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INTERFERENCE PHENOMENA IN RECORDING OF DECAYING PARTICLE PAIRS

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The ideas developed in[1] are applied to an analysis of interference phenomena during registration of pairs of nonidentical unstable particles. General formulas are derived for the probability of recording decaying particle pairs and resonances in correlation experiments. It is demonstrated that interference that depends on the proper time of flight of the particles and is due to nonorthogonality of the internal wave functions occurs when a single detector is used to record the decay products of two particles with identical spins and other conservable quantum numbers. If the particles have different conservable quantum numbers, their internal functions will be orthogonal to each other and there is never any interference during registration of such particles by a single detector.

1. INTRODUCTION

THE conditions for observing interference in the registration of two non-identical particles in correlation experiments were analyzed by us in detail earlier[1]. According to[1], for the same particles A and B, the presence or absence of interference depends on the type of the recording instruments. In particular, interference is possible if the particles are unstable, have identical decay modes, and they are registered by means of the products of these decays[1)].

The purpose of the present paper was a more detailed study of interference phenomena in the registration of pairs of unstable particles. Particular attention was paid here to the interference in the registration of two unstable particles with identical spins and other conserved quantum numbers by a single detector (see also[5]). In particular, we arrive at the conclusion that if the mass difference between the particles A and B tends to zero, the probability of registering non-identical particles by means of either 1 or 2 detectors can be described in many cases by the same formulas as the probability of registering identical particles.

2. CORRELATION FORMULAS FOR WAVE PACKETS OF PAIRS OF UNSTABLE PARTICLES

Assume that as a result of some process there is produced at some point of space a pair of unstable particles A and B, and that the first detector, located at a distance l_1 from the region where packets of particles A and B are produced, registers the decay products of these particles in the state $|n\rangle$, while a second detector, located at a distance l_2 from the packet-production region, records the decay products of the particles A and B in the state $|m\rangle$. We denote by a(1, 2) (a(2, 1)) the production or scattering amplitude corresponding to the emission of particle A in the direction of the first (second) detector and of particle B in the direction of the second (first) detector; we denote by $f_{A\to n}$ and $f_{B\to n}$ ($f_{A\to m}$, $F_{b\to m}$) the amplitudes of the transition of particles A and B into the state $|n\rangle$ ($|m\rangle$). Then, according to[1], the number of delayed coincidences in the registration of particles A and B by two detectors are proportional to the quantity

$$d^2P_{n,m} = |P^{(\pm)}_{n,m}(t_1, t_2)|^2, \qquad (1)$$

where

$$P^{(\pm)}_{n,m}(t_1,t_2) = a(1,2) f_{A\to n} f_{B\to m} \exp\left[-\left(im_A + \frac{\Gamma_A}{2}\right)t_1\right] \times \exp\left[-\left(im_B + \frac{\Gamma_A}{2}\right)t_2\right] \pm a(2,1) f_{B\to n} f_{A\to m} \times \exp\left[-\left(im_B + \frac{\Gamma_B}{2}\right)t_1\right]\exp\left[-\left(im_A + \frac{\Gamma_A}{2}\right)t_2\right]. \qquad (2)$$

Here m_A and m_B are the masses of the particles A and B, Γ_A and Γ_B are their widths, $t_1 = l_1/v_1\gamma_1$, and $t_2 = l_2/v_2\gamma_2$, where v_1 and v_2 are the group velocities of the wave packets moving respectively in the directions of the first and second detectors; $\gamma = (1 - v^2)^{-1/2}$ is the Lorentz factor. The sign "plus" in formula (2) corresponds to particles A and B with integer spin; the sign "minus" corresponds to particles with half-integer spins. Formulas (1) and (2), which are valid under the condition $\Delta m = |m_A - m_B| \ll m_A, m_B$,[2)] will be needed later on in the analysis of the question of registration of a pair of unstable particles by means of a single detector (see Sec. 4).

Let us consider now a more general case, when pairs of identical particles AA and BB are produced besides the pair of particles A and B. The state produced in this case has immediately following the generation act the form

$$|\psi(1,2)\rangle^{(\pm)} = a(1,2)|A\rangle^{(1)}\cdot|B\rangle^{(2)} \pm a(2,1)|A\rangle^{(2)}\cdot|B\rangle^{(1)} + F^{(\pm)}_{AA}(1,2)|A\rangle^{(1)}\cdot|A\rangle^{(2)} + F^{(\pm)}_{BB}(1,2)|B\rangle^{(1)}\cdot|B^{(2)}\rangle. \qquad (3)$$

Here $F_{AA}(1, 2)$ is the amplitude for the production of the pair AA, $F_{BB}(1, 2)$ is the amplitude for the production of the pair BB, with $F_{AA(BB)}(1, 2) =$

[1)]See in this connection also [2-4], which are devoted to an analysis of the properties of $K^0\bar{K}^0$ pairs.

[2)]We put $\hbar = c = 1$ throughout. The conditions for applicability of formula (2) and its corollaries are discussed in greater detail in [1]. In particular, if both detectors are identical ($|n\rangle = |m\rangle$), then when $t_1, t_2 \ll 1/|m_A - m_B|$ and $|t_1 - t_2| \ll 1/|\Gamma_A - \Gamma_B|$ the registration amplitude is $P^{(\pm)}_{n,m} \sim a(1,2) \pm a(2,1)$, i.e., it has the same form as in the case of identical particles.

$\pm F_{AA(BB)}(2, 1)$, but generally speaking $a(1, 2) \neq a(2, 1)$. It is easy to see that in this case the probability of registering the decays A, B → n and A, B → m by detectors connected for coincidence is proportional to the quantity $|\tilde{P}^{(\pm)}_{n,m}(t_1, t_2)|^2 dt_1 dt_2$, where

$$\tilde{P}^{(\pm)}_{n,m}(t_1, t_2) = F^{(\pm)}_{AA}(1.2) f_{A\to n} f_{A\to m} \exp\left[-\left(im_A + \frac{\Gamma_A}{2}\right)(t_1 + t_2)\right]$$
$$+ F^{(\pm)}_{BB}(1.2) f_{B\to n} f_{B\to m} \exp\left[-\left(im_B + \frac{\Gamma_B}{2}\right)(t_1 + t_2)\right] + P^{(\pm)}_{n,m}(t_1 + t_2), \quad (4)$$

with $P^{(\pm)}_{n,m}(t_1, t_2)$ determined from formula (2)[3)]. Formula (4) describes, in particular, the correlations in the decays of the $K^0\overline{K}^0$ pair (see[2-4]). The role of particle A is played here by the K_1^0 meson, and the role of particle B by the K_2^0 meson. Since the K mesons are bosons, the corresponding expression has the structure $\tilde{P}^{(+)}_n(t_1, t_2)$. The pair-production amplitudes satisfy in this case the relations

$$F^{(+)}_{K_1^0K_1^0}(1,2) = -F^{(+)}_{K_2^0K_2^0}(1.2), \qquad a(1,2) = -a(2.1). \quad (5)$$

Equation (5) corresponds to the fact that the combination $K_1^0K_0^0 - K_2^0K_2^0$ is produced when the pair $K^0\overline{K}^0$ is produced in states with even orbital angular momenta, and the combination $K_1^0K_2^0$ is produced in the case of odd orbital angular momenta.

3. INTERFERENCE IN DECAYS OF RESONANCE PAIRS

It is known that the effective-mass spectrum of the decay product of an unstable particle has a Breit-Wigner form:

$$dW \sim \frac{1}{2\pi}\left|\frac{f_{A\to n}}{M - m_A + i\Gamma_A/2}\right|^2 dM, \quad (6)$$

where $f_{A\to n}$ is the decay amplitude corresponding to a transition to the given state $|n\rangle$. Let us assume that as a result of some reaction there are produced two non-identical resonances with identical decay modes $|n\rangle$ and $|m\rangle$. The lifetimes of the resonances are so short that the observation of the time correlations considered in Sec. 2 becomes impossible. One cannot, however, raise the question of the distribution of the effective masses of the decay products of two resonances. Let the first detector register the decay products of two resonances A and B in the state $|n\rangle$ with total momentum p_1 and effective mass M_1, and the second detector register the decay products of the same resonances in the state $|m\rangle$ with total momentum p_2 and effective mass M_2. In this case the process can follow two indistinguishable paths: a) the state $|n\rangle$ is produced as the result of the decay of the resonance A, while the state $|m\rangle$ is due to the decay of the resonance B; b) the state $|n\rangle$ is produced as the result of the decay of the resonance B, and the state $|m\rangle$ as the result of the decay of the resonance A. The distribution over the effective masses M_1 and M_2 will take the form

$$d^2W^{(\pm)}_{n,m} \sim \frac{1}{(2\pi)^2}\left|\frac{f_{A\to n} f_{B\to m}\, a(\mathbf{p}_1, \mathbf{p}_2)}{(M_1 - m_A + i\Gamma_A/2)(M_2 - m_B + i\Gamma_B/2)} \pm \frac{f_{A\to m} f_{B\to n}\, a(\mathbf{p}_2, \mathbf{p}_1)}{(M_1 - m_B + i\Gamma_B/2)(M_2 - m_A + i\Gamma_A/2)}\right|^2 dM_1\, dM_2, \quad (7)$$

[3)] If the detectors do not register the projections of the spins of the decay products, then the quantities $|p_{n,m}|^2$ and $|\tilde{p}_{n,m}|^2$ must be summed over the polarizations of the final particles and averaged over the polarizations of particles A and B.

where $a(\mathbf{p}_1, \mathbf{p}_2)$ ($a(\mathbf{p}_2, \mathbf{p}_1)$) is the amplitude of production of the resonance A (B) with momentum $\mathbf{p}_1$ and of the resonance B (A) with momentum $\mathbf{p}_2$. The rule for choosing the sign in (7) is the same as in Sec. 2. If identical decay modes of the resonances A and B are registered (A → n, B → n), then in the limit as $m_A \to m_B$ and $\Gamma_A \to \Gamma_B$ we have

$$d^2W^{(\pm)}_{n,m} \sim |a(p_1, p_2) \pm a(p_2, p_1)|^2 \frac{dM_1\, dM_2}{[(M_1 - m)^2 + \Gamma^2/4][(M_2 - m)^2 + \Gamma^2/4]}. \quad (8)$$

Expression (8) coincides with the formula describing the correlations in the decays of a pair of identical resonances. Indeed, the quantity $a(\mathbf{p}_1, \mathbf{p}_2) \pm a(\mathbf{p}_2, \mathbf{p}_1)$ is symmetrical with respect to the substitution $\mathbf{p}_1 \rightleftarrows \mathbf{p}_2$ in the case of bosons (all the even orbital angular momenta take part in the process) and antisymmetrical in the case of fermions (only odd angular momenta take part).

In the case of production of a pair of identical resonances, the distributions of the effective masses of the decay products always have the form (8), regardless of whether the detectors register identical or different decay modes.

If pairs of identical resonances AA and BB are produced in the same process in addition to the non-identical resonances A and B, the same reasoning as in Sec. 2 leads to a general formula for the distribution of the effective masses

$$d^2W^{(\pm)}_{n,m} \sim \frac{1}{(2\pi)^2}\left|\frac{f_{A\to n} f_{B\to m}\, a(\mathbf{p}_1, \mathbf{p}_2)}{(M_1 - m_A + i\Gamma_A/2)(M_2 - m_B + i\Gamma_B/2)} \pm \frac{f_{A\to m} f_{B\to n}\, a(\mathbf{p}_2, \mathbf{p}_1)}{(M_1 - m_B + i\Gamma_B/2)(M_2 - m_A + i\Gamma_A/2)} + \frac{F^{(\pm)}_{AA}(\mathbf{p}_1, \mathbf{p}_2) f_{A\to n} f_{A\to m}}{(M_1 - m_A + i\Gamma_A/2)(M_2 - m_A + i\Gamma_A/2)} + \frac{F^{(\pm)}_{BB}(\mathbf{p}_1, \mathbf{p}_2) f_{B\to n} f_{B\to m}}{(M_1 - m_B + i\Gamma_B/2)(M_2 - m_B + i\Gamma_B/2)}\right|^2 dM_1\, dM_2. \quad (9)$$

Here $F^{(\pm)}_{AA}(\mathbf{p}_1, \mathbf{p}_2)$ and $F^{(\pm)}_{BB}(\mathbf{p}_1, \mathbf{p}_2)$ are the amplitudes of production of the pairs AA and BB, with $F^{(\pm)}(\mathbf{p}_1, \mathbf{p}_2) = \pm F^{(\pm)}(\mathbf{p}_2, \mathbf{p}_1)$.

4. INTERFERENCE PHENOMENA IN THE REGISTRATION OF PAIRS OF UNSTABLE PARTICLES BY ONE DETECTOR

It follows from the foregoing that in correlation experiments any non-identical particles A and B, having the same decay modes, can interfere. In particular, if the detectors register the decay products of the particles A and B in a narrow angle interval, interference is possible also in the case when the spins of these particles are different (the spin difference $|S_A - S_B|$ is assumed to be an integer). After averaging over the angles, the interference terms in formulas (1) and (7) vanish.

When we state that the particles A and B are not identical, we usually imply the existence of some method which makes it possible to distinguish between them. But if such a method exists, then, in accordance with the formalism of quantum mechanics, the states should be orthogonal to each other, i.e., $\langle A|B\rangle = 0$. It is therefore clear that no interference is observed in the correlation experiments if the detector register the distinguishable (orthogonal) states $|A\rangle$ and $|B\rangle$ themselves, and not their superpositions.

We note now that the concepts of non-identity and distinguishability are in complete correspondence without any stipulations only for stable particles. Of course, if the unstable particles have different quantum numbers (for example, charges or spins), which are conserved during the decay processes, then these particles are also distinguishable in principle, i.e., their states are orthogonal. Let us assume, however, that for the particles A and B all the conserved quantum numbers are the same, and that the particles differ in their masses and in their lifetimes. In the case of stable particles with identical internal quantum numbers, but with different albeit close masses, we have at our disposal, in principle, an infinitely long time to be able to determine the masses of the particles A and B and by the same token identify these particles.

In the case of unstable particles with identical internal quantum numbers, the situation is different: the time of mass measurement is limited to the lifetime, and the mass itself is not a fully defined quantity. One should therefore expect such particles not to be fully distinguishable, or, from the formal point of view, the states of such particles are not orthogonal. Indeed, it is easy to prove that the following relation holds true for unstable particles A and B:

$$\langle B|A\rangle = \frac{R_{AB}}{{}^1/_2(\Gamma_A+\Gamma_B)+i(m_A-m_B)}. \qquad (10)$$

Here

$$\Gamma_A = \sum_m |f_{A\to m}|^2; \quad \Gamma_B = \sum_m |f_{B\to m}|^2; \quad R_{AB} = \sum_m f_{A\to m} f^*_{B\to m};$$

$f_{A\to m}$ and $f_{B\to m}$ are the decay amplitudes defined in Sec. 2; the summation sign includes also integration with respect to the angles (see[8])[4)].

It is important that in the case of unstable particles with identical spins, parities, and other conserved quantum numbers, the quantity $R_{AB} \to 0$, i.e., $\langle B|A\rangle \neq 0$. Unstable particles for which $0 < |\langle A|B\rangle|^2 < 1$ will be called "quasi-identical." It is easy to see that if $|m_A - m_B| \ll \Gamma_A, \Gamma_B$ and $f_{A\to m} \to f_{B\to m}$, then $\langle A|B\rangle \to 1$. On the other hand, if the particles are stable, then in accordance with (10), for any non-zero mass difference, the states $|A\rangle$ and $|B\rangle$ are orthogonal.

We proceed now to analyze the process of registration of a pair of particles A and B by one detector (we shall assume first that the pairs A and B are not produced). It is easy to understand that if the particles A and B are in principle distinguishable, no interference takes place. Indeed, by observing, after the measurement act, a particle that has not fallen in the measuring instrument, we can identify it and by the same token uniquely indicate which of the particles, A or B, was registered by the instrument. From the formal point of view, the particles in the counter do not interfere if their states are orthogonal. It is important to emphasize that although in the registration of such particles in correlation experiments we can observe interference under certain conditions, in the case when the second detector is removed, the interference term becomes identically equal to zero no matter what the registration method employed.

As to the aforementioned "quasi-identical" particles, their states are not fully distinguishable, and we can therefore expect the presence of interference when pairs of these particles are registered by a single counter. For a more detailed analysis of this question, we return to the correlation formulas (1) and (2). Let us assume that we have taken away the second detector. It is clear that the registration of the decays $A \to n$ and $B \to n$ by one detector is equivalent to registration of the correlations of these decays with all the decays of the particles A and B at all distances in a kinematically-conjugate direction. Consequently, the probability of registration of the decays $A \to n$ and $B \to n$ by a detector located at a distance $l_1 = v_1\gamma_1 t_1$ from the region of production of the packets A and B is proportional to the quantity

$$dP_n = Q_n^{(\pm)}(t_1)\,dt_1,$$

where

$$Q_n^{(\pm)}(t_1) = \int_0^\infty \sum_m |P^{(\pm)}_{n,m}(t_1,t_2)|^2\,dt_2, \qquad (11)$$

and the expression $P^{(\pm)}_{n,m}(t_1, t_2)$ is determined from formula (2).

Elementary integration yields

$$Q_n^{(\pm)}(t_1) = |f_{A\to n}|^2|a(1.2)|^2 e^{-\Gamma_A t_1} + |f_{B\to n}|^2 |a(2.1)|^2 e^{-\Gamma_B t_1} \pm$$
$$\pm 2\,\mathrm{Re}\left\{a(1.2)a^*(2.1) f_{A\to n} f^*_{B\to n}\left[\frac{R_{AB}}{(\Gamma_A+\Gamma_B)/2 + i(m_B-m_A)}\right] \times e^{-i(m_A-m_B)t_1}\right\} e^{-(\Gamma_A+\Gamma_B)t_1/2}. \qquad (12)$$

The expression in the square brackets in the interference term is the measure of the non-orthogonality of the states $|A\rangle$ and $|B\rangle$ (see formula (10)). In particular, if we register the scattering of particles A and B in the c.m.s. ($a(1, 2) = f(\theta)$, $a(2, 1) = f(\pi - \theta)$, where θ is the scattering angle), then

$$Q_n^{(\pm)}(t_1) = |f_{A\to n}|^2 |f(\theta)|^2 e^{-\Gamma_A t_1} + |f_{B\to n}|^2 |f(\pi-\theta|^2 e^{-\Gamma_B t_1} \pm$$
$$\pm 2e^{-(\Gamma_A+\Gamma_B)t_1/2}\,\mathrm{Re}\{f(\theta)f^*(\pi-\theta) f_{A\to n} f^*_{B\to n} e^{-i(m_A-m_B)t_1}\langle A|B\rangle\}. \qquad (13)$$

The structure of formulas (12) and (13) does not depend on whether the decays of particles A and B are registered or their interactions. It follows from (13) that for "quasi-identical" particles the interference terms differ from zero. It is precisely such particles (and only such) which interfere when they are registered by a single detector.

In the limit, when $m_A - i\Gamma_A/2 \to m_B - i\Gamma_B/2$ we obtain the known formula for the scattering cross section of identical particles. We note that the cross section for the production or scattering of two unstable particles A and B is connected with the quantities $Q_n(t_1)$ and $P_{n,m}(t_1, t_2)$, which characterize the probabilities registration, by means of the simple relation

$$\sigma^{(\pm)} \sim \int_0^\infty dt_1 \int_0^\infty dt_2 \sum_n \sum_m P^{(\pm)}_{n,m}(t_1,t_2) = \int_0^\infty dt_1 \sum_n Q_n^{(\pm)}(t_1)$$
$$= |a(1,2)|^2 + |a(2,1)|^2 \pm 2|\langle A|B\rangle|^2\,\mathrm{Re}\,a(1.2)a^*(2.1). \qquad (14)$$

[4)]Formula (10) for neutral K mesons was obtained in [6] (see also [7]). If CP-parity is conserved, then $\sum_n f_{K_S\to n} f^*_{K_L\to n} = 0$ and $\langle K_S | K_L\rangle = 0$; when CP invariance is violated, generally speaking, we have $\langle K_L | K_S\rangle = 0$. From the manner in which it is derived, formula (10) is valid for all unstable particles.

When $\langle B|A\rangle \neq 0$, the quantity $\sigma \neq |a(1, 2)|^2 + |a(2, 1)|^2$. This means that in the case when a pair of unstable particles with identical conserved quantum numbers is produced, the interference term differs from zero no matter what the registration method. In this case, the concepts of non-identity and identity of the particles, obviously, no longer have that absolute significance as in the case of stable particles.

5. FURTHER DISCUSSIONS AND EXAMPLES

If the state (3) is produced as the result of the interaction act, then we obtain for the probability of registering the decays $A \to n$ and $B \to n$ by one detector the more general expression:

$$dP_n = \tilde{Q}_n^{(\pm)}(t_1)\,dt_1,$$

$$\tilde{Q}_n^{(\pm)}(t_1) = \sum_m \int_0^\infty |\tilde{P}_{n,m}^{(\pm)}(t_1, t_2)|^2\,dt_2, \tag{15}$$

where $\tilde{P}_{n,m}^{(\pm)}(t_1, t_2)$ is determined in accordance with (4).

We note that expression (15) with allowance for relations (5) describes the probability of registering the decays of the pair $K^0\overline{K}^0$ by means of one detector. The interference phenomena occurring in the registration of the $K^0\overline{K}^0$ pair with one detector were investigated earlier in[9]. In particular it was shown there that when CP invariance is violated, when the states K_L and K_S, generally speaking, are not orthogonal, the expression for the probability of registration of the decays of the system K_LK_S (which is produced, for example, in the process $\varphi \to K_SK_L$) contains an oscillating term proportional to the magnitude of the non-orthogonality $\langle K_L|K_S\rangle$, in full agreement with formula (12). It is easy to see that in the general case of a production of a $K^0\overline{K}^0$ pair, the sum of the readings of the two counters, which are not connected in coincidence and are located at equal distances from the point of pair production in kinematically-conjugate directions, does not contain an oscillating interference term if the states K_L and K_S are orthogonal. When $\langle K_L|K_S\rangle \neq 0$, the oscillating term is proportional to the quantity

$$\mathrm{Re}\{\langle K_L / K_S\rangle f^*_{K_S\to n} f_{K_L\to n} e^{i(m_L-m_S)t_1}\}\, e^{-(\Gamma_S+\Gamma_L)t_1/2}. \tag{15}$$

If the detector registers all the decay modes of the particles A and B, then the registration probability is obviously proportional to the quantity $\sum_n Q^{(\pm)}_{n(t_1)}$. It is easy to see that the expression for the registration probability contains oscillating interference terms, which are linear and quadratic in the quantity $\langle A|B\rangle$. In this case

$$\int_0^\infty \Big(\sum_n \tilde{Q}_n^{(\pm)}(t_1)\Big)dt_1 = \langle \psi^{(\pm)}(1,2)\,|\,\psi^{(\pm)}(1,2)\rangle, \tag{16}$$

where $|\psi^{(\pm)}(1, 2)\rangle$ is the vector of the state (3). If $F_{AA} = F_{BB} = 0$, we arrive at formula (14).

We proceed now to the case of production of pairs of short-lived unstable particles (resonances) with identical conserved quantum numbers. The states of such particles, according to (10), should be non-orthogonal. In Sec. 3 we have considered the correlations in the distribution of the effective masses of the $A, B \to n$ and $A, B \to m$ decay products. Let us find now this distribution in the case of registration with one detector. To this end, it is necessary to sum formula (7) over all the $A, B \to m$ decay modes with total momentum $\mathbf{p}_2$, and integrate over the effective masses M_2. The integration and the summation of the interference term in formula (7) yields

$$\sum_m \frac{1}{2\pi}\int_0^\infty \frac{f_{B\to m} f^*_{A\to m}\,dM_2}{(M_2 - m_B + i\Gamma_B/2)(M_2 - m_A + i\Gamma_A/2)}$$
$$= \frac{R_{AB}}{(\Gamma_A+\Gamma_B)/2 + i(m_B - m_A)} = \langle A|B\rangle.$$

As the result we obtain

$$dW_n^{(\pm)} = \sum_m \int_0^\infty d^2W_{n,m}^{(\pm)}\,dM_2$$
$$= \frac{1}{2\pi}\Big\{\frac{|f_{A\to n}|^2|a(\mathbf{p}_1,\mathbf{p}_2)|^2}{(M_1-m_A)^2+\Gamma_A^2/4} + \frac{|f_{B\to n}|^2|a(\mathbf{p}_2,\mathbf{p}_1)|^2}{(M_1-m_B)^2+\Gamma_B^2/4}$$
$$\pm 2\,\mathrm{Re}\Big(\langle A|B\rangle f_{A\to n} f^*_{B\to n}\frac{a(\mathbf{p}_1,\mathbf{p}_2)\,a^*(\mathbf{p}_2,\mathbf{p}_1)}{(M_1-m_A+i\Gamma_A/2)(M_1-m_B-i\Gamma_B/2)}\Big)\Big\}\,dM_1 \tag{17}$$

When $\langle A|B\rangle \neq 0$, the effective-mass spectrum (17) does not reduce to a sum of two Breit-Wigner terms. In the case of production of the resonance pairs AA, BB, and AB, the effective-mass spectrum of the $A, B \to n$ decay products is given by

$$d\tilde{W}_n^{(\pm)} = \sum_m \int_0^\infty d^2\tilde{W}_{n,m}^{(\pm)}\,dM_2, \tag{18}$$

where $d^2\tilde{W}_{m,n}^{(\pm)}$ is determined from formula (9).

Formula (18), the explicit form of which will not be presented because of its complexity (see[12]), describes, in particular, the distribution of the effective masses of two α particles in reactions with production of two excited states of the nuclei Be_1^{*8} and Be_2^{*8} with energies 16.6 and 16.9 MeV[5)]. An example may be the still uninvestigated process $\alpha + C^{12} \to 2Be_1^{*8} \to 4\alpha$. In this reaction there is produced a linear combination of the pairs $Be_1^{*8}Be_1^{*8}$, $Be_2^{*8}Be_2^{*8}$, and $Be_1^{*8}Be_2^{*8}$, with zero total isotopic spin. Since the Be_1^{*8} and Be_2^{*8} levels have the same spins and parities (2^+), and their widths are comparable with the energy difference, it follows from (10) that the degree of non-orthogonality is $\langle Be_1^{*8}|Be_2^{*8}\rangle \sim 1$, and the effect of the deviation of the effective-mass spectrum from the Breit-Wigner form should be appreciable (see[8]). We assume that the α particles produced in the case of decay of Be^{*8} nuclei moving in opposite directions in the c.m.s. are kinematically distinguishable. Such a situation takes place at sufficiently high energy of the primary α particles.

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