the theory of the so-called closure do-

[1)] In the theory of domain structures, the condition of continuity of H_n is often formulated as the condition of absence of surface magnetic charges. The term "magnetic charge" is obsolete and long out of use in other areas of physics. We also shall not use it.

mains in a uniaxial ferromagnet (see Sec. 4). The usually used conditions of magnetic-flux closure are incompatible with the thermodynamic boundary condition $\Phi_1'(H_t, B_n) = \Phi_2'(H_t, B_n)$. Actually the flux is not completely closed, and near the surface of the specimen there appears a magnetic field $\mathbf{H} \neq 0$, which orients the magnetization $\mathbf{M}$ in the closure domains almost perpendicular to the easy axis. It is shown also that near the surface of the specimen, the boundaries of the ferromagnetic and ferroelectric domains may in general be curved.

2. CONDITIONS FOR COEXISTENCE OF PHASES

As an illustration, we shall consider the model of a uniaxial ferromagnet described in the book of Landau and Lifshitz.[1] In this model, the thermodynamic potential of unit volume of the ferromagnet is

$$\tilde{\Phi} = \Phi_0(\mathbf{M}) - \frac{1}{4\pi}\int_0^{\mathbf{H}} \mathbf{B}\,d\mathbf{H} = \Phi_0(\mathbf{M}) - \mathbf{M}\mathbf{H} - \frac{H^2}{8\pi}. \qquad (2)$$

The integral in formula (2) is calculated at a fixed value of the magnetic moment $\mathbf{M}$, which must then be found by minimization of $\tilde{\Phi}$ at given $\mathbf{H}$, that is from the equation $(\partial\tilde{\Phi}/\partial\mathbf{M})_{\mathbf{H}} = 0$. The thermodynamic potential thus determined possesses the necessary property

$$\frac{\partial\tilde{\Phi}}{\partial\mathbf{H}} = -\frac{\mathbf{B}}{4\pi} = -\frac{\mathbf{H}}{4\pi} - \mathbf{M}(\mathbf{H}), \qquad (3)$$

where the differentiation is carried out with allowance for the dependence of $\mathbf{M}$ on $\mathbf{H}$.

The quantity $\Phi_0(\mathbf{M})$ is basically of exchange origin and in first approximation is isotropic. Anisotropy occurs only when relativistic interactions are taken into account:

$$\Phi_0(\mathbf{M}) = \Phi_0(|\mathbf{M}|) + U_{an}. \qquad (4)$$

The magnetic-anisotropy energy in the model considered is

$$U_{an} = {}^1/_2\beta M^2 \sin^2\varphi, \qquad (5)$$

where φ is the angle of inclination of the magnetic moment to the axis of easy magnetization (the z axis; $M_z = M\cos\varphi$). In the plane perpendicular to this axis, there is no anisotropy in the present approximation.

The relativistic origin of the anisotropy energy expresses itself in the fact that it is proportional to M^2. The constant β is in general by no means small. It can, in particular, be much greater than unity. The contrary case is also possible. The absolute value M of the magnetic moment may be considered invariant. In this case $\Phi_0(|\mathbf{M}|)$ is a constant, unimportant in the thermodynamics, and we shall disregard it hereafter.

On minimizing $\tilde{\Phi}$ at given $\mathbf{H}$, we obtain the equation that determines the orientation of the magnetic moment:

$$\beta M \sin\varphi\cos\varphi = -H_z\sin\varphi + H_x\cos\varphi; \qquad (6)$$

the magnetic field $\mathbf{H}$ is located in the xz plane.

In the case $H_x^{2/3} + H_z^{2/3} < (\beta M)^{2/3}$, the thermodynamic potential $\tilde{\Phi}$ as a function of the angle φ has two minima, one of which corresponds to an absolutely stable and the other to a metastable state. Thus in this case two different values of $\mathbf{M}$ are possible for the same $\mathbf{H}$. In the range $H_x^{2/3} + H_z^{2/3} > (\beta M)^{2/3}$, metastable states are impossible, and consequently the direction of the magnetic moment $\mathbf{M}$ at given $\mathbf{H}$ is determined uniquely.

If the magnetic field $\mathbf{H}$ is perpendicular to the axis of easy magnetization ($H_z = 0$) and less in absolute value than βM, then two equally stable states are possible ($M_{z1} = -M_{z2}$; $M_{x1} = M_{x2}$; $M_{y1} = M_{y2}$), with identical thermodynamic potentials $\tilde{\Phi}_1 = \tilde{\Phi}_2$. This means that in a field perpendicular to the axis of easy magnetization, coexistence of phases is possible. The boundary separating the phases must then be parallel to the axis of easy magnetization (this follows from the conditions of continuity of the components of the magnetic field $\mathbf{H}$ tangential to the separating boundary and of the normal component of the magnetic induction $\mathbf{B}$). The orientation of the separation boundary in the plane perpendicular to the axis of easy magnetization can be arbitrary and is in no way related to the orientation of the magnetic field $\mathbf{H}$.

Thus on the boundary separating the phases, besides the electrodynamic conditions $H_t = \text{const}$ and $B_n = \text{const}$, a thermodynamic condition for coexistence of phases must be satisfied. In the case of phase coexistence considered above, the complete system of boundary conditions can be written in the following form:

$$H_{t1} = H_{t2}, \quad B_{n1} = B_{n2}, \quad \tilde{\Phi}_1 = \tilde{\Phi}_2. \qquad (7)$$

Instead of the conditions $H_{n1} = H_{n2}$ and $\tilde{\Phi}_1 = \tilde{\Phi}_2$, it is possible to require that the boundary separating the phases shall be parallel to the axis of easy magnetization, and the magnetic field $\mathbf{H}$ perpendicular to this axis. These two conditions follow from the system of equations (7). On the other hand, they together with the electrodynamic conditions $H_{t1} = H_{t2}$ and $B_{n1} = B_{n2}$ are equivalent to the system (7). The system (7) contains five relations, of which only three are purely electrodynamic. The two additional conditions ($H_{n1} = H_{n2}$ and $\tilde{\Phi}_1 = \tilde{\Phi}_2$) are extremely restrictive, so that the problem of the stratification of a specimen of arbitrary form into domains in general has no solution, if we require that the relations (7) shall be satisfied on the separation boundaries. We shall elucidate this by the following example.

If we fix the position of the boundaries, it is possible to solve the problem of the distribution of the magnetic field in a system of contiguous magnets, if we do not require fulfillment on the boundary of any boundary conditions other than the electrodynamic. In order that the two additional conditions may be satisfied on the boundaries, it is necessary that the function $z = z(x, y)$ determining the position of the boundaries shall satisfy the two equations

$$H_{n1}(x, y, z(x, y)) = H_{n2}(x, y, z(x, y)),$$
$$\tilde{\Phi}_1(x, y, z(x, y)) = \tilde{\Phi}_2(x, y, z(x, y)), \qquad (8)$$

which is in general impossible. In the case of superconductors, by an analogous method, only the single equation

$$H(x, y, z(x, y)) = H_c = \text{const}$$

is obtained.

A solution of this problem, satisfying the five boundary conditions (7), exists in the case of a plane-parallel ferromagnetic plate cut perpendicular to the axis of easy magnetization, in the absence of an external field ($\mathbf{H} = 0$).[1] The existence of a solution in this case is

accidental; one of the boundary conditions, namely the parallelism of the separation boundary to the axis of easy magnetization, is fulfilled because of the symmetry of the problem.

Such a situation (absence of a solution satisfying the five boundary conditions (7)) has already been encountered in the theory of domain structure of nonferromagnetic metals.[4] It was established that actually, on the boundaries separating the phases, besides the three electrodynamic boundary conditions, only one thermodynamic condition needs to be satisfied, and not two, as in (7). The same is true in ferromagnets also. The derivation presented below of the condition for coexistence of phases is not dependent on the specific nature of the ferromagnet and is valid also for other cases of coexistence of magnetic phases, for example for nonferromagnetic metals.

The continuity of $\mathbf{H}_t$ and B_n on the boundary separating the phases plays the same role in our case as does the equality of the temperature and pressure in a liquid-vapor system. It is natural that equilibrium should require equality of thermodynamic potentials in the variables $\mathbf{H}_t$ and B_n. Such a thermodynamic potential is the potential

$$\Phi' = \tilde{\Phi} + \frac{1}{4\pi} H_n B_n. \tag{9}$$

For given $\mathbf{H}_t$ and B_n, it has a minimum at equilibrium.

Since we are not taking account of magnetostriction, we may speak of equality of the thermodynamic potentials of unit volume rather than of unit mass. Thus the condition for phase equilibrium has the form

$$\Phi_1'(\mathbf{H}_t, B_n) = \Phi_2'(\mathbf{H}_t, B_n), \quad \Phi' = -\frac{1}{4\pi}\int_0^{\mathbf{H}} \mathbf{B}\,d\mathbf{H} + \frac{H_n B_n}{4\pi}. \tag{10}$$

The equality $\Phi_1' = \Phi_2'$ means that the boundary is in a position of neutral equilibrium with respect to a shift in the direction parallel to it. Such an equilibrium condition was obtained earlier for nonferromagnetic metals.[4] For ferroelectrics, the analogous condition has the form

$$\int_0^{\mathbf{E}} \mathbf{D}\,d\mathbf{E} - D_n E_n = \text{const}. \tag{11}$$

In the case of coexistence of the superconducting and normal phases, the boundary condition obtained gives nothing new, since in the superconducting phase $\mathbf{B} = 0$. The problem of the intermediate state differs essentially from other problems of the theory of domain structures. In the superconducting phase it is generally convenient not to introduce the vector $\mathbf{H}$ (see [1]). With such a mode of description, boundary conditions are imposed only in the normal phase, in which $\mathbf{H} = \mathbf{B}$. On the boundary with the superconductor, $H_n = 0$ (electrodynamic condition) and $H = H_c = \text{const}$ (thermodynamic condition). The problem of the intermediate state was solved by Landau (see, for example, [1]).

The equilibrium equations (10) permit, as a special case, the coexistence of phases described by the relations (7). In nonferromagnetic metals this is possible when $B_n = 0$; but in the model of a uniaxial ferromagnet considered above, it is possible only when the boundary separating the phases is parallel to the axis of easy magnetization.

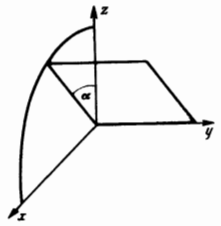

FIG. 1

We shall now discuss in more detail the conditions for coexistence of phases in ferromagnets. We shall consider first the case $\beta = \infty$. In this case, the anisotropy energy is zero, and

$$\Phi' = -M_z H_z - \frac{H^2}{8\pi} + \frac{H_n B_n}{4\pi}, \quad M_{z1} = -M_{z2} = M.$$

Let the separating boundary make an angle α with the z axis, and let the y axis be chosen in the plane of the boundary (Fig. 1).

By taking into account that

$$-\frac{H^2}{8\pi} + \frac{H_n B_n}{4\pi} = -\frac{H_t^2}{8\pi} + \frac{B_n^2 - (4\pi M_n)^2}{8\pi},$$

it is easy to see that this quantity is the same on both sides of the separating boundary. Therefore the condition $\Phi_1' = \Phi_2'$ takes the form $\mathbf{M}_{z1}\mathbf{H}_{z1} = \mathbf{M}_{z2}\mathbf{H}_{z2}$, whence it follows that

$$H_{z1} + H_{z2} = 0 \quad (\beta = \infty). \tag{12}$$

Since $\mathbf{H}_z = \mathbf{H}_{t(x,z)}\cos\alpha - (\mathbf{B}_n - 4\pi\mathbf{M}_n)\sin\alpha$, where $\mathbf{H}_{t(x,z)}$ is the projection of $\mathbf{H}_t$ on the xz plane, this condition is equivalent to the following:

$$H_{t(x,z)} = B_n \operatorname{tg}\alpha \quad (\beta = \infty). \tag{13}$$

Either of these two relations ((12) or (13)) can be used as the thermodynamic boundary condition.

Thus if the separating boundary is inclined to the axis of easy magnetization, then on this boundary $\mathbf{H}_{z1}$ and $\mathbf{H}_{z2}$ do not vanish and $\mathbf{H}_{x1} \neq \mathbf{H}_{x2}$. The converse assertion is also correct. It can be shown that in a ferromagnetic plate with $\beta = \infty$, cut perpendicular to the easy axis, in an external field $\mathbf{H}_0 \neq 0$ parallel to this axis, the domain boundaries near the surface of the specimen must curve in the same way as in superconductors (see [1]). The distortion is of the order of $\mathbf{H}_0/4\pi\mathbf{M}$ (there are no other parameters in the problem, since $\beta = \infty$).

A ferromagnet with $\beta = \infty$ has a great resemblance to ferroelectrics, in which the direction and magnitude of the polarization vector $\mathbf{P}$ do not change even in a very strong field. In a uniaxial ferroelectric, the thermodynamic boundary condition has the form

$$E_{z1} + E_{z2} = 0, \tag{12'}$$

or

$$E_{t(x,z)} = D_n \operatorname{tg}\alpha. \tag{13'}$$

The boundaries of ferroelectric domains may also curve on exit to the surface.

We shall now consider a ferromagnet with extremely small anisotropy ($\beta \to 0$), in which the magnetization $\mathbf{M}$ in the presence of a magnetic field $\mathbf{H}$ is oriented along

H. In the absence of a field, M may be directed arbitrarily; that is, not necessarily along the axis of easy magnetization. This means that actually the magnetic moment M is oriented by an infinitely small field $H \sim \beta M$. In the isotropic case

$$\Phi' = -MH - \frac{H^2}{8\pi} + \frac{H_n B_n}{4\pi}.$$

By taking into account that $B_n = H_n + 4\pi M H_n/H$, we transform Φ' to the form

$$B_n = H_n + 4\pi M H_n / H,$$

$$\Phi' = -M\frac{H_t^2}{H} + \frac{H^2}{8\pi} - \frac{H_t^2}{4\pi}.$$

On equating Φ_1' and Φ_2', we get

$$H_1 H_2 (H_1 - H_2)(H_1 + H_2) = M H_t^2 (H_2 - H_1),$$

whence it follows that $H_1 = H_2$. This equation is compatible with the conditions $H_{t1} = H_{t2}$ and $B_{n1} = B_{n2}$ only in case $H_{n1} = H_{n2}$. If H_1 and H_2 do not vanish, then the conditions obtained lead to the equality $M_1 = M_2$ and do not correspond to phase equilibrium. The latter is possible only in case

$$H_1 = H_2 = 0 \quad (\beta \to 0). \tag{14}$$

The orientation of the separating boundary may then be arbitrary. The magnetizations M_1 and M_2 may be inclined to the surface at an arbitrary angle α (Fig. 2). The inclination of M to the axis of easy magnetization means, as already indicated, that the magnetization is oriented by an infinitely small field $H \sim \beta M$.

The equilibrium condition obtained differs from that which would be obtained from (7) by setting $\beta \to 0$ only by the fact that the separation boundary can be inclined to the axis of easy magnetization.

We shall present without derivation the boundary condition for arbitrary β, but for small angles of inclination α of the boundary (see Fig. 2), obtained by taking account of terms of order α^2. It has the form

$$H_{z1} + H_{z2} = 0, \quad (\alpha \to 0), \tag{15}$$

that is, the same as in the case $\beta = \infty$ (see [12]), except that now it is correct only to the second order in α.

We shall now consider the problem of the transition layer between domains, assuming that the width of this layer is large in comparison with the distance between atoms (such a situation occurs in all cases of the coexistence of magnetic phases). It will be shown, in particular, that the problem of the transition layer has a solution only when the condition (10) for coexistence of phases is satisfied. This problem is one-dimensional, and in consequence of Maxwell's equations the values of H_t and B_n do not change in the transition layer. The values of B_t and H_n may change in the direction perpendicular to the separation boundary (along the ξ axis).

FIG. 2

Far from the separation boundary (for $\xi \to \pm\infty$), they must approach the asymptotic values $B_t^{\pm}$ and $H_n^{\pm}$. The orientation of the separation boundary with respect to the crystallographic axes we here consider arbitrary.

The thermodynamic potential in the case considered is a functional of the distribution $B_t(\xi)$ and $H_n(\xi)$:

$$\tilde{\Phi} = \int_{-\infty}^{\infty} d\xi \tilde{\Phi}\{B_t(\xi), H_n(\xi)\}. \tag{16}$$

The specific form of this functional will not be needed.

We introduce also the thermodynamic potentials

$$\Phi = \tilde{\Phi} + \frac{1}{4\pi}\int_{-\infty}^{\infty} d\xi\, \mathbf{H}(\xi)\mathbf{B}(\xi) \tag{17}$$

$$\Phi' = \tilde{\Phi} + \frac{1}{4\pi}\int_{-\infty}^{\infty} d\xi H_n(\xi) B_n(\xi). \tag{18}$$

The values of B_n and H_t are related to the potentials $\tilde{\Phi}$ and Φ as follows:

$$B_n(\xi)/4\pi = -(\delta\tilde{\Phi}/\delta H_n(\xi))_{H_t(\xi)}$$
$$H_t(\xi)/4\pi = (\delta\Phi/\delta B_t(\xi))_{B_n(\xi)}. \tag{19}$$

It is easy to see that these equations are the Euler-Lagrange equations for the functional Φ' under the additional conditions $B_n = \text{const}$ and $H_t = \text{const}$. If there is a functional relation between $B_t(\xi)$ and $H_n(\xi)$, then one of these equations is a consequence of the other two. In order that the functional Φ' may have an extremum, it is necessary that the integrand in (18) take the same values for $\xi \to \pm\infty$. Thus we again obtain the condition for coexistence of phases $\Phi_1'(H_t, B_n) = \Phi_2'(H_t, B_n)$.

The investigation made shows that the structure of the transition layer can be found by minimization of the thermodynamic potential Φ'. If phases coexist with different thermodynamic potentials $\tilde{\Phi}_1$ and $\tilde{\Phi}_2$, then the surface tension on the separating boundary is expressed in terms of the thermodynamic potential $\tilde{\Phi}$:

$$\Delta = \int_{-\infty}^{\infty} d\xi [\tilde{\Phi}\{B_t(\xi), H_n(\xi)\} - \tilde{\Phi}(\xi = \pm\infty)], \tag{20}$$

where $\tilde{\Phi}(\xi = \pm\infty) = \tilde{\Phi}_1 = \tilde{\Phi}_2$.

In ferroelectrics far from the Curie point, the change of D_t and E_n in the boundary separating the phases occurs over distances of the order of interatomic distances. In this case, the above statement of the problem of the transition layer for ferroelectrics has no meaning.

3. INTERNAL PROPERTIES OF DOMAIN STRUCTURES

If we neglect the thin surface layer in which there is a distortion of the domain structure, then in specimens of ellipsoidal form the separation boundaries between domains may be considered plane, and the orientation of all the separation boundaries the same. The volume properties of domain structures are characterized in addition by the values of H_t and B_n and the concentrations of the phases c_1 and c_2 ($c_1 + c_2 = 1$). These quantities satisfy the three equations (1), where $\langle H \rangle = c_1 H_1 + c_2 H_2$ and $\langle B \rangle = c_1 B_1 + c_2 B_2$, and the condition for coexistence of phases (10); that is, the number of equations is less by two than the number of parameters that

determine the properties of the domain structure. These parameters must be found from the condition that the total thermodynamic potential of unit volume

$$\tilde{\Phi}_n(\mathbf{H}_0) = \tilde{\Phi}_n(\mathbf{H}_0 = 0) - \int_0^{\mathbf{H}_0} \langle \mathbf{M} \rangle \, d\mathbf{H}_0. \tag{21}$$

shall be a minimum.

a) We consider first a ferromagnetic ellipsoid in the absence of an external field ($\mathbf{H}_0 = 0$). It is easily shown that if $\mathbf{H}_0 = 0$, that is if the external currents $\mathbf{j}$ vanish, then the thermodynamic potentials

$$\tilde{\Phi} = -\frac{1}{4\pi}\int d^3\mathbf{x} \int_0^{\mathbf{H}(\mathbf{x})} \mathbf{B} d\mathbf{H} \quad \text{and} \quad \Phi = \tilde{\Phi} + \frac{1}{4\pi}\int d^3\mathbf{x}\mathbf{H}(\mathbf{x})\mathbf{B}(\mathbf{x})$$

coincide, independently of the nature and geometry of the specimen. In fact, $\mathbf{H} \cdot \mathbf{B} = \operatorname{div}[\mathbf{A} \times \mathbf{H}] + \mathbf{A} \operatorname{curl} \mathbf{H}$, where $\mathbf{A}$ is the vector potential. The second term in this expression vanishes, since $\mathbf{j} = 0$, and consequently the integral of $\mathbf{H} \cdot \mathbf{B}$ reduces to the integral of a divergence.

An analogous situation occurs in ferroelectrics if $\mathbf{E}_0 = 0$; that is, if the external charges and charges on conductors vanish. In this case $\mathbf{E} \cdot \mathbf{D} = -\operatorname{div}(\varphi \mathbf{D}) + \varphi \operatorname{div} \mathbf{D}$ (φ = scalar potential), wherein $\operatorname{div} \mathbf{D} = 0$ and on the surfaces of conductors $D_n = 0$.

The thermodynamic potential Φ in a ferromagnet is equal to $U_{an} + H^2/8\pi$. It is obvious that when $\mathbf{H}_0 = 0$, the minimum of the thermodynamic potential is attained when, far inside the specimen, $\mathbf{H} = 0$, $\mathbf{M}_1 = -\mathbf{M}_2$ (then $U_{an} = 0$), and $c_1 = c_2$; that is, when the boundary conditions (10) are satisfied. This result is independent of the model.

Analogous statements are valid also for ferroelectrics.

b) We consider, further, a uniaxial ferromagnet with $\beta = \infty$ in an arbitrary external field $\mathbf{H}_0 \neq 0$. We fix the position of a boundary, which is determined by two parameters: the angle α and the direction of the x axis in the plane perpendicular to the axis of easy magnetization (see Fig. 1). From the conditions $H_{t1} = H_{t2}$ and $B_{n1} = B_{n2}$ we obtain

$$\begin{gathered} H_{y1} = H_{y2} = H_y, \\ (H_{z1} - H_{z2})\cos\alpha = (H_{x2} - H_{x1})\sin\alpha, \\ (H_{z1} - H_{z2})\sin\alpha = (H_{x1} - H_{x2})\cos\alpha - 8\pi M \sin\alpha. \end{gathered} \tag{22}$$

On taking into account that $H_{z1} + H_{z2} = 0$ (see (12)), we find

$$H_{z1} = -H_{z2} = -4\pi M \sin^2\alpha, \tag{23}$$

$$H_{x1} - H_{x2} = 4\pi M \sin 2\alpha. \tag{24}$$

It is convenient to rewrite equations (1) in the following form:

$$H_{0i} = \langle H_i \rangle + 4\pi n_{ik} \langle M_k \rangle, \tag{25}$$

or in expanded form

$$\begin{aligned} H_{0z} &= 4\pi(n_{zz} - \sin^2\alpha)\langle M_z \rangle, \\ H_{0x} &= c_1 H_{x1} + c_2 H_{x2} + 4\pi n_{xz}\langle M_z \rangle, \\ H_{0y} &= H_y + 4\pi n_{yz}\langle M_z \rangle. \end{aligned} \tag{26}$$

In the first of these relations, use has been made of the facts that $\langle H_Z \rangle = -4\pi M(c_1 - c_2)\sin^2\alpha$ and that $\langle M_Z \rangle = M(c_1 - c_2)$. For a given position of the separation boundary, a domain structure is possible only if

$$-4\pi M < \frac{H_{0z}}{n_{zz} - \sin^2\alpha} < 4\pi M. \tag{27}$$

If the external field $\mathbf{H}_0 = 0$, then $\langle M_Z \rangle = 0$, that is $c_1 = c_2 = 1/2$, and it follows from the relations (24) and (26) that

$$H_y = 0, \quad H_{x1} = -H_{x2} = 2\pi M \sin 2\alpha. \tag{28}$$

A simple calculation leads to the following formula for $\langle \tilde{\Phi} \rangle$ when $\mathbf{H}_0 = 0$:

$$\langle \tilde{\Phi} \rangle_{\mathbf{H}_0 = 0} = \tilde{\Phi}_1 = \tilde{\Phi}_2 = -M_z H_z - \frac{H^2}{8\pi} = 2\pi M^2 \sin^2\alpha. \tag{29}$$

The total thermodynamic potential at $\mathbf{H}_0 \neq 0$ is easily calculated by means of the first of the relations (26). It is

$$\tilde{\Phi}_n = 2\pi M^2 \sin^2\alpha - \frac{H_{0z}^2}{8\pi(n_{zz} - \sin^2\alpha)} \tag{30}$$

and, for given orientation of the boundaries, depends only on H_{0Z}; this is a result of the fact that in the model considered, $M_X = M_Y = 0$. The coefficient n_{ZZ} depends on the ratio of the ellipsoid semiaxes and on the orientation of the axis of easy magnetization with respect to the ellipsoid. It can vary over the range $0 < n_{ZZ} < 1$.

The total thermodynamic potential $\tilde{\Phi}_n$, considered as a function of $\sin^2\alpha$ in the region

$$|n_{zz} - \sin^2\alpha| < \frac{|H_{0z}|}{4\pi M}, \quad 0 \leqslant \sin^2\alpha \leqslant 1,$$

attains its smallest value when $\sin^2\alpha = 0$. This means that in the interior of the specimen the boundary conditions (7) are fulfilled on the boundaries that separate the phases.

Similar properties are possessed by the model of a uniaxial ferroelectric mentioned above, if the external field $\mathbf{E}_0$ is given; that is, if the charges that produce this field are given. In this case there must be a minimum of the thermodynamic potential Φ_n (see [1]):

$$\Phi_n = \langle \Phi(\mathbf{E}_0 = 0) \rangle - \int_0^{\mathbf{E}_0} \langle \mathbf{P} \rangle \, d\mathbf{E}_0,$$

$$\Phi = -\mathbf{PE} - \frac{E^2}{8\pi} + \frac{\mathbf{ED}}{4\pi} = \frac{E^2}{8\pi}.$$

We are supposing that $P_Z = \pm P = \text{const}$, so that the formulas for $\mathbf{E}$ and $\langle \mathbf{P} \rangle$ are analogous to the corresponding formulas for $\mathbf{H}$ and $\langle \mathbf{M} \rangle$. It is easy to see that the thermodynamic potential Φ_n is equal to

$$\Phi_n = 2\pi P^2 \sin^2\alpha - \frac{E_{0z}^2}{8\pi(n_{zz} - \sin^2\alpha)} \tag{30'}$$

and has a minimum at $\alpha = 0$.

c) We shall show further that in the general case also, that is in an arbitrary external field $\mathbf{H}_0$ and independently of the model, the conditions (7) for coexistence of phases require an extremum of the total thermodynamic potential $\tilde{\Phi}_n$ (we at first neglect surface effects).

We shall start from the following expression for $\tilde{\Phi}_n$, which is equivalent to (21):

$$\tilde{\Phi}_n = \langle \tilde{\Phi} \rangle + \frac{H_0^2}{8\pi} - \frac{1}{8\pi V}\int_{V'} (H^2 - H_0^2) dV'. \tag{31}$$

Here V is the volume of the specimen, and dV′ is an

element of volume of the space outside the body. The quantity $\mathbf{H} - \mathbf{H}_0$ is the field produced by the magnetic moment of the ellipsoid. The term expressed as an integral, which we shall designate by φ, of course depends only on $\mathbf{H}_0$, $\langle \mathbf{M} \rangle$, and the parameters of the ellipsoid: $\varphi = \varphi(\langle \mathbf{M} \rangle, \mathbf{H}_0)$.

The field $\mathbf{H} - \mathbf{H}_0$ decreases slowly at infinity (as $1/r^3$), and the integral of the product $\mathbf{H}_0(\mathbf{H} - \mathbf{H}_0)$ converges only as the result of an averaging over angles. It then depends on the manner of approach of the limits of integration to infinity. It would be possible to calculate $\varphi(\langle \mathbf{M} \rangle, \mathbf{H}_0)$ by giving the field $\mathbf{H}_0(\mathbf{x})$ in such a form that at infinity $\mathbf{H}_0(\mathbf{x}) \to 0$. Only such a case has physical meaning. Under these conditions, $\mathbf{H}_0(\mathbf{x})$ must satisfy the equation div $\mathbf{H}_0(\mathbf{x}) = 0$. It would be possible to calculate analogously the corresponding integral for ferroelectrics, by requiring that curl $\mathbf{E}_0(\mathbf{x}) = 0$. Of course a different dependence of the integral on the external field and the moment is obtained for magnets and for ferroelectrics (see below). We shall, however, circumvent the above-mentioned difficulty by calculating the integral by another method.

We represent the thermodynamic potential of a ferromagnet with $\beta = \infty$ in the uniform state (without domains) in the form

$$\tilde{\Phi}_n^{\text{unif}} = -\mathbf{MH}_0 - \left(\mathbf{MH} + \frac{H^2}{8\pi}\right)_{\mathbf{H}_0=0} + \varphi(\mathbf{M}, \mathbf{H}_0 = 0), \qquad (32)$$

where for $\mathbf{H}_0 = 0$ we have $\mathbf{H}_i = -4\pi n_{ik}\mathbf{M}_k$. It follows from (31) that

$$\tilde{\Phi}_n^{\text{unif}} = -\left(\mathbf{MH} + \frac{H^2}{8\pi}\right)_{\mathbf{H}_0 \neq 0} + \frac{H_0^2}{8\pi} + \varphi(\mathbf{M}, \mathbf{H}_0), \qquad (33)$$

where $\mathbf{H}_i = \mathbf{H}_{0i} - 4\pi n_{ik}\mathbf{M}_k$. On comparing (32) and (33), we get after simple calculations

$$\varphi(\mathbf{M}, \mathbf{H}_0) = -n_{ik}H_{0i}M_k + \varphi(\mathbf{M}, \mathbf{H}_0 = 0). \qquad (34)$$

We now use the fact that if in formula (26) for the thermodynamic potential of a domain structure we set $\alpha = 0$ and $\mathbf{H}_{0Z} = 4\pi \mathbf{M} n_{ZZ}$, we get the thermodynamic potential of the uniform state, $\Phi_n^{\text{unif}} = -2\pi n_{ik}\mathbf{M}_i\mathbf{M}_k$. On comparing this quantity with (32), where it is also necessary to set $\mathbf{H}_{0Z} = 4\pi \mathbf{M} n_{ZZ}$, we find $\varphi(\mathbf{M}, \mathbf{H}_0)$. Finally we get

$$\tilde{\Phi}_n = \langle \tilde{\Phi} \rangle + \frac{H_0^2}{8\pi} - n_{ik}H_{0i}\langle M_k \rangle - 2\pi(n_{ik} - n_{li}n_{lk})\langle M_i \rangle \langle M_k \rangle. \qquad (35)$$

We shall now determine, in a linear approximation, the change of $\tilde{\Phi}_n$ in the case in which the conditions for coexistence of phases differ insignificantly from the condition (7) (this means a small change of all quantities, including $\langle \mathbf{H}_i \rangle$, $\langle \mathbf{M}_i \rangle$, the orientation of the separation boundary, etc. at given $\mathbf{H}_0$). By means of the relations (25) it is easy to show that

$$\delta\varphi = -n_{ik}\langle H_i \rangle \delta\langle M_k \rangle + (\langle H_i \rangle - H_{0i})\delta\langle M_i \rangle.$$

Making use of (1), we get

$$\delta\varphi = -n_{ik}\langle B_i \rangle \delta\langle M_k \rangle.$$

It follows from (25) that

$$-4\pi n_{ik}\delta\langle M_k \rangle = \delta\langle H_i \rangle.$$

Therefore the change of $\tilde{\Phi}_n$ can be expressed in the form

$$\delta\tilde{\Phi}_n = \delta\langle \tilde{\Phi} \rangle + \frac{1}{4\pi}\langle \mathbf{B} \rangle \delta\langle \mathbf{H} \rangle = \frac{1}{4\pi}(\langle \mathbf{B} \rangle \delta\langle \mathbf{H} \rangle - \langle \mathbf{B}\delta\mathbf{H} \rangle). \qquad (36)$$

In the second term ($\langle \mathbf{B}\delta\mathbf{H} \rangle$), the average is carried out with the concentrations of the zero-order approximation, since in this approximation $\tilde{\Phi}_1 = \tilde{\Phi}_2$.

We shall denote by $\mathbf{H}_t$ and $\mathbf{H}_n$ the components of the magnetic field (and similarly for the induction) tangential and normal to the separation boundary of the zero-order approximation. Then it is obvious that $\langle B_n \delta H_n \rangle = B_n \delta\langle H_n \rangle$. On the other hand, the projections $\delta H_{t'_1}$ and $\delta H_{t'_2}$ on the new separation boundary coincide. All that differ from each other are the projections $\delta H_{n_1}'$ and $\delta H_{n_2}'$, and these may be disregarded in the calculation of δH_{t_1} and δH_{t_2}, since the angle between these separation boundaries is small. Thus in the linear approximation, $\delta H_{t_1} = \delta H_{t_2}$. Consequently, $\langle \mathbf{B}\delta\mathbf{H} \rangle = \langle \mathbf{B} \rangle \delta\langle \mathbf{H} \rangle$; that is, in the linear approximation $\delta\tilde{\Phi}_n = 0$. This means that the total thermodynamic potential has an extremum if the conditions (7) for coexistence of phases are satisfied.

We cannot show in general that this extremum is a minimum. This assertion has been tested in a few special cases. Besides the examples considered above, this property is possessed by the isotropic model of a nonferromagnetic metal in the case investigated in [4]: a plate in an external field perpendicular to it.

It is evident from formula (36) that all structures satisfying the conditions (7) have the same thermodynamic potential $\tilde{\Phi}_n$. Equations (1) and (7) allow a one-parameter family of structures. The actual orientation of the separation boundaries, like the width of the layers, is determined by the condition that the sum of the energy of exit of domains to the surface and the energy of surface tension on the boundaries separating the phases shall be a minimum.

In ferroelectrics in a given external field $\mathbf{E}_0$, there must be a minimum of the thermodynamic potential

$$\Phi_n = \frac{1}{V}\int d^3\mathbf{x}\left[\Phi(\mathbf{x}) - \frac{\mathbf{E}_0^2}{8\pi}\right] = \Phi_n(\mathbf{E}_0 = 0) - \int_0^{\mathbf{E}_0} \langle \mathbf{P} \rangle d\mathbf{E}_0, \qquad (37)$$

where

$$\Phi = -\frac{1}{4\pi}\int_0^{\mathbf{E}} \mathbf{D}d\mathbf{E} + \frac{\mathbf{ED}}{4\pi}. \qquad (38)$$

In the case of an ellipsoidal specimen, this can be reduced to a form analogous to (35).

On comparing the formulas for the thermodynamic potential of a ferroelectric with $\mathbf{P} = \text{const}$, we get

$$\left(\frac{E^2}{8\pi}\right)_{\mathbf{E}_0=0} + \varphi(\mathbf{P}, \mathbf{E}_0 = 0) - \mathbf{PE}_0 = \left(\frac{E^2}{8\pi}\right)_{\mathbf{E}_0 \neq 0} - \frac{E_0^2}{8\pi} + \varphi(\mathbf{P}, \mathbf{E}_0),$$

whence

$$\varphi(\mathbf{P}, \mathbf{E}_0) = \varphi(\mathbf{P}, \mathbf{E}_0 = 0) + n_{ik}E_{0i}P_k - \mathbf{PE}_0. \qquad (39)$$

The integral $\varphi(\mathbf{P}, \mathbf{E}_0 = 0)$ converges and is independent of the method of approaching the limit. Therefore it can be obtained from the corresponding formula for magnets by a change of sign and the substitution $\mathbf{M} \to \mathbf{P}$. We finally get

$$\Phi_n = \langle \Phi \rangle - \frac{E_0^2}{8\pi} + n_{ik}E_{0i}\langle P_k \rangle - \langle \mathbf{P} \rangle \mathbf{E}_0 + 2\pi(n_{ik} - n_{li}n_{lk})\langle P_i \rangle \langle P_k \rangle.$$

The change of Φ_n has the form

$$\delta\Phi_n = \delta c_1(\Phi_1 - \Phi_2) + \frac{1}{4\pi}(\langle \mathbf{E}\delta\mathbf{D}\rangle - \langle \mathbf{E}\rangle\,\delta\,\langle \mathbf{D}\rangle). \qquad (41)$$

Here use has been made of

$$\delta\,\langle\Phi\rangle = \delta c_1(\Phi_1 - \Phi_2) + \frac{1}{4\pi}\langle \mathbf{E}\delta\mathbf{D}\rangle.$$

If in the zero-order approximation $\mathbf{E}_1 = \mathbf{E}_2 = \mathbf{E}$ and $\widetilde{\Phi}_1 = \widetilde{\Phi}_2$, then since

$$\Phi_1 - \Phi_2 = \frac{1}{4\pi}\mathbf{E}(\mathbf{D}_1 - \mathbf{D}_2) \text{ and } \delta\mathbf{D} = \delta c_1(\mathbf{D}_1 - \mathbf{D}_2) + \langle\delta\mathbf{D}\rangle,$$

the quantity $\delta\Phi_n$ vanishes; that is, Φ_n has an extremum. Just as in magnets, in the approximation considered the orientation of the separation boundaries remains undetermined.

4. CLOSURE DOMAINS IN A UNIAXIAL FERROMAGNET

In this section, a study is made of the domain structure in a ferromagnetic plate whose plane is perpendicular to the axis of easy magnetization. For simplicity, we consider only the case $\mathbf{H}_0 = 0$. We shall consider the anisotropy of the ferromagnet to be small ($\beta \ll 1$). As is well known, in this case so-called closure domains are formed near the surface of the plate.[1,3] In the limit $\beta \to 0$, the structure near the surface has the form depicted in Fig. 3, and the magnetic field **H** vanishes both inside and outside the plate. On the boundaries of the closure domains, the conditions (10) for coexistence of phases are satisfied (see (14) and Fig. 2), but the more rigid conditions (7) are not satisfied. The latter fact is obvious, since the separation boundaries form an angle of $\pi/4$ with the axis of easy magnetization.

In [1,3] it is assumed that at sufficiently small but finite β ($\beta < \beta_k \sim 1$), this structure is unchanged, and, in particular, **H** as before vanishes, while **M** in the closure domains is perpendicular to the axis of easy magnetization. Such a state of a closure domain is absolutely unstable, since it corresponds to a maximum and not a minimum of the thermodynamic potential when **H** = 0. Furthermore, on the separation boundaries the conditions (10) for coexistence of phases are not satisfied, and the problem of the transition layer has no solution corresponding to a transition from a stable to an absolutely unstable state. From our point of view, the picture proposed in [1,3] is impossible. It will be shown below that for small but finite β, there exists in the closure domains and in adjoining regions a magnetic field **H**, which, in particular, orients the magnetic moment perpendicular to the axis of easy magnetization.

It is obvious that in the first order in β, the field **H** in the closure domains is perpendicular to the axis of easy magnetization, parallel to the magnetic moment, and equal to $\pm\beta\mathbf{M}$. On the boundaries of the triangles in the regions I and II (Fig. 3), the field is antiparallel to the magnetic moment **M** and is equal to $\pm\beta\mathbf{M}$. The

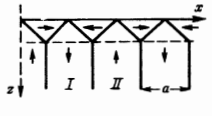

FIG. 3

latter fact follows from the condition of continuity of $\mathbf{H}_t$. We here neglect possible movement of the separation boundaries, requiring fulfillment of the boundary conditions on motionless boundaries.

Knowing the field in the closure domains, we can easily find the field outside the specimen. It must satisfy the boundary condition

$$H_x^{(e)}(x, z=0) = \begin{cases} -\beta M \text{ for } -a < x < 0 \\ \beta M \text{ for } 0 < x < a \end{cases} =$$

$$= \frac{4\beta M}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\sin\frac{(2n+1)\pi x}{a}. \qquad (42)$$

The solution of this problem has the form

$$H_x^{(e)} = \frac{4\beta M}{\pi}\sum_{n=0}^{\infty}\frac{1}{2n+1}\exp\left\{-\frac{(2n+1)\pi z}{a}\right\}\sin\frac{(2n+1)\pi x}{a},$$

$$H_z^{(e)} = -\frac{4\beta M}{\pi}\sum_{n=0}\frac{1}{2n+1}\exp\left\{-\frac{(2n+1)\pi z}{a}\right\}\cos\frac{(2n+1)\pi x}{a}. \qquad (43)$$

The presence at z = 0 of a component $H_z^{(e)}$ perpendicular to the surface leads to a small (of order β) inclination of the magnetic moment near the surface to the x axis.

We pass on to the calculation of the field in regions I and II. It is obvious that, by virtue of the symmetry of the problem, the boundaries between these regions do not curve. For $z \to 0$ the field **H** must approach zero. By virtue of Maxwell's equations, the component H_x at finite z must be nonzero (but small). From equation (6) it is easy to obtain the dependence of **M** on **H** for small H_x but finite H_z:

$$M_x^{(1)} \approx \frac{MH_x}{\beta M + H_z}, \quad M_x^{(2)} \approx \frac{MH_x}{\beta M - H_z}, \quad M_z^{(1)} \approx -M_z^{(2)} \approx M. \qquad (44)$$

Maxwell's equations in this case have the form

$$\frac{\partial H_z}{\partial x} = \frac{\partial H_x}{\partial z}, \quad \frac{\partial H_z}{\partial z} + \frac{\partial}{\partial x}\left[\left(1 + \frac{4\pi M}{\beta M \pm H_z}\right)H_x\right] = 0. \qquad (45)$$

After the substitution

$$H_z = \beta MV, \quad H_x = \beta^{3/2}M(1 \pm V)U \qquad (46)$$

they transform to the following:

$$\frac{\partial V}{\partial x} = \sqrt{\beta}\frac{\partial}{\partial z}[(1 \pm V)U], \quad \sqrt{\beta}\frac{\partial V}{\partial z} + \frac{\partial}{\partial x}\{[4\pi + \beta(1 \pm V)]U\} = 0. \qquad (47)$$

In the closure domains the relations analogous to (44)–(47) have the form

$$M_z = -\frac{MH_z}{\beta M \pm H_x}, \quad M_x \approx \mp M, \qquad (44')$$

$$\frac{\partial H_x}{\partial z} = \frac{\partial H_z}{\partial x}, \quad \frac{\partial H_x}{\partial x} + \frac{\partial}{\partial z}\left[\left(1 - \frac{4\pi M}{\beta M \pm H_x}\right)H_z\right] = 0, \qquad (45')$$

$$H_x = \beta MV', \quad H_z = \beta^{3/2}M(1 \pm V')U', \qquad (46')$$

$$\frac{\partial V'}{\partial z} = \sqrt{\beta}\frac{\partial}{\partial x}[(1 \pm V')U'], \quad \sqrt{\beta}\frac{\partial V'}{\partial x} = \frac{\partial}{\partial z}\{[4\pi - \beta(1 \pm V')]U'\}. \qquad (47')$$

In the limit $\beta \to 0$ the equations (47) and (47′) admit the solution $V' = \pm 1$, $U' = 0$; $V = \pm 1$, $U = 0$ in the triangular parts of domains I and II and $V = U = 0$ outside these triangles. Thus in the closure domains and in the triangular parts of domains I and II, the field is practically uniform and equal to $\pm\beta\mathbf{M}$. Below, the field rapidly decreases to zero. The transition occurs in a narrow layer of width $\delta z \sim \sqrt{\beta a}$ near the dotted line connecting

the vertices of the triangles in Fig. 3. The field H in domains I and II does not exceed the limits of metastability. In the closure domains, near the lower vertices of the triangles, at distances $\delta z \sim \sqrt{\beta a}$, the direction of magnetization may deviate significantly from the x axis; this leads to a pronounced curving of the separation boundaries in the transition region. This region makes a small contribution to the energy of exit of the domains to the surface.

The region outside the specimen makes a contribution to the exit energy that is proportional to β^2 and may be neglected. The energy density in the closure domains is equal to the sum of the anisotropy energy $\beta M^2/2$ and the energy $-\mathbf{MH} = -\beta M^2$ (we neglect the term $-H^2/8\pi$). This sum is negative.

In regions I and II, the anisotropy energy is insignificant, while $-\mathbf{MH}$ is equal either to βM^2 or to zero. The regions in which $-\mathbf{MH} = \beta M^2$ have the same volume as the closure domains. Thus in the approximation adopted, the exit energy coincides with the anisotropy energy, as was assumed in [1,3]. Per unit area of the plate (with allowance for the two sides of the plate), the energy of exit of domains to the surface is

$$E_{\text{exit}} = {}^1/_4 \beta M^2 a. \tag{48}$$

This result can be obtained by another method, by using the fact that in the case $\mathbf{H}_0 = 0$ the thermodynamic potential $\tilde{\Phi}$ is (see Sec. 3, Item a)

$$\tilde{\Phi} = \Phi = \int d^3\mathbf{x}(U_{\text{an}} + H^2/8\pi).$$

In the limit $\beta \to 0$, the second term makes a small contribution to the exit energy ($\sim\beta^2$). The latter is determined by the anisotropy energy alone, as is also reflected in formula (48). This statement is correct also in the case in which the axis of easy magnetization is inclined to the surface of the plate (this case also was considered by Landau and Lifshitz[3]).

When a current j flows in the direction perpendicular to the layers, the boundaries of the layers should move[4] with speed $v = j/n|e|$ (here e is the charge of the electron, and n is the difference between the numbers of electrons and of holes; it is assumed that $n \neq 0$).

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