ELECTRIC AND MAGNETIC PROPERTIES OF A SEMICONDUCTOR IN THE FIELD OF A STRONG ELECTROMAGNETIC WAVE

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The electric and magnetic properties of a semiconductor in a strong electromagnetic field with a frequency exceeding the width of the forbidden band are investigated. Interaction between the electrons and the strong electromagnetic field is rigorously taken into account and an exact solution of the problem is obtained by means of a canonical transformation. Quasiparticles with a new dispersion law subsequently appear. The interaction of the quasiparticles with each other and with the phonons is taken into account on the basis of perturbation theory and this permits one to derive the kinetic equations for the quasiparticles by the standard method. A characteristic feature of the quasiparticle spectrum is the presence of a gap. In this connection, the electromagnetic properties of the system are analyzed by methods used in superconductivity theory. It is demonstrated that there is no anomalous diamagnetism in the stationary state. On the other hand, the absorption of the additional electromagnetic wave of frequency lower than the gap width decreases sharply. It is also shown that the presence of a gap changes significantly the shape of the current-voltage characteristic of the transition between a semiconductor and a semiconductor in the saturation state.

INTRODUCTION

THE recent development of quantum generators has raised a new set of problems, which can be arbitrarily called electrodynamics of strong electromagnetic waves. From the purely theoretical point of view, these problems are distinguished by the large value of the interaction between the charged particles in the wave and by the impossibility of using perturbation theory with respect to this interaction. The general approach to these problems should obviously consist of a rigorous allowance for the interaction with the wave and the use of different approximations for the interaction of the charged particles with other objects. This is precisely the approach used in many papers pertaining to electrodynamics in vacuum[1] and to resonant interaction with two-level molecules[2].

In the present paper we develop a similar approach as applied to the interaction of a strong electromagnetic wave with a semiconductor. The most essential factors here are the electronic transitions between the valence band and the conduction band under the influence of the electromagnetic field. By virtue of the momentum conservation law, the matrix elements of the transition connect only two states with each other (one in the valence band and one in the conduction band). The problem is therefore analogous to the problem of a two-level system in an alternating external field[3] and can be solved rigorously. This solution is obtained by means of a canonical transformation, which gives rise to quasiparticle, with a new dispersion law. The interaction of these quasiparticles with one another and with the phonons can be taken into account by perturbation theory, making it possible to obtain by a standard method a kinetic equation for the distribution functions of the quasiparticles. In the employed approximations, all the observed quantities are expressed in terms of the distribution function of the quasiparticles and of the parameters of the canonical transformation.

In this paper we develop the appropriate technique, we determine the quasiparticle dispersion law, and we construct the kinetic equations. In the particular case of zero temperature and absence of recombination, we obtain the stationary state of the system and calculate the interband absorption coefficient.

A characteristic feature of the energy spectrum of the quasiparticles is the presence of a gap that depends on the angle between the directions of the momentum and of the electric field of the wave. In this connection, it is of interest to examine the electromagnetic properties of the system; this is done with the aid of methods usually employed in superconductivity theory[4]. It is shown that in the stationary state there is no anomalous diamagnetism. However, the presence of a gap changes the character of the absorption of the additional weak low-frequency electromagnetic wave, and greatly changes the current-voltage characteristic of the tunnel transition between the semiconductor and the semiconductor in the saturated state.

The proposed method is valid, obviously, when the frequency of the transitions between the bands, under the influence of the incident wave, is larger than the collision frequency:

$$^1/_2 e\mathbf{A}_0 \mathbf{v}_{cv} \equiv \lambda \gg 1/\tau, \tag{1}$$

$\mathbf{v}_{cv}$ is the matrix element of the interband transition. τ in the right side of the inequality (1) should be taken to mean the smallest of the times of electron-electron or electron-phonon collision or the recombination time τ_R. The inequality $\lambda\tau_R \gg 1$ is the necessary condition for saturation, and is always satisfied in quantum generators. We neglect in this paper also the nonreso-

nant part of the interaction in the interband transitions as well as the oscillations of the electron momentum under the influence of the field within the limits of one band. We use a system of units with $\hbar = c = 1$.

1. FORMULATION OF PROBLEM AND CANONICAL TRANSFORMATION

Let us consider a semiconductor placed in the field of a strong electromagnetic wave with vector potential

$$\mathbf{A} = \mathbf{A}_0 \cos(\omega t - \mathbf{kr}), \quad \mathbf{kA}_0 = 0. \tag{2}$$

The wave frequency ω is close to the distance Δ between the conduction band and the valence band

$$0 < \omega - \Delta \ll \Delta. \tag{3}$$

The electrons in the crystal are described by Bloch wave functions

$$\psi_{lp}(\mathbf{r}) = e^{i\mathbf{pr}} u_{lp}(\mathbf{r}),$$

where $\mathbf{p}$ is the quasimomentum and l is the number of the band.

By virtue of condition (3), the principal role in the calculation of the wave function of the crystal in the field is played by resonant transitions of electrons between the top of the valence band and the bottom of the conduction band, for which the condition $E_C(\mathbf{p}) - E_V(\mathbf{p}) - \omega = 0$ is satisfied. Accordingly, we include the resonant part of the interaction with the electromagnetic field in the zeroth-approximation Hamiltonian, the eigenstates of which will be determined exactly:

$$H_0(t) = \sum_{\mathbf{p}} E(\mathbf{p})(a_{\mathbf{p}}^+ a_{\mathbf{p}} + b_{-\mathbf{p}}^+ b_{-\mathbf{p}}) + \lambda(\mathbf{p})(a_{\mathbf{p}}^+ b_{-\mathbf{p}}^+ e^{-i\omega t} + b_{-\mathbf{p}} a_{\mathbf{p}} e^{i\omega t}). \tag{4}$$

Here $a_{\mathbf{p}}^+$ and $b_{\mathbf{p}}^+$ are the operators for the production of an electron in the conduction band and a hole in the valence band,

$$\lambda(\mathbf{p}) = \frac{1}{2} e\mathbf{A}_0 \mathbf{v}_{cv}(\mathbf{p}), \qquad \mathbf{v}_{cv}(\mathbf{p}) = \frac{1}{m}\int u_{c\mathbf{p}}^* \hat{\mathbf{p}} u_{v\mathbf{p}}\, d^3r. \tag{5}$$

The matrix element $\mathbf{v}_{CV}(\mathbf{p})$ can be regarded, without loss of generality, as real. For simplicity we shall henceforth consider a quadratic dispersion law for the electrons and holes, with identical effective masses:

$$E(\mathbf{p}) = \Delta/2 + p^2/2m. \tag{6}$$

As is well known, at the frequency $\omega \sim \Delta$, we can neglect the wave vector $\mathbf{k}$ compared with the electron quasimomentum, as was indeed done in (4) and (5).

We first use the unitary transformation

$$U(t) = \exp\left\{-\frac{\omega t}{2}\sum_{\mathbf{p}} a_{\mathbf{p}}^+ a_{\mathbf{p}} + b_{\mathbf{p}}^+ b_{\mathbf{p}}\right\} \tag{7}$$

to change over to a representation in which the Hamiltonian (4) does not depend on the time:

$$H_0 = U^+(t) H_0(t) U - iU^+ \frac{\partial}{\partial t} U = \sum_{\mathbf{p}} \xi(\mathbf{p})(a_{\mathbf{p}}^+ a_{\mathbf{p}} + b_{\mathbf{p}}^+ b_{\mathbf{p}}) \times \lambda(\mathbf{p})(a_{\mathbf{p}}^+ b_{-\mathbf{p}}^+ + b_{-\mathbf{p}} a_{\mathbf{p}}), \tag{8}$$

where

$$\xi(\mathbf{p}) = \frac{p^2 - p_0^2}{2m}, \qquad \frac{p_0^2}{2m} = \frac{\omega - \Delta}{2}.$$

Henceforth all the operators and the wave functions will be considered in the representation (7).

By means of the canonical transformation

$$a_{\mathbf{p}} = u_{\mathbf{p}} \alpha_{\mathbf{p}} + v_{-\mathbf{p}} \beta_{-\mathbf{p}}^+, \qquad b_{-\mathbf{p}} = u_{\mathbf{p}} \beta_{-\mathbf{p}} - v_{-\mathbf{p}} \alpha_{\mathbf{p}}^+ \tag{9}$$

the Hamiltonian (8) is reduced to diagonal form

$$H_0 = \sum_{\mathbf{p}} \varepsilon(\mathbf{p})(\alpha_{\mathbf{p}}^+ \alpha_{\mathbf{p}} + \beta_{-\mathbf{p}}^+ \beta_{-\mathbf{p}}). \tag{10}$$

the new operators satisfy the usual anticommutation relations $\{\alpha_{\mathbf{p}}^+ \alpha_{\mathbf{p}}\} = \{\beta_{\mathbf{p}}^+ \beta_{\mathbf{p}}\} = 1$, and the remaining anticommutators are equal to zero. The functions $u_{\mathbf{p}}$ and $v_{-\mathbf{p}}$ should satisfy here the system of equations

$$u_{\mathbf{p}}^2 + v_{-\mathbf{p}}^2 = 1, \qquad 2\xi(\mathbf{p}) u_{\mathbf{p}} v_{-\mathbf{p}} + \lambda(\mathbf{p})(u_{\mathbf{p}}^2 - v_{-\mathbf{p}}^2) = 0,$$

the solution of which we choose in the form

$$u_{\mathbf{p}}^2 = \frac{1}{2}\left(1 + \frac{\xi(\mathbf{p})}{\varepsilon(\mathbf{p})}\right), \quad v_{-\mathbf{p}}^2 = \frac{1}{2}\left(1 - \frac{\xi(\mathbf{p})}{\varepsilon(\mathbf{p})}\right), \quad u_{\mathbf{p}} v_{-\mathbf{p}} = -\frac{\lambda(\mathbf{p})}{2\varepsilon(\mathbf{p})}, \tag{11}$$

where

$$\varepsilon(\mathbf{p}) = \sqrt{\xi^2(\mathbf{p}) + \lambda^2(\mathbf{p})}. \tag{12}$$

The operators $\alpha_{\mathbf{p}}$ ($\alpha_{\mathbf{p}}^+$) and $\beta_{\mathbf{p}}$ ($\beta_{\mathbf{p}}^+$) are the operators of annihilation (creation) of quasiparticles of type α and β. It is seen from (9) and (11) that when $\xi < 0$ and $|\xi| \gg |\lambda|$ the creation of a pair of quasiparticles (α, β) means annihilation of an electron-hole pair; when $\xi > 0$ and $\xi \gg |\lambda|$, the creation of a pair of quasiparticles means creation of an electron-hole pair.

The function $\epsilon(\mathbf{p})$ is the energy of a quasiparticle with momentum $\mathbf{p}$. As follows from (12), the minimal value of the energy is

$$\varepsilon_{min} = |\lambda(\mathbf{p})|,$$

and this mean value is reached at a momentum $|\mathbf{p}| = p_0$. Thus, the energy spectrum of the quasiparticles has a gap. From the definition of $\lambda(\mathbf{p})$ (5) we see that the magnitude of the gap depends on θ—the angle between the direction of the momentum and the direction of the electric field in the wave. In the isotropic model assumed by us, the matrix element $\mathbf{v}_{CV}(\mathbf{p})$ is directed along the momentum $\mathbf{p}$, and consequently

$$\lambda(\mathbf{p}) = \lambda \cos\theta. \tag{13}$$

We note that there exists directions from which the gap vanishes.

Let us find the state produced in our system after the passage of the leading front of the wave. If the width t_0 of this front satisfies the inequality $t_0 \gg 1/\lambda$, then the passage of the front can be regarded as an adiabatic switching-on of the field. At each instant of time the state of the system is described in terms of the operators $\alpha_{\mathbf{p}}$ and $\beta_{\mathbf{p}}$, which are defined by the transformations (9) with instantaneous value λ. In the adiabatic approximation, the conserved quantities are the quantum numbers, i.e., in our case the occupation numbers $\alpha_{\mathbf{p}}^+ \alpha_{\mathbf{p}}$ and $\beta_{\mathbf{p}}^+ \beta_{\mathbf{p}}$. Consequently, their values after switching on the interaction coincides with the values in the absence of the field ($\lambda = 0$). Assuming that in the absence of the field the crystal is in the ground state, we have

$$\alpha_{\mathbf{p}}^+ \alpha_{\mathbf{p}} = \begin{cases} a_{\mathbf{p}}^+ a_{\mathbf{p}} = 0, & p > p_0 \\ b_{-\mathbf{p}} b_{-\mathbf{p}}^+ = 1, & p < p_0 \end{cases} \tag{14}$$

For $\beta_{\mathbf{p}}^+ \beta_{\mathbf{p}}$ we obtain the same result. Thus, in the state produced after the passage on the leading front of the wave, the distribution of the quasiparticle has the form of a Fermi step with limiting momentum p_0.

2. KINETIC EQUATIONS FOR QUASIPARTICLES

The Fermi distributions for the quasiparticles (14) are valid only in the absence of collisions. The presence of collisions leads to relaxation of these distributions. The relaxation is described by the kinetic equations for the quasiparticle distribution function. The most appreciable contribution to the collision integral of the kinetic equations is made by the electron-electron and electron-phonon interactions. However, as we shall show below, the result of the relaxation (the stationary distribution function) is physically obvious in the absence of recombination and does not depend on the form of the collisions. We shall therefore obtain the kinetic equation for the simplest case of electron-phonon interaction.

The Hamiltonian of the interaction between the electrons or holes or the phonons is of the form

$$H_{ef} = \sum_{\mathbf{pp'q}} G(\mathbf{p}, \mathbf{p'}, \mathbf{q})\,(a_{\mathbf{p}}^{+}a_{\mathbf{p'}} + b_{-\mathbf{p}}b_{-\mathbf{p'}}^{+})\,c_{\mathbf{q}}^{+} + \text{h.c.} \qquad (15)$$

where $G(\mathbf{p}, \mathbf{p'}, \mathbf{q}) = g(\mathbf{q})\delta(\mathbf{p} + \mathbf{q} - \mathbf{p'})$, and for simplicity we assume $g(\mathbf{q})$ to be the same for both hands; $c_{\mathbf{q}}^{+}$ is the phonon creation operator.

By subjecting the Hamiltonian (15) to the transformation (7) and going over to quasiparticle operators, we get

$$H_{ef} = \sum_{nn'\mathbf{q}} \Phi(n, n', \mathbf{q})\,\alpha_n^{+}\alpha_{n'}\,c_{\mathbf{q}}^{+} + \text{h.c.} \qquad (16)$$
$$\Phi(1\mathbf{p}, 1\mathbf{p'}, \mathbf{q}) = \Phi(2\mathbf{p}, 2\mathbf{p'}, \mathbf{q}) = G(\mathbf{p}, \mathbf{p'}, \mathbf{q})\,(u_{\mathbf{p}}u_{\mathbf{p'}} + v_{-\mathbf{p}}v_{-\mathbf{p'}}),$$
$$\Phi(1\mathbf{p}, 2\mathbf{p'}, \mathbf{q}) = -\Phi(2\mathbf{p}, 1\mathbf{p'}, \mathbf{q}) = G(\mathbf{p}, \mathbf{p'}, \mathbf{q})\,(u_{\mathbf{p}}v_{-\mathbf{p'}} - u_{\mathbf{p'}}v_{-\mathbf{p}}).$$

To abbreviate the notation, we have introduced here the symbol $n \equiv (i, \mathbf{p})$, $i = 1, 2$, with $\alpha_{1\mathbf{p}} = \alpha_{\mathbf{p}}$ and $\alpha_{2\mathbf{p}} = \beta_{-\mathbf{p}}^{+}$.

The Hamiltonian (16) describes the processes of emission and absorption of phonons by quasiparticles, and also the annihilation of a pair of quasiparticles into a phonon and the inverse process of creation of a pair by a phonon. It is precisely the pair creation and annihilation processes which lead to a result that is at first glance unexpected, namely the relaxation of the Fermi distributions.

Assuming the interaction to be small, it is easy to calculate in first order of perturbation theory the probabilities of the different processes. The kinetic equations for the distribution functions

$$f^{\alpha}(\mathbf{p}) = \langle \alpha_{\mathbf{p}}^{+}\alpha_{\mathbf{p}} \rangle, \quad f^{\beta}(\mathbf{p}) = \langle \beta_{\mathbf{p}}^{+}\beta_{\mathbf{p}} \rangle$$

are now written in standard fashion. For example,

$$\frac{\partial}{\partial t} f^{\alpha}(\mathbf{p}) = 2\pi \sum_{\mathbf{p'q}} |\Phi(1\mathbf{p}, 1\mathbf{p'}, \mathbf{q})|^2 \delta(\varepsilon(\mathbf{p}) + \omega(\mathbf{q}) - \varepsilon(\mathbf{p'}))$$
$$\times \{f^{\alpha}(\mathbf{p'})\,(1 - f^{\alpha}(\mathbf{p})) + N(\mathbf{q})\,(f^{\alpha}(\mathbf{p'}) - f^{\alpha}(\mathbf{p}))\} -$$
$$- |\Phi(1\mathbf{p'}, 1\mathbf{p}, \mathbf{q})|^2 \delta(\varepsilon(\mathbf{p'}) + \omega(\mathbf{q}) - \varepsilon(\mathbf{p}))$$
$$\times \{f^{\alpha}(\mathbf{p})\,(1 - f^{\alpha}(\mathbf{p'})) + N(\mathbf{q})\,(f^{\alpha}(\mathbf{p}) - f^{\alpha}(\mathbf{p'}))\} - |\Phi(1\mathbf{p'}, 2\mathbf{p}, \mathbf{q})|^2$$
$$\times \delta(\varepsilon(\mathbf{p}) + \varepsilon(-\mathbf{p'}) - \omega(\mathbf{q}))\,\{f^{\alpha}(\mathbf{p}) f^{\beta}(-\mathbf{p'}) + N(\mathbf{q})\,(f^{\alpha}(\mathbf{p}) + f^{\beta}(-\mathbf{p'}) - 1)\}. \qquad (17)$$

Here $N(\mathbf{q})$ is the phonon distribution function, which we shall henceforth assume to be in equilibrium with the temperature T. Equations (17) should be supplemented by the electroneutrality condition

$$\sum_{\mathbf{p}} f^{\alpha}(\mathbf{p}) = \sum_{\mathbf{p}} f^{\beta}(\mathbf{p}),$$

which follows from the condition that the number of electrons and holes must be equal.

When $\lambda \to 0$ Eqs. (17) go over into the ordinary kinetic equations for the electrons and holes interacting with the phonons. When λ differs from zero, Eqs. (17) differ from the ordinary kinetic equation in that the electromagnetic field enters not in the left, dynamic part of the equation, but in the collision integral, in the amplitudes of the Φ and δ functions. The latter circumstance, in the case of λ comparable with the Debye frequency ω_D indicates apparently that the field influences the collision act. We note that the δ function in the third term of (17) leads in quasiparticle-annihilation processes to certain hindrances connected with the gap in the spectrum. This, however, does not influence the form of the stationary distribution function when $1/\tau_R = 0$.

Let us find the stationary distribution of the quasiparticles. From the condition that the collision integrals must vanish it follows that the functions f^{α} and f^{β} should be the Fermi and chemical potentials μ_{α} and μ_{β}, satisfying the condition $\mu_{\alpha} + \mu_{\beta} = 0$. The electroneutrality condition leads to the requirement that the chemical potentials be equal, $\mu_{\alpha} = \mu_{\beta}$, from which it follows that each of them must vanish, so that

$$f^{\alpha}(\mathbf{p}) = f^{\beta}(-\mathbf{p}) = [\exp(\varepsilon(\mathbf{p}) / T) + 1]^{-1}. \qquad (18)$$

This result is physically obvious. It follows directly from our approach, in which the interaction leads only to transitions of the quasiparticles or to annihilation. The stationary distribution is in this case the Gibbs distribution with Hamiltonian (11), and the phonons play the role of the thermostat. The Gibbs distribution together with the electroneutrality condition leads, as is well known, to the functions (18). These general considerations show that the final result of the relaxation, i.e., the form of the distribution functions (18), does not depend on the character of the collisions. A specially simple result is obtained when T = 0:

$$f^{\alpha}(\mathbf{p}) = f^{\beta}(\mathbf{p}) = 0. \qquad (19)$$

The distribution of the electrons and holes has in this case the form

$$f^{e}(\mathbf{p}) = f^{h}(-\mathbf{p}) = v_{-\mathbf{p}}^{2},$$

i.e., the form of a step which is smeared out over a width $m\lambda/p_0$ at momentum values near p_0 (when $p_0^2 \gg m\lambda$, as is henceforth assumed. In the case $\lambda \to 0$ this result can be readily understood from simple physical considerations. Indeed, by virtue of the conservation laws, the electromagnetic wave realizes transitions of electrons with momentum p_0 from the lower band to the upper one; in the case of phonon emission (T = 0!) the electrons only lose energy, filling all the states with lower momenta.

As seen from (19), at zero temperatures the system relaxes to a state with zero occupation numbers with respect to the quasiparticles. In the subsequent sections we shall consider the properties of our system in the state Ψ_0 defined by the conditions

$$\alpha_{\mathbf{p}}\Psi_0 = \beta_{-\mathbf{p}}\Psi_0 = 0 \qquad (20)$$

This state will henceforth be called the saturation state.

3. COEFFICIENT OF ABSORPTION OF A STRONG ELECTROMAGNETIC WAVE

We define the absorption coefficient by the relation

$$\varkappa = Q / I,$$

where $Q = \overline{\langle \mathbf{j} \cdot \mathbf{E}(t) \rangle}$ is the energy dissipated in the crystal per unit time and per unit volume; the bar denotes averaging over the time; I is the intensity of the incident wave. That part of the current operator which is in phase with the electric field is given by

$$\hat{\mathbf{j}} = -ie \sin \omega t \sum_{\mathbf{p}} \mathbf{v}_{cv}(\mathbf{p}) (\alpha_{\mathbf{p}}^{+} \beta_{-\mathbf{p}}^{+} - \beta_{-\mathbf{p}} \alpha_{\mathbf{p}}). \tag{21}$$

In the state (20), the mean value of the current operator (21) vanishes, i.e., we deal with saturation of the interband absorption coefficient.

Allowance for the electron-electron and electron-phonon interactions in the saturation state does not lead to absorption. This is a natural result, for in this state the system is in equilibrium, in accordance with the results of the preceding section. A nonzero value of the absorption coefficient is obtained when recombination is taken into account. Let us consider the case of radiative recombination and let us take the interaction Hamiltonian in the form

$$H_R = \sum_{\mathbf{p}\mathbf{p}'\mathbf{k}\delta} \varphi(\mathbf{p}, \mathbf{p}', \mathbf{k}) e^{-i\omega t} b_{-\mathbf{p}} a_{\mathbf{p}'} d_{\mathbf{k}\delta}^{+} + \text{h.c.};$$

here

$$\varphi(\mathbf{p}, \mathbf{p}', \mathbf{k}) = e\sqrt{2\pi / \omega_{\mathbf{k}}} \mathbf{e}_{\mathbf{k}\delta} \mathbf{v}_{cv}(\mathbf{p}) \delta(\mathbf{p} + \mathbf{k} - \mathbf{p}'),$$

$\mathbf{e}_{\mathbf{k}\delta}$ is the quantum polarization vector, and $d_{\mathbf{k}\delta}^{+}$ is its creation operator.

After determining the corrections to the wave function (20) in second order of perturbation theory in H_R, we obtain for the absorption coefficient

$$\varkappa = \frac{\omega}{I} \sum_{\mathbf{p}} \frac{1}{\tau} u_{\mathbf{p}}^2 v_{-\mathbf{p}}^4. \tag{22}$$

Here $1/\tau_R = (4/3) e^2 \omega |\mathbf{v}_{cv}|^2$ is the probability of dipole radiation of the photon in the interband transition in the absence of a strong wave. In (22) we have neglected the wave vector of the quantum compared with the electron quasimomentum. Calculation of the integral in (22) leads to

$$\varkappa = \frac{m\omega p_0 \lambda}{16\pi I \tau_R} \qquad p_0^2 \gg m\lambda.$$

This result agrees with the corresponding result of[5], where the saturation effect was analyzed by a different method. Thus, the state (20) is indeed a saturation state.

To calculate the absorption coefficient at $T \neq 0$ we used the density-matrix formalism. With the aid of the standard procedure[6] equations were obtained for the single-particle quasiparticle density matrix $\langle \alpha_n^{+} \alpha_n \rangle$. Using the small parameter $1/\lambda\tau \ll 1$, these equations were solved by successive approximations. The zeroth approximation yields the kinetic equations (17) for the diagonal elements. In the next approximation, the density-matrix nondiagonal elements needed to calculate the absorption coefficient are expressed in terms of the solution of the kinetic equations. Not being able to present the results in the present paper, we note only that the foregoing notions concerning the saturation state remain unchanged.

4. DIAMAGNETIC PROPERTIES

In this section we consider the magnetic properties of a system in the saturation state in order to check on the presence of the Meissner effect. This analysis is stimulated by the presence of a gap in the quasiparticle spectrum. Naturally, the presence of a gap still does not mean that the system acquires superconducting properties, since the appearance of these properties requires that the gap exist in a reference frame in which the electrons as a whole are at rest, and this system can move arbitrarily relative to the lattice (the current state). In our case the gap is rigidly coupled to the lattice and a stable current state is impossible. By the same token, the appearance of the Meissner effect is impossible. It should be recognized, however, that the usual arguments that relate to the Meissner effect with the superconducting properties are valid for a closed system. In our case, the energy introduced into the system by the electromagnetic wave can compensate for the energy dissipation due to the screening currents. Therefore the absence of the Meissner effect is not obvious beforehand.

To consider the diamagnetic properties of the system, it is necessary to calculate the average value of the current produced under the influence of the constant (time-independent) magnetic field. Bearing in mind the fact that we shall also need in the future the response of the system to a weak electromagnetic field of frequency Ω, we present here the results for the general case, when the system is acted upon by a field with a vector potential

$$\mathbf{A} = \sum_{\mathbf{q}} \mathbf{A}(\mathbf{q}) e^{i\mathbf{q}\mathbf{r} - i\Omega t}, \qquad \mathbf{q}\mathbf{A}(\mathbf{q}) = 0. \tag{23}$$

Following the transformation (10), the Hamiltonian of the interaction of the electrons and of the holes with the field (23) takes the form

$$H_{eA} = \frac{e}{2m} \sum_{\mathbf{p}\mathbf{p}'\mathbf{q}} \delta(\mathbf{p} + \mathbf{q} - \mathbf{p}') \mathbf{A}(\mathbf{q}) \{\mathbf{P}(1\mathbf{p}, 1\mathbf{p}') a_{\mathbf{p}}^{+} a_{\mathbf{p}'} + \mathbf{P}(2\mathbf{p}, 2\mathbf{p}') b_{-\mathbf{p}} b_{-\mathbf{p}'}^{+} + \mathbf{P}(1\mathbf{p}, 2\mathbf{p}') a_{\mathbf{p}}^{+} b_{-\mathbf{p}}^{+} e^{i\omega t} + \mathbf{P}(2\mathbf{p}, 1\mathbf{p}') b_{-\mathbf{p}} a_{\mathbf{p}'} e^{-i\omega t}\} e^{-i\Omega t}, \tag{24}$$

where the matrix elements

$$\mathbf{P}(i\mathbf{p}, j\mathbf{p}') = \int d^3r \{e^{i\mathbf{p}\mathbf{r}} u_{i\mathbf{p}}^{*}(i\nabla) e^{-i\mathbf{p}\mathbf{r}} u_{j\mathbf{p}'} + e^{i\mathbf{p}'\mathbf{r}} u_{i\mathbf{p}}^{*}(i\nabla) e^{-i\mathbf{p}'\mathbf{r}} u_{j\mathbf{p}'}\}. \tag{25}$$

The current Fourier-component operator is

$$\hat{\mathbf{j}}(\mathbf{q}) = \hat{\mathbf{j}}^{(D)}(\mathbf{q}) + \hat{\mathbf{j}}^{(t)}(\mathbf{q}),$$

where

$$\hat{\mathbf{j}}^{(D)}(\mathbf{q}) = -\frac{e^2}{m} \mathbf{A}(\mathbf{q}) \sum_{\mathbf{p}} a_{\mathbf{p}}^{+} a_{\mathbf{p}} + b_{-\mathbf{p}} b_{-\mathbf{p}}^{+},$$

$$\hat{\mathbf{j}}^{(P)}(\mathbf{q}) = -\frac{e}{2m} \sum_{\mathbf{p}\mathbf{p}'} \delta(\mathbf{p} + \mathbf{q} - \mathbf{p}') \{a_{\mathbf{p}}^{+} a_{\mathbf{p}'} \mathbf{P}(1\mathbf{p}, 1\mathbf{p}') + b_{-\mathbf{p}} b_{-\mathbf{p}'}^{+} \mathbf{P}(2\mathbf{p}, 2\mathbf{p}')$$

$$\times a_{\mathbf{p}}^{+} b_{-\mathbf{p}'}^{+} \mathbf{P}(1\mathbf{p}, 2\mathbf{p}') e^{i\omega t} + b_{-\mathbf{p}} a_{\mathbf{p}'} \mathbf{P}(2\mathbf{p}, 1\mathbf{p}') e^{-i\omega t}\}.$$

Using standard perturbation-theory methods, we obtain the average value of the Fourier component of the current in the first approximation in the weak field

$$\mathbf{j}(\mathbf{q}) = -\frac{Ne^2}{m} \mathbf{A}(\mathbf{q}) + \frac{e^2}{2m^2} \sum_{\mathbf{p}\mathbf{p}'} \delta(\mathbf{p} + \mathbf{q} - \mathbf{p}') \cdot$$

$$\times\Big\{\frac{[\mathbf{A}(\mathbf{q})\mathbf{P}(1\mathbf{p},1\mathbf{p}')]\,u_{\mathbf{p}}v_{-\mathbf{p}'}-[\mathbf{A}(\mathbf{q})\mathbf{P}(2\mathbf{p},2\mathbf{p}')]u_{\mathbf{p}'}v_{-\mathbf{p}}}{\varepsilon(\mathbf{p})+\varepsilon(\mathbf{p}')-\Omega-i\delta}$$
$$\times[\mathbf{P}^*(1\mathbf{p},1\mathbf{p}')u_{\mathbf{p}}v_{-\mathbf{p}'}-\mathbf{P}^*(2\mathbf{p},2\mathbf{p}')u_{\mathbf{p}'}v_{-\mathbf{p}}]$$
$$\times\frac{[\mathbf{A}(\mathbf{q})\mathbf{P}(1\mathbf{p},2\mathbf{p}')]\,\mathbf{P}^*(1\mathbf{p},2\mathbf{p}')u_{\mathbf{p}}{}^2u_{\mathbf{p}'}{}^2}{\varepsilon(\mathbf{p})+\varepsilon(\mathbf{p}')+\omega-\Omega-i\delta}$$
$$\times\frac{[\mathbf{A}(\mathbf{q})\mathbf{P}(2\mathbf{p},1\mathbf{p}')]\,\mathbf{P}^*(2\mathbf{p},1\mathbf{p}')v_{-\mathbf{p}}^2v_{-\mathbf{p}'}^2}{\varepsilon(\mathbf{p})+\varepsilon(\mathbf{p}')-\omega-\Omega-i\delta}\Big\}, \qquad (26)$$

N is the electron concentration in a completely filled band.

Putting $\Omega = 0$ in (26), we obtain the current produced under the influence of the constant magnetic field. The expression for the current can be transformed by using the sum rule[7]. Then, as $q \to 0$, the last two terms in (26) add up together with the first term to make up the diamagnetic current corresponding to the electrons in the conduction band and to the holes in the valence band:

$$\mathbf{j}(\mathbf{q})=\mathbf{j}^{(D)}(\mathbf{q})+\mathbf{j}^{(P)}(\mathbf{q})=-\frac{(2n)e^2}{m}\mathbf{A}(\mathbf{q})+\frac{e^2}{2m^2}\sum_{\mathbf{p}\mathbf{p}'}\delta(\mathbf{p}+\mathbf{q}-\mathbf{p}')$$
$$\times\frac{[\mathbf{A}(\mathbf{q})\mathbf{P}(1\mathbf{p},1\mathbf{p}')]\,u_{\mathbf{p}}v_{-\mathbf{p}'}-[\mathbf{A}(\mathbf{q})\mathbf{P}(2\mathbf{p},2\mathbf{p}')]\,u_{\mathbf{p}'}v_{-\mathbf{p}}}{\varepsilon(\mathbf{p})+\varepsilon(\mathbf{p}')}$$
$$\times[\mathbf{P}^*(1\mathbf{p},1\mathbf{p}')u_{\mathbf{p}}v_{-\mathbf{p}'}-\mathbf{P}^*(2\mathbf{p},2\mathbf{p}')u_{\mathbf{p}'}v_{-\mathbf{p}}], \qquad (27)$$

where $n = 8\pi p_0^3/3(2\pi)^3$.

In a semiconductor with quadratic electron and hole dispersion we have for the matrix elements (25)

$$\mathbf{P}(1\mathbf{p},\,1\mathbf{p}')=\mathbf{p}+\mathbf{p}',\quad P(2\mathbf{p},\,2\mathbf{p}')=-(\mathbf{p}+\mathbf{p}'). \qquad (28)$$

Substituting (28) in (27) and symmetrizing, we obtain for the paramagnetic current

$$\mathbf{j}^{(P)}(\mathbf{q})=\frac{2e^2}{m^2}\sum_{\mathbf{p}}(\mathbf{A}(\mathbf{q})\mathbf{p})\mathbf{p}\frac{(u_{\mathbf{p}+\mathbf{q}/2}v_{-\mathbf{p}+\mathbf{q}/2}+u_{\mathbf{p}-\mathbf{q}/2}v_{-\mathbf{p}-\mathbf{q}/2})^2}{\varepsilon(\mathbf{p}+\mathbf{q}/2)+\varepsilon(\mathbf{p}-\mathbf{q}/2)}. \qquad (29)$$

We take the limit as $q \to 0$ in (29). Taking into account the transversality of the potential $\mathbf{A}(\mathbf{q})$, we obtain after some calculations

$$\mathbf{j}^{(P)}(\mathbf{q})\,|_{\mathbf{q}=0}=\frac{2ne^2}{m}\mathbf{A}(\mathbf{q}).$$

We see therefore that the paramagnetic current $\mathbf{j}^{(P)}$ cancels the diamagnetic current $\mathbf{j}^{(D)}$. Consequently, there is no Meissner effect in the model under consideration. The formal cause of the absence of the anomalous diamagnetism lies in the fact that the numerator of (29) contains a plus sign, in contrast to superconductivity theory. This is apparently connected with the fact that the states of the particle and of the hole are correlated in the system. There is a definite analogy here with the appearance of anomalous properties following pairing in semimetals[8]. We note also that deviations from the dispersion laws (6) and (28) can uncover new possibilities.

5. INTRABAND ABSORPTION

The properties of a superconductor in the saturated state placed in a high-frequency field can be explained in the same manner as in the case of a constant field. The expression for the mean value of the current is given by (26). We confine ourselves here to the imaginary part of the current (26), which determines the absorption of the weak electromagnetic wave with frequency $\Omega \ll \omega$ by electrons and holes. This absorption will be called intraband absorption, and the corresponding absorption coefficient is defined by

$$\mathrm{Im}\,j_i(\mathbf{q}\Omega)=A_j(\mathbf{q})K_{ij}(\mathbf{q}\Omega). \qquad (30)$$

The expression for the absorption coefficient simplifies if account is taken of the transversality of the potential (23) and if the most interesting case $\mathbf{A}(\mathbf{q}) \parallel \mathbf{A}_0$ is considered; we then obtain

$$K_{ij}(\mathbf{q}\Omega)=K(\mathbf{q}\Omega)\delta_{ij},$$
$$K(\mathbf{q}\Omega)=\frac{e^2}{(2\pi)^3m^2}\int d^3p\cos^2\theta\,\delta(\varepsilon_++\varepsilon_--\Omega)\frac{1}{2}\Big(1-\frac{\xi_+\xi_--\lambda^2(p)}{\varepsilon_+\varepsilon_-}\Big)$$

where

$$\xi_\pm=\xi\pm qv_0\sin\theta\cos\varphi,\quad v_0=p_0/m,$$
$$\varepsilon_\pm=\sqrt{\xi_\pm{}^2+\lambda^2(\mathbf{p})},\quad \lambda(\mathbf{p})=\lambda\cos\theta; \qquad (31)$$

the z axis is directed along $\mathbf{A}_0$, θ is the angle between the momentum direction and the z axis, and φ is the azimuthal angle.

We change over in (31) to new variables $y = v_0(p - p_0)$, $x = \cos\theta$, and $t = \cos\varphi$:

$$K(\mathbf{q}\Omega)=\frac{2e^2p_0^3}{\pi^2m}\int_0^1\frac{dt}{\sqrt{1-t^2}}\int_0^1x^2\,dx\int_0^\infty dy\,\delta(\varepsilon_++\varepsilon_--\Omega)\Big(1-\frac{\xi_+\xi_--\lambda^2x^2}{\varepsilon_+\varepsilon_-}\Big). \qquad (32)$$

In the absence of a gap ($\lambda \to 0$), formula (32) should describe the behavior of free electrons and holes having a distribution function (19). Taking the limit as $\lambda \to 0$ and calculating the integrals, we obtain for $qv_0 \gg \Omega$:

$$K(\mathbf{q}\Omega)=\frac{2ne^2}{m}\frac{3\pi}{4}\frac{\Omega}{qv_0}.$$

We see therefore that in this limiting case the intraband absorption is determined by the Landau damping, which is not surprising, since in our approximation $\lambda\tau \gg 1$ or $v_0/\lambda \ll v_0\tau = l$, i.e., the mean free path is much larger than v_0/λ. Further calculations are best carried out for certain concrete situations.

We consider first the "London" case, when $qv_0 \ll \Omega$. In a semiconductor, this approximation is of greatest interest, since the depth of the skin layer $\delta = (2\tau\sigma\Omega)^{-1}$ is usually large, owing to the relatively small conductivity of the semiconductor σ. Putting in (32) $q = 0$, we obtain after integration

$$K(\Omega)=\begin{cases}\dfrac{ne^2}{m}\dfrac{9\pi^2}{16}\Big(\dfrac{\Omega}{2\lambda}\Big)^3, & \Omega<2\lambda\\[2ex] \dfrac{ne^2}{m}\Big(\dfrac{2\lambda}{\Omega}\Big)^2 3\pi\displaystyle\int_0^1\frac{x^4\,dx}{\sqrt{1-x^2(2\lambda/\Omega)^2}}, & \Omega>2\lambda\end{cases} \qquad (33)$$

In the last expression we have when $\Omega \gg 2\lambda$

$$K(\Omega)=\frac{ne^2}{m}\frac{3\pi}{5}\Big(\frac{2\lambda}{\Omega}\Big)^2. \qquad (34)$$

It is seen from (33) that when $\Omega \approx 2\lambda$ the absorption coefficient reaches a maximum and decreases with decreasing frequency in proportion to Ω^{-3}. Such a $K(\Omega)$ dependence is due to the existence of a gap in the quasiparticle spectrum (12). Indeed, when $\Omega < 2\lambda$, the energy of the quantum is insufficient to produce a pair of quasiparticles. The difference between the absorption coefficient and zero (compare with[4] at $T = 0$) is due to the anisotropy of the gap $\lambda(p)$ (13).

Calculation of the absorption for another orientation of the fields $\mathbf{A}$ and $\mathbf{A}_0$, for example $\mathbf{A} \perp \mathbf{A}_0$, yields

$$K(\Omega)\sim\Omega/2\lambda,\quad \Omega<2\lambda. \qquad (35)$$

Consequently, the intraband absorption coefficient depends strongly on the angle between **A** and $\mathbf{A}_0$. The presence of anisotropy, and also the specific dependence of $K(\Omega)$ on the frequency, give reasons for hoping that the effect will become distinctly observable experimentally, and that the magnitude of the gap λ will be determined.

Let us consider further absorption in the "Pippard" case, when $qv_0 \gg \Omega$. We present the results of the calculation for $\Omega < 2\lambda$:

$$K(\Omega) = \frac{ne^2}{m}\frac{3\pi^2}{16}\left(\frac{\Omega}{2\lambda}\right)^3 \frac{\Omega}{qv_0}. \qquad (36)$$

This expression differs from (33) by the factor $3\Omega/qv_0$.

6. TUNNEL CURRENT THROUGH A "SEMICONDUCTOR IN THE SATURATED STATE" JUNCTION

It is known[9] that a study of the gap in the spectrum of a superconductor is best carried out by investigating the current-voltage characteristics of tunnel junctions. In our case, the analog of the "metal-superconductor" junction is the "semiconductor–semiconductor in the saturated state" junction. The currents through such a junction is given, in accordance with the phenomenological theory of tunnel junctions[9], by the expression

$$J = C\int_{-1}^{1} dx \int_{-V}^{0} \rho_N(\varepsilon)\,d\varepsilon, \quad \rho_N(\varepsilon) = \frac{mp_0|\varepsilon|}{2\pi^2\sqrt{\varepsilon^2 - \lambda^2 x^2}} \qquad (37)$$

where C is a constant, $\rho_N(\epsilon)$ is the density of the states of the semiconductor in the saturated state, and V is the external voltage. Integrating in (37), we get

$$J = C\frac{p_0 m}{8\pi}\frac{V^2}{\lambda}. \qquad (38)$$

We see therefore that the dependence of the current J on the voltage V becomes quadratic, and the value of λ can be determined from the shape of the current-voltage characteristic.

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