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EXPERIMENTAL INVESTIGATION OF INEQUALITY OF THE OPTICAL PATHS OF OPPOSITE WAVES IN A 3.39 MICRON RING LASER

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Experiments with a 3.39 micron ring laser are described which show the possibility of existence of opposite waves of unequal power. A consequence is the inequality of the optical paths and frequency splitting. A qualitative analysis of the phenomenon is carried out. At a wavelength of 0.63 microns the effect may somewhat change the splitting frequency induced by rotation.

A laser with a ring resonator (RRL) can be used to measure the speed of rotation around an axis perpendicular to the ring plane. It is important here that the optical paths of the rays propagating over the ring in opposite directions be equal to each other in the absence of rotation.[1] Owing to the nonlinear properties of the active medium, however, this equality is violated when the powers of waves 1 and 2 are different.

To investigate the operation of an RRL operating at 0.63 μ, a special non-reciprocal device was used in [1] to introduce different losses for waves 1 and 2, thereby making their powers unequal to one degree or another. The splitting frequency (the difference between the frequencies of waves 1 and 2), due to the rotation of the system,[2] varied strongly with the length of the resonator; the same took place also in the absence of the non-reciprocal element. The use of a device that automatically equalized the power of the wave for any setting of the resonator stabilized the splitting frequency.

This raises the question of what causes the inequality of the powers of the waves in the absence of special non-reciprocal elements, and the appearance of the splitting frequency (or of the change in this value). The present article is devoted to this question. We note that in some papers (see, for example, [2,3]) it is simply postulated that the resonator has different values of Q for waves 1 and 2 (and this causes the difference in power), but the cause of the inequality is not discussed.

The present investigation was made with an RRL laser operating at 3.39 μ. Owing to the large gain of the active medium for this transition, the investigated effect is manifest more strongly, and it is easier to observe than at wavelengths 0.63 or 1.15 μ. We note that it was already mentioned in [4] that the introduction of an inhomogeneity (the point of a needle, etc.) into the resonator of a stationary 3.39 μ RRL causes frequency splitting; no such effect was apparently observed at other wavelengths.[3]

We present first a formula relating the splitting frequency in the absence of rotation of an RRL operating in the single-mode regime with the difference between the wave powers P_1 and P_2. Assuming that the natural radiation width of the atoms $\Delta\nu_N$ is much smaller than the Doppler width of the line $\Delta\nu_D$, we can, considering the propagation of wave 1, assume for the imaginary component of the dielectric susceptibility of the medium the approximation

$$\varkappa_2 = \frac{Ae^{-x^2}}{\sqrt{1+\gamma P_1+\varepsilon P_2}} \approx Ae^{-x^2}\left[1-\frac{\gamma}{2}P_1-\frac{\varepsilon}{2}P_2\right], \qquad (1)$$

where $x = (\nu - \nu_L)/0.6\,\Delta\nu_D$ is the relative detuning of the generation frequency ν from the center of the line ν_L; A, γ, and ϵ are parameters of the medium. For wave 2, it is necessary to interchange the quantities P_1 and P_2. The usual assumptions concerning the parameters of the inversion dips make it possible to determine, when the approximation (1) is used, the additional phase difference occurring in the propagation of the waves in the active medium, as well as the corresponding corrections to the generation frequency and their difference, which turns out to be in this case

$$F = \nu_1 - \nu_2 \approx 2\pi\nu\frac{l}{L}Ae^{-x^2}\frac{\nu-\nu_{\text{л}}}{\Delta\nu_N[1+4(\nu-\nu_{\text{л}})^2/\Delta\nu_N{}^2]}\gamma(P_2-P_1), \qquad (2)$$

where l and L are the lengths of the active medium and of the perimeter of the resonator.[4]

It should be kept in mind that expression (2) for F is quantitatively quite approximate since it is derived on the basis of a number of idealizations that agree only qualitatively with the experimental data (see, for example, [7] concerning the parameters of the inversion dip).

It follows from (2) that F = 0 when $\nu = \nu_L$ and has a maximum at approximately $|\nu - \nu_L| \approx \Delta\nu_N/2$ (it is assumed that $P_2 - P_1$ changes little near such values of $\nu - \nu_L$). We note also that the values of A at 3.39 μ are larger approximately by two orders of magnitude than at the wavelength 0.63 μ. This circumstance causes the frequency-splitting effect to become "smeared" at 0.63 μ by the known locking phenomenon.

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[1] For brevity, we shall henceforth designate the waves (rays) propagating clockwise and counter clockwise, as well as the parameters pertaining to them, by the symbols 1 and 2 respectively.

[2] The rotation was imitated by a special device.

[3] Incidentally, frequency splitting of an RRL at a wavelength 0.63 μ is noted in [5,6] and is attributed by the authors to the motion of the gas in the discharge tube (the "Langmuir effect"). It seems to us that in this case the effect analyzed in the present paper could also have taken place.

[4] A similar expression using other parameters at l = L can be obtained from the equations of [2].

EXPERIMENTAL RESULTS AND THEIR DISCUSSION

A block diagram of the principal elements of the experimental setup is shown in Fig. 1. The resonator ($L \approx 75$ cm) is made up of three mirrors I, II, and III. Mirror III is mounted on a piezocylinder, making it possible to control the resonator frequency with the aid of an electric voltage. The experiments were performed both in the case when all three mirrors were flat, and with one of the mirrors replaced by a spherical one ($R = 1.2$ m). The discharge tube, with Brewster plates, 3 mm in diameter and with a discharge length of 18 cm, was fed from a dc source. The pressure of the helium-neon mixture (ratio 5.6:1) was 1.2 torr; a tube with natural neon and a tube with the isotope Ne^{20} were used.

Two diaphragms D_1 and D_2 were located inside the resonator; both their diameters and their positions could be regulated. Near mirror II was placed a setup I producing interference between the rays emerging from the resonator. The radiation was recorded with a photodiode Ph (InSb cooled with liquid nitrogen), with a lens L placed in front of it; the photodiode output was fed (after amplification) to an oscilloscope and other instruments. Provision was made for heterodyning the RRL oscillations; the emission of an additional 3.39 μ laser G could also be fed to the photodiode with the aid of a semitransparent plate P, and the entire system was adjusted in such a way that this emission interfered with the rays from the investigated RRL.

The experiments were performed with the RRL operating in a single-frequency mode (TEM_{00}). At small discharge currents, this was produced naturally, and at large currents this was produced by decreasing the aperture of one of the diaphragms. By applying a low frequency (~200 Hz) alternating voltage to the piezocylinder and to the oscilloscope speed, it was possible to observe on the oscilloscope screen the generation region (i.e., the dependence of the power on the tuning of the resonator relative to ν_L). By covering one of the mirrors of the setup I, the generation regions of rays 1 and 2 could be observed separately.

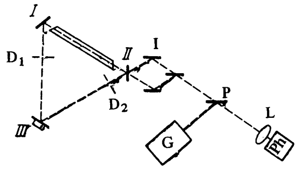

FIG. 1.

The very first experiments have shown that as a rule frequency splitting occurs at some value of $\nu - \nu_L$. Only by careful adjustment of the mirrors (and of the diaphragm position if the latter was used to limit the radiation beam) was it possible to eliminate frequency splitting in the entire generation region (approximately ±180 MHz from ν_L). Photographs obtained of the generation region of ray 1 are shown in Fig. 2a for the case of a natural neon mixture and in Fig. 2b for the case of Ne^{20}. (The abscissas in these figures represent the generation frequency ν reckoned from ν_L, while the ordinates represent the values of $P_1(\nu)$). In a region extending approximately ±10 MHz from ν_L, a "competition" of the opposing waves is observed. This phenomenon was considered in a number of papers (see, for example [1,3]), and therefore this region of ν will not be considered or discussed here. The remaining generation region (of both ray 1 and ray 2) is approximately symmetrical relative to ν_L.

If following the indicated adjustment of the resonator one of the diaphragms (inside or outside the resonator) is moved a distance on the order of a fraction of a mm at an aperture of ~1 mm, then the pictures of the generation region are distorted. This effect was particularly pronounced in the case when one of the resonator mirrors was spherical. Typical pictures are shown in Figs. 3a and b for natural neon and in Figs. 4a and b for the isotope Ne^{20} (a and b correspond to rays 1 and 2). The figures show clearly the presence of a power difference $P_1 - P_2$, and the sign of this difference changes when ν goes through the value of ν_L; the $P_2(\nu)$ curve is approximately the mirror image, relative to ν_L, of the $P_1(\nu)$ curve. The sum $P_1 + P_2$ remains approximately symmetrical relative to ν_L, and at not very small diaphragm displacements it exceeds by 20–40% its prior value (at $P_1 = P_2$).

The appearance of the difference $P_1 - P_2$ is accompanied by a splitting of the wave frequencies. For quantitative measurements, the ac voltage was removed from the piezocylinder, leaving only the slow change of the resonator length due to self-heating. With the aid of an additional photodiode (not shown in Fig. 1), the value of P_1 was marked on the chart of the automatic recorder, and the output of the main photodiode was fed

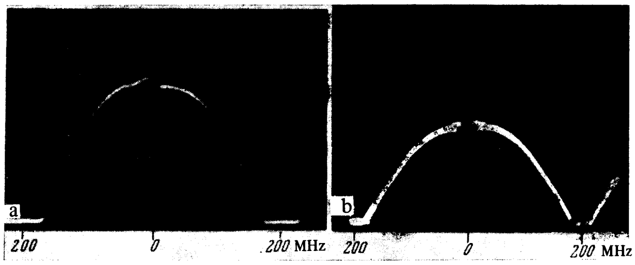

FIG. 2.

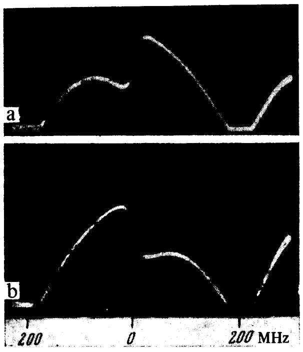

FIG. 3.

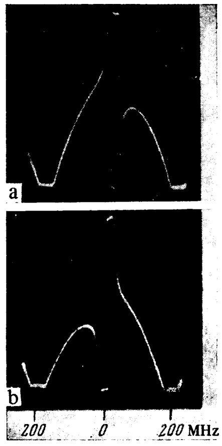

FIG. 4.

to a frequency meter of type ChZ-9, which recorded the splitting frequency F. Typical results are shown in Figs. 5a, b, where the dashed curves show the values of P_1 (in arbitrary scale), and the solid curves the values of F. We note that the asymmetry (relative to ν_L) of the $P_1(\nu)$ curve in Fig. 5b is not readily seen visually; nevertheless, it produces a readily observed frequency splitting. Therefore the correct setting of the apparatus elements to eliminate splitting in the entire generation region is a much more sensitive device than setting to obtain symmetry of the $P_1(\nu)$ curve (or $P_2(\nu)$).

Reversal of the sign of $\nu - \nu_L$ was always accompanied by reversal of the sign of $P_1 - P_2$; therefore, in accordance with (2), the sign of $\nu_1 - \nu_2$ should not change in the entire generation region. This was confirmed experimentally by heterodyning both rays of the RRL with an additional laser G. The distances between the maxima of F were 140–200 MHz; as indicated above, this quantity determines approximately $\Delta\nu_N$. Estimates of the values of F on the basis of formula (2), which were only approximate in character, have shown satisfactory agreement with the experimental data.

An increase of the discharge current of the tube led to an increase of the values of $|P_1 - P_2|$ and to larger values of F. The largest value of F in our experiments with flat mirrors was ~500 kHz; substitution of a spherical mirror for one of the flat ones increased this value by approximately three times.5) The minimum values of F that could be observed in the experiment were 0.8–1.2 kHz; these values determine the locking band in our setup.

We note also the following experimental facts:

The asymmetry produced by displacing the diaphragm D_2 inside the resonator was of a sign opposite to the asymmetry produced by a similar displacement of D_1, i.e., the pictures of rays 1 and 2 changed places.

In the presence of the described asymmetry of the power plots, displacement of the free diaphragm (i.e., the one not taking part in the production of the asymmetry) can eliminate the asymmetry and the frequency splitting in the entire generation region. The powers of the waves remain practically unchanged in this case.

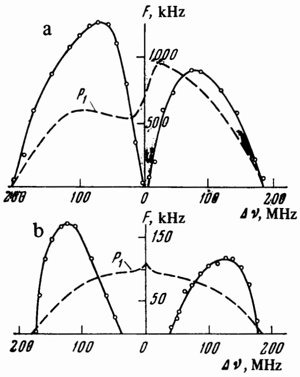

FIG. 5.

Frequency splitting leads to a slight amplitude modulation, with frequency F, of the power of each wave separately, the phases of the modulation being opposite in the two rays. This fact has already been noted in the literature.[8]6)

The use of a high frequency discharge in the tube (~40 MHz) left the described picture of the phenomena practically unchanged.

The production of asymmetry of the power curves by displacing the diaphragm was used in our experiment as a simple means of obtaining a regular reproducible effect. But the phenomenon could also be observed by smoothly introducing into the resonator a thin wire, etc. The situation is somewhat more complicated when it comes to the influence of slight misalignment of the resonator mirrors: if one of the mirrors was spherical, then slight rotation of any of the mirrors also led to the described asymmetry. On the other hand, if all the resonator mirrors were flat, then by adjusting one of the mirrors it was possible to obtain both the above-described asymmetry, and an asymmetry of a different kind, the character of which is seen from the photograph of the generation region of ray 1 in Fig. 6. Here we have a significant broadening of the frequency region $\nu > \nu_L$, and a narrowing of the region $\nu < \nu_L$. But the picture for ray 2 has the same form; therefore the difference $P_1 - P_2$ does not arise (in any case, it can be eliminated by an additional slight adjustment of the elements of the setup), and such an asymmetry, which is the same in both cases, produces no frequency splitting. We note also that such an asymmetry is accompanied by an appreciable decrease in the powers of both waves (by a factor 3–4).

DISCUSSION OF EXPERIMENTAL RESULTS

The splitting of the ray frequencies, due to the difference between their powers, can be readily explained; the dependence of this splitting (i.e., of the frequency F) on the parameters of the system agrees qualitatively (and also quantitatively when it comes to order of magnitude) with the results of the approximate analysis. The situation is more complicated when it comes to determining the causes of the power difference $P_1 - P_2$ (in the absence of special non-reciprocal elements) and of the reversal of the sign of this difference when the generation frequency passes through the center of the transition line ν_L. The entire phenomenon occurs in a frequency region amounting, in order of magnitude, to 10^{-6} of the average generation frequency, and the system does not contain any special elements whose operation has such sharp selectivity. One can hardly be satisfied

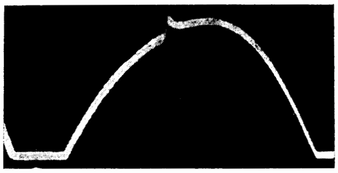

FIG. 6.

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5)It is possible that the possibility of obtaining a frequency splitting of two waves with easily adjustable magnitude is of practical interest for some problems.

6)Figures 3 and 4 do not show the indicated modulation, since it is smeared out by the scanning and by the use of a narrow band amplifier.

with the simple statement that the Q factors of the resonator are different for rays 1 and 2, since it is not clear why the difference between the Q factors reverses sign when the generation frequency goes through ν_L. A hypothesis was advanced that the cause is a parasitic coupling between the opposing waves (which explains the locking phenomenon). To verify this hypothesis, an additional mirror was placed outside the resonator to reflect the wave 2 back into the resonator, and the magnitude of the reflection could be regulated by means of a continuous attenuator in front of the mirror. Experiment has shown that this procedure increases P_1 and decreases P_2 in the entire generation region, but the asymmetry of the $P_1(\nu)$ curve (and of $P_2(\nu)$) decreases.

Our attempts to explain all the experimental facts on the basis of the one-dimensional model customarily used in the analysis of laser operation has not led to any success.[7)] Apparently, in this case it is important to take into account the three-dimensional character of the system. A quantitative analysis of the three-dimensional problem of the propagation of opposing waves in the system, with allowance for the nonlinear properties of the active medium and for the complicated boundary conditions, is practically impossible, and we present only a qualitative discussion of the question.

We note first that the uneven distribution of the "activity" (of the gain) of the medium over the cross section of the discharge tube (see, for example, [9]) leads also to non-uniformity of the effective refractive index n over the cross section. Therefore, if the radiation propagates in the medium not parallel to the tube axis, then the refraction will take place, the path of the ray will bend, and will be deflected in one direction or the other, depending on the gradient of n. The sign of this gradient is reversed when ν passes through ν_L. It follows therefore that the optical paths for frequencies lower than ν_L and higher than ν_L may differ somewhat. Of course, this effect is small (the bending of the path, according to rough estimates, can reach 1–3 minutes of angle). For its experimental observation, the following procedure was used: a diaphragm (not shown in Fig. 1) was installed near the lens L of the photodiode. Its aperture was first large and did not limit the radiation incident on the photodiode. After photographing the picture of the generation region of one of the rays, the aperture was decreased, and the diaphragm was shifted smoothly to the left and to the right insofar as it was possible without causing the picture on the screen to be distorted by the apparatus noise (such a displacement of the diaphragm weakened the signal and it was necessary to increase the gain of the amplifier); the pictures were then again photographed. The results of the experiment are shown in Fig. 7, where a corresponds to the absence of the influence of the diaphragm and b and c correspond to displacing the diaphragm to the left and to the right respectively. The considerable difference between these photographs confirms the assumption that the path of the ray changes somewhat with changing generation frequency. It is important to note that when there was no appreciable asymmetry of the generation region, the displacement of the diaphragm near the photodiode did not reveal a change in the picture; it can be assumed that in this case the radiation propagates without being deflected from the tube axis, and its path remains practically unchanged for all values of ν.

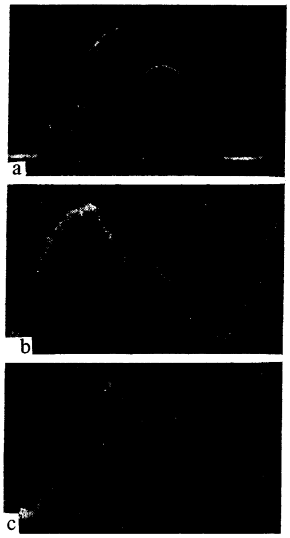

FIG. 7.

The gain of the wave increases if the ray is deflected towards the tube axis and decreases if it is deflected away from the axis. We note that to explain the observed asymmetrical pictures of the generation regions, it is sufficient to assume that the mean value of A (see the approximation (1)) changes by 1.5–3%. Estimates show that a value of $\Delta A/A$ of this order of magnitude can occur when the ray is deflected by approximately one minute of angle.

The foregoing explains the generation-region picture shown in Fig. 6, namely, at frequencies $\nu > \nu_L$ the ray is deflected towards the tube axis and this frequency range broadens, while the range $\nu < \nu_L$ narrows. In this case, however, as noted above, the same asymmetry of the generation region is observed in ray 2 as in ray 1, and no splitting of the frequencies occurs as the result of the inequality of the ray powers.

Allowance for refraction alone is insufficient to explain the ray-power asymmetry that leads to the frequency splitting (see Figs. 3 and 4). It is necessary to take into consideration the fact that the nonuniformity of n over the cross section of the medium is connected not only with the nonuniformity of its "activity," but also with the distribution of the opposing wave intensity over the cross section. Such a mutual influence of the waves may not be equal; the difference is connected with the geometry of the fields in the entire resonator (location of the diaphragms, setting of the mirrors etc.). A complicated wave-interaction picture is obtained, a phenomenon that can be arbitrary called non-reciprocal focusing (or defocusing) of the opposing waves. In any case, its result is that the radiation beams do not overlap completely inside the active medium; the consequence, when speaking of certain central lines of the beams, is that rays 1 and 2 follow somewhat different

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[7)] A small asymmetry of the generation region may be due to the Langmuir effect; in this case, however, in order for the difference $P_1 - P_2$ to reverse sign, it is necessary to reverse the direction of the discharge current. It follows from all the foregoing that the Langmuir effect was of no significance in our experiments.

paths.[8] A direct experimental proof of the foregoing is difficult to obtain, but two facts offer evidence in favor of such an explanation:

As noted above, when asymmetry is obtained by a slight displacement of the diaphragm, the total power $P_1 + P_2$ increases, and this can be attributed to the increase of the working cross section of the active medium. When working with the Ne^{20} isotope, a competition of the two RRL waves is clearly seen (Fig. 2b). When asymmetrical power pictures are obtained, this region decreases to some degree or another, and this may be the result of a partial spatial separation of the radiations of the opposing waves.

In a discussion of the results of the experiments, K. A. Goronina pointed out one other possible cause of the asymmetry of the generation region: it is natural to assume that the nonuniform distribution of the number of active atoms over the cross section causes the distribution of the velocities of these atoms off the tube axis to be different in the direction towards the axis from that away from the axis. It is then easy to show that if the ray propagates in the medium at a certain angle to the tube axis, then an asymmetry will take place in the number of "working" atoms, as well as the observed power asymmetry. It seems to us that this can hardly explain fully the phenomenon, since the radiation can propagate in the medium at an angle not exceeding several tenths of one degree.

CONCLUSION

We have described experiments demonstrating the possible existence of a power difference between the opposing waves of an RRL (in the absence of special non-reciprocal elements), and the resultant frequency splitting of these waves. The power difference was produced either by introducing into the resonator objects diffracting the radiation, or by slightly misaligning the mirrors. The cause of the effect is apparently the propagation of the radiation not strictly along the discharge-tube axis, and the consequent refraction and the interaction of the opposing radiation beams.

The experiments were performed at a wavelength 3.39 μ, but the influence of the effect under consideration can be expected also at a wavelength 0.63 μ, where it can appear in the form of a distortion of the splitting of the wave frequencies as the result of the rotation of the system.[9]

We are grateful to Yu. K. Kazarin for help with the experiments.

Note added in proof (May 30 1969). The experiments performed by us with an RRL at a wave 1.15 μ (with a discharge tube diameter 1.5 mm) have revealed a splitting of the frequencies of opposite waves caused by the interaction between the effects described here and the Langmuir effect.

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[8] According to the foregoing, these paths change somewhat when ν is varied within the limits of the generation region (Fig. 7).

[9] This does not at all deny the possibility of other causes of the indicated distortion in RRL operating at 0.63 μ.