FLUCTUATIONS IN SEMICONDUCTORS IN A STRONG ELECTRIC FIELD AND THE SCATTERING OF LIGHT BY "HOT" ELECTRONS

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The fluctuations of the current density and of the electron concentration in a nonequilibrium state, which is created in a semiconductor due to the action of a strong electric field, are investigated theoretically. A kinetic equation is derived for the correlator of the fluctuations of the distribution function with both the electron-phonon scattering and the electron-electron interaction taken into consideration. An expression is obtained for the spectral density of the fluctuations in the case of small frequency ω and small wave vector $\mathbf{q}$. It is established that inter-electron collisions lead to an additional correlation between the electrons in a nonequilibrium state, violating the direct relation between the correlator of the currents and the diffusion coefficient of the "hot" electrons, a relation which exists when such collisions are neglected. A theory of the scattering of light by "hot" electrons is constructed. It is established that the intensity of the scattering may increase in the presence of an electric field, especially if the momentum transferred to the light during the scattering is directed along the electron drift. This is associated with the possibility that the effective screening radius may increase in the presence of a strong electric field, thanks to which the intensity of the long wavelength fluctuations of the electron concentration is increased.

1. INTRODUCTION

IN the present article the electron fluctuations in a nonequilibrium stationary state, arising in a semiconductor upon the imposition of a constant electric field, are investigated theoretically. In such a system a direct connection between the fluctuations of physical quantities and the reaction of the system to a weak external influence, similar to the connection which was established by Callen and Welton[1] for equilibrium systems,[1)] no longer exists.

However, it is more or less natural to expect that even in a nonequilibrium state both the reaction of the system and the fluctuations are described in a certain sense by similar kinetic equations. In fact, one can express the fluctuations of the current density, of the electron concentration, and of other physical quantities in terms of the fluctuations of the electron distribution function. If it is assumed that at each instant of time t the electron distribution function $F_{\mathbf{p}} + \delta F_{\mathbf{p}}(\mathbf{r}, t)$ fluctuates around the average value $F_{\mathbf{p}}$, then the fluctuations of the distribution function will be characterized by the correlators $\langle F_{\mathbf{p}}(\mathbf{r}, t+\tau)\delta F_{\mathbf{p}_1}(\mathbf{r}', t)\rangle$. Here the angular brackets denote an averaging over all instants of time t for a fixed value of τ. In what follows we shall frequently omit the argument t over which the averaging takes place.

We will show that the correlator $\langle \delta F_{\mathbf{p}}(\mathbf{r}, \tau)\delta F_{\mathbf{p}_1}(\mathbf{r}', 0)\rangle$ satisfies a kinetic equation with respect to the variables $\mathbf{p}$, $\mathbf{r}$, and τ. This, of course, constitutes the similarity of both problems which was mentioned above, since the change of the electron distribution function associated with an external influence on the system is determined from the same kinetic equation. However, in order to solve the kinetic equation it is necessary to specify the initial condition, i.e., the value of the desired function, for example, at $\tau = 0$. For a problem about fluctuations this means that it is necessary to know the quantity $\langle \delta F_{\mathbf{p}}(\mathbf{r})\delta F_{\mathbf{p}_1}(\mathbf{r}')\rangle$, which differs from the equal-time binary distribution function of the electrons by the additive term $F_{\mathbf{p}}F_{\mathbf{p}_1}$.

In the case of thermodynamic equilibrium the binary function can easily be expressed in terms of the one-particle function, and as a result the problem about the reaction of the system and about the fluctuations turn out to be completely equivalent. In the case of a nonequilibrium system, however, in order to find the binary equal-time distribution function it is necessary to solve the special kinetic equation, which this function satisfies, with respect to two pairs of variables: $\mathbf{p}$, $\mathbf{r}$ and $\mathbf{p}_1$, $\mathbf{r}'$. Thus, the theory of fluctuations in a nonequilibrium stationary state is described not by one equation but by two. It is precisely this aspect which makes the problem of fluctuations more complicated than the problem of the system reaction.

Our goal is to derive equations for the kinetics of the fluctuations under the same assumptions for which the kinetic equation for the one-particle electron distribution function in a strong electric field is usually derived. Later these equations are solved by us for the fluctuations with sufficiently small frequency ω and small wave vector q. More precisely, in this case one is able to express the correlator of the fluctuations in terms of such parameters, characterizing the nonequilibrium electron system, as the differential conduc-

[1)] A number of theoretical articles (see [2-9]) and also the review[10] and subsequent articles by Lax (see [11]) have been devoted to questions about fluctuations near a stationary nonequilibrium state in various cases. Spatially-inhomogeneous fluctuations in a nonequilibrium state in semiconductors were recently treated by the Langevin method by Kogan and Shul'man.[19]

tivity σ_{ik}, the drift velocity $\mathbf{V}$, and the diffusion coefficient D_{ik}. One more quantity, which has the physical meaning of the correlator of the current fluctuations for $q = 0$ and $\omega \to 0$, enters into this expression. It should be noted that according to Lax[10] (see also[3]) even in a nonequilibrium state this correlator is not an independent quantity, but can be expressed in terms of the tensor D_{ik}. However, as was shown in the authors' article,[8] electron-electron collisions in a nonequilibrium state lead to an additional correlation, and as a consequence the direct connection between these two quantities is lost, so that for us they play the role of independent parameters of the theory.

In conclusion we consider the problem of the scattering of light by "hot" electrons in semiconductors, caused by the fluctuations of the electron concentration. An expression is obtained which describes the angular distribution of the scattered light and its spectral composition. In this connection the following interesting property is observed.

Electron fluctuations taking place in volumes with linear dimensions of the order of the wavelength of the light play a role in the scattering of light at not too small angles. In this connection, if the electron concentration is large enough so that the Debye radius is much smaller than this characteristic size, the fluctuations of the electron concentration turn out to be strongly suppressed due to Coulomb repulsion of the electrons. As noted by Wolff,[12] this property reduces the intensity of the scattered light.

It is shown by us that in a strong field the field-dependent effective screening radius, which is proportional to the root of the ratio of the diffusion coefficient to the differential conductivity, enters into the expression for the intensity of the fluctuations. In a number of important cases this ratio may increase appreciably due to the effect of the electric field, which must lead to a substantial increase in the scattering intensity.

2. DETERMINATION OF THE CORRELATORS. A DIAGRAM TECHNIQUE

Let us consider a semiconductor located in a strong electric field. Let the density matrix of the system at the moment of time $t = 0$ be ρ_0. Then the fluctuations of the current density are determined by the following averages:

$$\langle \delta j_i(\mathbf{r}'+\mathbf{r}, t+\tau)\,\delta j_k(\mathbf{r}', t)\rangle \equiv \frac{1}{2}\overline{[j_i(\mathbf{r}'+\mathbf{r}, t+\tau) j_k(\mathbf{r}', t)} + \overline{j_k(\mathbf{r}', t) j_i(\mathbf{r}'+\mathbf{r}, t+\tau)}] - \bar{j}_i\bar{j}_k. \tag{2.1}$$

Here the bars denote an averaging over the density matrix ρ_0 of the Heisenberg operators $j_i(t)$ and their products;

$$j(t) = S^+(t) j S(t), \tag{2.2}$$

where j is the Schrödinger operator for the current density, $S(t)$ is the evolution operator:

$$iS = (H_0 + \hat{V})S, \tag{2.3}$$

$H_0 + \hat{V}$ is the total Hamiltonian of the system, and $\hbar = 1$. Both the (strong) electric field and the interaction are included in $\hat{V}$; thus H_0 in Eq. (2.3) is the Hamiltonian of the free particles without the field. We shall assume that in the system after a sufficiently long time interval (formally as $t \to \infty$, and in practice after a time interval of the order of the relaxation time) a stationary state is established which does not depend on the state at $t = 0$. Precisely such systems in a nonequilibrium state are described by a kinetic equation[2] (see[13]). In view of this, the averages in formula (2.1) do not depend on t for large values of t. The fact that the final state is independent of the initial state allows us to express ρ_0 in the form

$$\rho_0 = \exp\left(-\frac{H_0 - \mu\hat{N}}{T}\right) \tag{2.4}$$

and to put $S(0) = 1$. ($\hat{N}$ in Eq. (2.4) denotes the operator for the number of particles, μ is the chemical potential, and T is the temperature in energy units). The spectral density of the fluctuations is defined by the Fourier transform of (2.1) with respect to $\mathbf{r}$ and τ:

$$\langle \delta j_i(\tau)\,\delta j_k\rangle_{\omega\mathbf{q}} = \frac{1}{2\pi\mathcal{V}}\int_{-\infty}^{\infty} d\tau \int d^3\mathbf{r}\, e^{i\omega\tau - i\mathbf{q}\mathbf{r}} \langle \delta j_i(\mathbf{r}, \tau)\,\delta j_k\rangle. \tag{2.5}$$

We note that in the presence of a homogeneous electric field the average (2.1) does not depend on $\mathbf{r}'$. It is convenient, however, to calculate not (2.5) but the following expression:

$$[j_i(\tau) j_k]_{\omega\mathbf{q}} = \frac{1}{\mathcal{V}}\int_0^{\infty} d\tau \int d^3\mathbf{r}\, e^{i\omega\tau - i\mathbf{q}\mathbf{r}}\, \overline{j_i(\mathbf{r}, t+\tau) j_k(0, t)}. \tag{2.6}$$

The total Fourier component of (2.1) is expressed in terms of the quantities (2.6) in the following way:

$$\langle \delta j_i(\tau)\,\delta j_k\rangle_{\omega\mathbf{q}} = \frac{1}{4\pi}\{[j_i(\tau) j_k]_{\omega\mathbf{q}} + [j_i j_k(\tau)]_{-\omega,-\mathbf{q}} + [j_k j_i(\tau)]_{\omega\mathbf{q}} + [j_k(\tau) j_i]_{-\omega,-\mathbf{q}}\} - \bar{j}_i\bar{j}_k\delta(\omega)\delta_{\mathbf{q},0}. \tag{2.7}$$

The correlator $[j_i(\tau) j_k]_{\omega\mathbf{q}}$ can be expressed in terms of the two-particle distribution function $g_{\mathbf{q}}^{\omega}(\mathbf{p}\mathbf{p}_1)$, which is defined as follows:

$$g_{\mathbf{q}}{}^{\omega}(\mathbf{p}\mathbf{p}_1) = \int_0^{\infty} g_{\mathbf{q}}(\mathbf{p}, \tau|\mathbf{p}_1) e^{i\omega\tau} d\tau,$$
$$g_{\mathbf{q}}(\mathbf{p}, \tau|\mathbf{p}_1) = \lim_{t\to\infty} G_{\mathbf{q}}(\mathbf{p}, t+\tau|\mathbf{p}_1, t) \tag{2.8}$$

where

$$G_{\mathbf{q}}(\mathbf{p}, t+\tau|\mathbf{p}_1, t) = (\mathrm{Sp}\,\rho_0)^{-1}\,\mathrm{Sp}\,\rho_0 S^+(t+\tau) a^+_{\mathbf{p}-\mathbf{q}/2} a_{\mathbf{p}+\mathbf{q}/2} S(t+\tau) S^+(t) a^+_{\mathbf{p}_1+\mathbf{q}/2} a_{\mathbf{p}_1-\mathbf{q}/2} S(t). \tag{2.9}$$

We have

$$[j_i(\tau) j_k]_{\omega\mathbf{q}} = \frac{e^2}{\mathcal{V}^2}\sum_{\mathbf{p}\mathbf{p}'} v_i v_k' g_{\mathbf{q}}{}^{\omega}(\mathbf{p}\mathbf{p}'). \tag{2.10}$$

Here $a^+_{\mathbf{p}}$ and $a_{\mathbf{p}}$ are the electron creation and annihilation operators, e is the charge, v is the electron's velocity, and $\mathcal{V}$ is the volume of the system (for brevity we do not take the electron spin into consideration).

Let us derive an equation for the function $g_{\mathbf{q}}^{\omega}(\mathbf{p}\mathbf{p}_1)$. We consider both scattering by phonons and the electron-electron interaction. In the derivation of the equation we shall use the diagram technique of Konstantinov and Perel',[14] and also the electric field will be taken into account to all orders of perturbation theory,

[2] The stationary state of a semiconductor in a strong electric field may turn out to be unstable (for example, in the presence of the Gunn effect). Our investigation does not pertain to such cases.

neglecting however the influence of the electric field on collisions.

We carry out the calculation to the lowest approximation in the interaction constants, confining our attention to the case of nondegenerate electrons. In addition, we shall assume that the concentration of electrons in the conduction band does not change under the action of the electric field. We restrict our attention to the case when the phonon distribution function remains an equilibrium distribution[3] whereas the electron distribution function F_p naturally may be strongly nonequilibrium. We shall assume that the usual criteria for the existence of a kinetic equation are satisfied:

$$\frac{1}{\varepsilon_p \tau_e} \ll 1, \quad \frac{\omega}{\varepsilon_p} \ll 1, \quad \frac{q}{p} \ll 1. \tag{2.11}$$

Here ϵ_p and p are the characteristic energy and momentum of the electron, respectively, and τ_e is the relaxation time of the electrons.

We define the double Laplace transform of the functions $G_q(p, t+\tau \mid p_1, t)$ like this:

$$G_q^{s\sigma}(pp_1) = \int_0^\infty d\tau \int_0^\infty dt\, e^{-st-\sigma\tau} G_q(p, t+\tau \mid p_1, t). \tag{2.12}$$

In connection with the evaluation of the two-particle distribution function $G_q(pp_1)$ on the contour of Konstantinov and Perel', a second pair of terminal points appear (see Fig. 1).[4] Correspondingly two types of sections appear, an s section and a σ section. The remaining rules of correspondence between diagrams and analytic expressions are completely retained, and therefore we shall not deduce them here. Let us carry out a classification of the diagrams. Following[14], we shall collect the diagrams containing a given number of free sections (in s and in σ) to lowest order in the interaction constants. Similarly[14 8] we obtain the graphical equation shown in Fig. 2 for $G_q^{s\sigma}(pp_1)$.

The first term in Fig. 2 corresponds to the sum of the diagrams containing a unique (always existing) free section between the pairs of terminal points p and p_1.

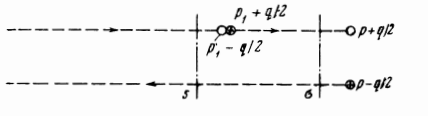

FIG. 1.

FIG. 2.

[3] This holds under the hypothesis of a sufficiently large heat capacity of the phonon system and a sufficiently small relaxation time for the long wavelength phonons with which the electrons interact directly.

[4] Due to the averaging over the matrix ρ_0, the contour shown in Fig. 1 is simpler than in [14] (the vertical parts are absent). Such a method of averaging was used by Keldysh [13] in order to develop a diagram technique for nonequilibrium processes. However, all of the following derivation would become only slightly more complicated if ρ_0 were chosen in the form $\exp(-(H_1 - \mu\hat{N})/T)$, where H_1 is the total Hamiltonian of the system in the absence of the field. Such a choice for ρ_0 would correspond to a representation involving equilibrium of the system up to the moment t = 0 when the field is switched on.

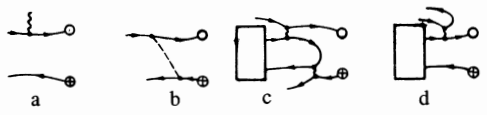

FIG. 3.

In the second term the crosshatched rectangle represents the "irreducible part," calculated to lowest order in the interaction. Diagrams of the type depicted in Fig. 3 are contained in this part. Diagrams a and b in Fig. 3 (and the diagrams analogous to them) will give the terms $e\mathbf{E}\cdot(\partial/\partial \mathbf{p})$ and I^{lat} ($\mathbf{E}$ is the electric field, and I^{lat} is the usual electron-phonon collision operator) in the equation. Diagrams c and d describe the electron-electron interaction, where diagram c (and the diagrams analogous to it) describe pair collisions, and diagram d (and the diagrams analogous to it) describe the interaction via the self-consistent field. The blocks depicted in diagrams c and d are collected on the entire contour, and in the second term on the right hand side of Fig. 2 they should be represented by having the appropriate function G superimposed on the block. The one-particle distribution function

$$F_p = (\mathrm{Sp}\,\rho_0)^{-1}\,\mathrm{Sp}\,\rho_0 S^+(t) a_p^+ a_p S(t). \tag{2.13}$$

obviously corresponds to these blocks. We note that in connection with an evaluation of expression (2.13) the diagrams of type d give zero contribution.[5]

Now let us consider the first term on the right hand side of the equation shown in Fig. 2. It is not difficult to see that in this term one can neglect the diagrams in which there are points lying between pairs of terminals (i.e., in the cross-hatched region). The rejected diagrams for an equal number of free sections contain the interaction constants to a higher power. The sum of the remaining diagrams is nothing else than the two-particle distribution function at one instant of time, $G_q(p, t \mid p_1, t)$. Having rearranged the creation and annihilation operators, it is convenient to represent this function in the following form:

$$G_q(p, t \mid p_1, t) = F_{p-q/2}\delta_{pp_1} + \Phi_q(pp_1), \tag{2.14}$$

$$\Phi_q(pp_1) = (\mathrm{Sp}\,\rho_0)^{-1}\,\mathrm{Sp}\,\rho_0 S^+(t) a^+_{p-q/2} a^+_{p_1+q/2} a_{p_1-q/2} a_{p+q/2} S(t), \tag{2.15}$$

In our approximation, one is able to obtain a closed equation (see Fig. 4) for the function $\Phi_q(pp_1)$. In this figure (we assume $q \neq 0$) the block A describes the interaction of an electron p from an isolated pair containing a "phonon" (field + scatterer), and block B describes the same interaction for the electron p_1. Diagrams C and D describe the interaction of electrons of

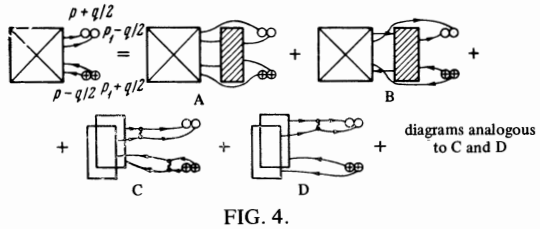

FIG. 4.

[5] Just as in considerations of spatially homogeneous fluctuations.[8]

an isolated pair with each other, where diagram C (and the diagrams analogous to it) correspond to scattering, and diagram D (and the diagrams analogous to it) correspond to the long-range Coulomb interaction.

Thus, we have deduced our equations graphically. In the following Section we reduce these equations to analytic form for $s \to 0$ and $\sigma \to -i\omega$. We note that to the approximation adopted by us

$$G_q(\mathbf{p}, t+\tau|\mathbf{p}_1, t) = G_q(\mathbf{p}_1, t|\mathbf{p}, t+\tau), \tag{2.16}$$

so that the correlators on the right hand side of Eq. (2.7) are equal in pairs.

3. EQUATIONS FOR THE KINETICS OF THE FLUCTUATIONS

The system of equations for the functions $g_{\mathbf{q}}^{\omega}(\mathbf{pp}_1)$ and $\Phi_{\mathbf{q}}(\mathbf{pp}_1)$ (corresponding to unequal-time and equal-time binary distribution functions), which is developed in the preceding section, is:[6)]

$$(-i\omega + i\mathbf{qv} + \mathcal{I}_{\mathbf{p}})g_{\mathbf{q}}^{\omega}(\mathbf{pp}_1) - iqU_q\frac{\partial F_{\mathbf{p}}}{\partial \mathbf{p}}\sum_{\mathbf{p}'} g_{\mathbf{q}}^{\omega}(\mathbf{p}'\mathbf{p}_1) = F_{\mathbf{p}}\delta_{\mathbf{pp}_1} + \Phi_{\mathbf{q}}(\mathbf{pp}_1), \tag{3.1}$$

$$[iq(\mathbf{v}-\mathbf{v}_1) + \mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1}]\Phi_{\mathbf{q}}(\mathbf{pp}_1) = iqU_q\left\{\frac{\partial F_{\mathbf{p}}}{\partial \mathbf{p}}\left[F_{\mathbf{p}_1} + \sum_{\mathbf{p}'}\Phi_{\mathbf{q}}(\mathbf{p}'\mathbf{p}_1)\right] - \frac{\partial F_{\mathbf{p}_1}}{\partial \mathbf{p}_1}\left[F_{\mathbf{p}} + \sum_{\mathbf{p}_1}\Phi_{\mathbf{q}}(\mathbf{pp}_1')\right]\right\} - I^{ee}_{\mathbf{pp}_1}\{F, F\}. \tag{3.2}$$

To this system one should add the equation for the stationary one-particle electron distribution function:

$$\left(e\mathbf{E}\frac{\partial}{\partial \mathbf{p}} + I_{\mathbf{p}}^{\text{lat}}\right)F_{\mathbf{p}} + I_{\mathbf{p}}^{ee}\{F, F\} = 0. \tag{3.3}$$

Here I^{lattice} is the electron-phonon collision operator, $I_{\mathbf{p}}^{ee}(F, F)$ is the term describing the electron-electron collisions (quadratic in F). It should be noted that if electron-electron collisions play an essential role then it is necessary to simultaneously take the scattering of electrons by ionized impurities into account; in this case we shall assume that the corresponding collision operator is included in I^{lat}. The sum of the operators corresponding to the field and to the linearized collision terms of the kinetic equation (3.3) is denoted by $\mathcal{I}$:[7)]

$$\mathcal{I}_{\mathbf{p}} \equiv e\mathbf{E}\frac{\partial}{\partial \mathbf{p}} + I_{\mathbf{p}}^{\text{lat}} + I_{\mathbf{p}}^{ee}\{F\}. \tag{3.4}$$

Equation (3.1) has the following obvious physical interpretation. Let us suppose that a small correction $f(\mathbf{r}, \mathbf{p})$ to the steady-state electron distribution function appears at the instant of time $\tau = 0$. Then the time evolution of this quantity, more precisely, of its Fourier component $f_{\mathbf{q}}(\mathbf{p}) = \int e^{-i\mathbf{q}\cdot\mathbf{r}} f(\mathbf{r}, \mathbf{p})d^3r$ will, according to (3.1), be exactly described by the same equation which the function $g(\mathbf{p}, \tau|\mathbf{p}_1)$ satisfies:

$$\left(\frac{\partial}{\partial \tau} + i\mathbf{qv} + \mathcal{I}_{\mathbf{p}}\right)f_{\mathbf{q}}(\mathbf{p}, \tau) - iU_q\mathbf{q}\frac{\partial F_{\mathbf{p}}}{\partial \mathbf{p}}\sum_{\mathbf{p}'} f_{\mathbf{q}}(\mathbf{p}', \tau) = 0. \tag{3.5}$$

The last term in (3.5) describes the change of the distribution function due to the action of the self-consistent field. This field is determined from the solution of Poisson's equation and equals $-iU_q\mathbf{q}\sum_{\mathbf{p}'} f_{\mathbf{q}}(\mathbf{p}', \tau)$, where

$$U_q = 4\pi e^2 / \varepsilon_{ik}q_iq_k\mathcal{V}. \tag{3.6}$$

Here ϵ_{ik} is the dielectric tensor of the crystal.

Then the fact that the function $g(\mathbf{p}, \tau|\mathbf{p}_1)$ also satisfies Eq. (3.5) indicates the validity of Onsager's principle in connection with the case we are considering. The initial condition for the function $g_{\mathbf{q}}(\mathbf{p}, \tau|\mathbf{p}_1)$—the right hand side of Eq. (3.1)—is given by

$$g_{\mathbf{q}}(\mathbf{pp}_1) \equiv g_{\mathbf{q}}(\mathbf{p}, \tau|\mathbf{p}_1)|_{\tau=0} = F_{\mathbf{p}}\delta_{\mathbf{pp}_1} + \Phi_{\mathbf{q}}(\mathbf{pp}_1). \tag{3.7}$$

The quantity $g_{\mathbf{q}}(\mathbf{pp}_1)$ represents the equal-time binary distribution function; Eq. (3.2) serves as its definition. This equation describes the onset of a correlation of a distinct pair of electron states (with momenta $\mathbf{p}$ and $\mathbf{p}_1$) under the action of the external field $\mathbf{E}$, of the self-consistent field of the fluctuations, of the interaction of the electrons with the scatterers, and also of the interaction between the electrons in state $\mathbf{p}$ and those in state $\mathbf{p}_1$. The latter interaction comes about by means of both the long-range Coulomb interaction (the terms proportional to $F_{\mathbf{p}_1}\partial F_{\mathbf{p}}/\partial \mathbf{p}$ and $F_{\mathbf{p}}\partial F_{\mathbf{p}_1}/\partial \mathbf{p}_1$ on the right hand side of Eq. (3.2)) and pair collisions, which are described by the expression[8)]

$$I^{ee}_{\mathbf{pp}_1}\{F, F\} = \sum_{\mathbf{p}'\mathbf{p}_1'}(W^{\mathbf{pp}_1}_{\mathbf{p}'\mathbf{p}_1'}F_{\mathbf{p}}F_{\mathbf{p}_1} - W^{\mathbf{p}'\mathbf{p}_1'}_{\mathbf{pp}_1}F_{\mathbf{p}'}F_{\mathbf{p}_1'}). \tag{3.8}$$

It is not difficult to verify that in the equilibrium case when $F_{\mathbf{p}} = \exp\{(\mu - \epsilon_{\mathbf{p}})/T\}$, as a consequence of the fact that $I^{ee}_{\mathbf{pp}_1}\{F, F\} = 0$[9)] and $\partial F_{\mathbf{p}}/\partial p_i = -v_iF_{\mathbf{p}}/T$ Eq. (3.2) satisfies the following expression:

$$\Phi_{\mathbf{q}}^{eq}(\mathbf{pp}_1) = -\frac{1}{N}\frac{F_{\mathbf{p}}F_{\mathbf{p}_1}}{1 + q^2/\varkappa_{eq}^2}, \tag{3.9}$$

where $N = \sum_{\mathbf{p}} F_{\mathbf{p}} \equiv n_0\mathcal{V}$ is the total number of particles,

$$\varkappa_{eq}^2 = 4\pi n_0e^2q^2/\varepsilon_{ik}q_iq_kT. \tag{3.10}$$

In the general case of a nonequilibrium system existing in a stationary state, the entire theory of fluctuations is contained in the system of equations (3.1)

[6)] Similar equations for the binary distribution functions, but without an explicit isolation of the collision terms, were obtained (by Bogolyubov's method) in a number of articles (see [15-17]).

[7)] We present an explicit expression for the linearized operator $I_{\mathbf{p}}^{ee}\{F\}$:

$$I_{\mathbf{p}}^{ee}\{F\}\psi = \sum_{\mathbf{p}'\mathbf{p}_1\mathbf{p}_1'}[W^{\mathbf{pp}_1}_{\mathbf{p}'\mathbf{p}_1'}(F_{\mathbf{p}}\psi_{\mathbf{p}_1} + F_{\mathbf{p}_1}\psi_{\mathbf{p}}) - W^{\mathbf{p}'\mathbf{p}_1'}_{\mathbf{pp}_1}(F_{\mathbf{p}'}\psi_{\mathbf{p}_1'} + F_{\mathbf{p}_1'}\psi_{\mathbf{p}'})],$$

where W is the probability of electron-electron scattering (the upper indices correspond to the initial state of the scattered particles). In the Born approximation the cross section for Coulomb scattering diverges for small momentum transfer. This divergence would be eliminated by taking higher approximations into consideration (compare with [18]).

[8)] We note that the expression (3.8) represents the pair collision term without including one summation, so that the usual pair collision term in (3.3) is given by

$$I_{\mathbf{p}}^{ee}\{F, F\} = \sum_{\mathbf{p}_1} I^{ee}_{\mathbf{pp}_1}\{F, F\}.$$

[9)] In the Born approximation which we are using, the existence of this equality is obvious. Incidentally, it is preserved even from the exact probabilities as a consequence of the Stueckelberg property [19]

$$\sum_{\mathbf{p}_1\mathbf{p}_1'}(W^{\mathbf{pp}_1}_{\mathbf{p}'\mathbf{p}_1'} - W^{\mathbf{p}'\mathbf{p}_1'}_{\mathbf{pp}_1}) = 0.$$

and (3.2). The main peculiarity of the nonequilibrium case consists in the fact that here (in contrast to the equilibrium case) it is generally impossible to represent the binary distribution function $\Phi_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1)$ in the form (3.9) and the solution of Eq. (3.2) is an independent problem. The system (3.1)–(3.2) turns out to be more complicated than the kinetic equation which describes the reaction of a system of electrons in a weak external variable field of the form $\mathcal{E}_{\mathbf{k}} = \mathcal{E}_{0\mathbf{k}} e^{-i\omega t + i\mathbf{q}\cdot\mathbf{r}}$, which creates a variable current $j_i = s_{ik}(\omega, \mathbf{q})\,\mathcal{E}_k$. This is one of the reasons why the fluctuation-dissipation theorem is not satisfied in nonequilibrium systems, i.e., the fluctuations in a semiconductor are not related to the tensor $s_{ik}(\omega, \mathbf{q})$ by any general relationship whatsoever.

4. CALCULATION OF THE EQUAL-TIME BINARY DISTRIBUTION FUNCTION FOR $q l_e \ll 1$

Equations (3.1) and (3.2) can be solved comparatively easily in the important case when $\omega\tau_e \ll 1$ and $q l_e \ll 1$. Here τ_e is the characteristic relaxation time of the electrons,[10] and l_e is the corresponding mean free path. Under these conditions the principal terms in Eqs. (3.1) and (3.2) will be the terms containing the operator $\mathcal{I}$. In order to solve the problem, it is necessary for us to find the inverse operators to $\mathcal{I}_{\mathbf{p}}$ and $(\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})$. Since

$$\mathcal{I}_{\mathbf{p}} \partial F_{\mathbf{p}} / \partial N = 0 \tag{4.1}$$

(it is not difficult to verify this by differentiating the kinetic equation (3.3) with respect to N), the solution of the equation

$$\mathcal{I}_{\mathbf{p}} x = y_{\mathbf{p}} \tag{4.2}$$

is determined to within a term of the form const $\partial \times F_{\mathbf{p}}/\partial N$.[11] In order to make the inverse operator single-valued, it is necessary to impose one additional condition on it. As such we require

$$\sum_{\mathbf{p}} \mathcal{I}_{\mathbf{p}}^{-1} y \equiv 0. \tag{4.3}$$

In addition, we note that for the functions we are considering, one always has $\sum_{\mathbf{p}} \mathcal{I}_{\mathbf{p}} x_{\mathbf{p}} = 0$, so the operator $\mathcal{I}_{\mathbf{p}}^{-1}$ is defined only with respect to functions possessing the property

$$\sum_{\mathbf{p}} y_{\mathbf{p}} \equiv 0. \tag{4.4}$$

Correspondingly, for the operator $(\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})^{-1}$ we have

$$(\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1}) \frac{\partial F_{\mathbf{p}}}{\partial N} \frac{\partial F_{\mathbf{p}_1}}{\partial N} = 0, \quad \sum_{\mathbf{p}\mathbf{p}_1} (\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})^{-1} y_{\mathbf{p}\mathbf{p}_1} \equiv 0, \quad \sum_{\mathbf{p}\mathbf{p}_1} y_{\mathbf{p}\mathbf{p}_1} \equiv 0. \tag{4.5}$$

Thus, the defined operator $(\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})^{-1}$ possesses the following property (which we will need later on):

$$\sum_{\mathbf{p}_1} (\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})^{-1} y_{\mathbf{p}\mathbf{p}_1} = \mathcal{I}_{\mathbf{p}}^{-1} \sum_{\mathbf{p}_1} y_{\mathbf{p}\mathbf{p}_1}. \tag{4.6}$$

[10] For convenience of discussion, we shall assume here and below that only one such time occurs in the problem. If several such times occur, then in the corresponding criteria one must insert the largest of the times.

[11] We are considering the case when the kinetic equation (3.3) (or (4.1)) has only one solution.

One can easily prove property (4.6) by using Eqs. (4.2) through (4.5).

Now let us consider the solution of the equation for $\Phi_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1)$ under the assumption $q l_e \ll 1$. For this purpose let us transform (3.2) to a form which is convenient for iterations with respect to $q l_e$:

$$\Phi_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1) = A \frac{\partial F_{\mathbf{p}}}{\partial N} \frac{\partial F_{\mathbf{p}_1}}{\partial N} + \varphi_{\mathbf{p}\mathbf{p}_1} + i\mathbf{q} U_{\mathbf{q}} (\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})^{-1} \Big\{ \frac{\partial F_{\mathbf{p}_1}}{\partial \mathbf{p}_1} \Big[F_{\mathbf{p}} + \sum_{\mathbf{p}'_1} \Phi_{\mathbf{q}}(\mathbf{p}\mathbf{p}'_1) \Big] - \frac{\partial F_{\mathbf{p}}}{\partial \mathbf{p}} \Big[F_{\mathbf{p}_1} + \sum_{\mathbf{p}'} \Phi_{\mathbf{q}}(\mathbf{p}'\mathbf{p}_1) \Big] \Big\} - i\mathbf{q} (\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})^{-1} (\mathbf{v} - \mathbf{v}_1) \Phi_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1). \tag{4.7}$$

The function $\varphi_{\mathbf{p}\mathbf{p}_1}$ is the solution of the equation

$$(\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1}) \varphi_{\mathbf{p}\mathbf{p}_1} = -I^{ee}_{\mathbf{p}\mathbf{p}_1}\{F, F\}, \tag{4.8}$$

with the following properties:[8]

$$\sum_{\mathbf{p}_1} \varphi_{\mathbf{p}\mathbf{p}_1} = N \frac{\partial F_{\mathbf{p}}}{\partial N} - F_{\mathbf{p}}, \quad \sum_{\mathbf{p}\mathbf{p}_1} \varphi_{\mathbf{p}\mathbf{p}_1} = 0. \tag{4.9}$$

We note that in equilibrium $\varphi_{\mathbf{p}\mathbf{p}_1}$ is obviously equal to zero. In order to determine the constant A we shall use "the equation of continuity"

$$i\mathbf{q} \sum_{\mathbf{p}\mathbf{p}_1} (\mathbf{v} - \mathbf{v}_1) \Phi_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1) = 0 \tag{4.10}$$

which is obtained by a summation of the original equation (3.2) over $\mathbf{p}$ and $\mathbf{p}_1$. Let us represent $\Phi_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1)$ in the form

$$\Phi_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1) = \Phi^{(0)}_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1) + \Phi^{(1)}_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1) + \ldots \tag{4.11}$$

For $\Phi^{(0)}$ let us choose the first two terms in Eq. (4.7)

$$\Phi^{(0)}_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1) = A \frac{\partial F_{\mathbf{p}}}{\partial N} \frac{\partial F_{\mathbf{p}_1}}{\partial N} + \varphi_{\mathbf{p}\mathbf{p}_1}. \tag{4.12}$$

Then, with (4.9) taken into consideration, one finds

$$\Phi^{(1)}_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1) = i\mathbf{q} U_q (N + A) (\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})^{-1} \Big[\frac{\partial F_{\mathbf{p}}}{\partial \mathbf{p}} \frac{\partial F_{\mathbf{p}_1}}{\partial N} - \frac{\partial F_{\mathbf{p}}}{\partial N} \frac{\partial F_{\mathbf{p}_1}}{\partial \mathbf{p}_1} \Big] - i\mathbf{q} (\mathcal{I}_{\mathbf{p}} + \mathcal{I}_{\mathbf{p}_1})^{-1} (\mathbf{v} - \mathbf{v}_1) \Phi^{(0)}_{\mathbf{q}}(\mathbf{p}\mathbf{p}_1). \tag{4.13}$$

We substitute this expression into (4.10). Exchanging the indices of summation $\mathbf{p}$ and $\mathbf{p}_1$ and using the property (4.6), we obtain the following expression for A:

$$A = -N \Big(1 + \frac{q^2}{\varkappa^2} \frac{\Theta_{\parallel}}{D_{\parallel}} \Big) \Big(1 + \frac{q^2}{\varkappa^2} \Big)^{-1}, \tag{4.14}$$

where

$$\varkappa^2 = 4\pi\sigma_{\parallel} / \varepsilon_{\parallel} D_{\parallel}, \tag{4.15}$$

and also

$$\sigma_{\parallel} = \sigma_{ik} q_i q_k / q^2 \tag{4.16}$$

and analogously for the other tensors. (We shall assume that $\kappa l_e \ll 1$. Only under this assumption does it make sense to retain terms of the type q^2/κ.)

The vector $\mathbf{V}$ and the tensors σ_{ik}, D_{ik}, and Θ_{ik} are defined in the following way:

$$V_i = \sum_{\mathbf{p}} v_i \frac{\partial F_{\mathbf{p}}}{\partial N}, \tag{4.17}$$

$$\sigma_{ik} = -\frac{e^2}{\mathcal{V}} \sum_{\mathbf{p}} v_i \mathcal{I}^{-1} \frac{\partial F_{\mathbf{p}}}{\partial p_k}, \tag{4.18}$$

$$D_{ik} = \sum_{\mathbf{p}} v_i \mathcal{I}^{-1} (v_k - V_k) \frac{\partial F_{\mathbf{p}}}{\partial N}, \tag{4.19}$$

$$\Theta_{ik}=\frac{1}{N}\sum_{pp'} v_i\mathscr{J}_p^{-1}(v_k-v_k')\varphi_{pp'}. \quad (4.20)$$

The first three of these quantities have the meanings (see the Appendix), respectively, of the differential drift velocity, the differential conductivity, and the diffusion coefficient. The tensor Θ_{ik} associated with the function $\varphi_{pp'}$ is the specific quantity which characterizes the additional correlation of the components of the current density vector associated with pair collisions.

Thus, neglecting terms of order ql_e the equal-time binary distribution function has the form

$$g_q(pp_1)=F_p\delta_{pp_1}+\varphi_{pp_1}-N\frac{\partial F_p}{\partial N}\frac{\partial F_{p_1}}{\partial N}\frac{1+q^2\Theta_\parallel/\varkappa^2 D_\parallel}{1+q^2/\varkappa^2}. \quad (4.21)$$

For $q \ll \kappa$ this expression goes over into

$$g_q(pp_1)=F_p\delta_{pp_1}+\varphi_{pp_1}-N\frac{\partial F_p}{\partial N}\frac{\partial F_{p_1}}{\partial N}, \quad (4.22)$$

i.e., the expression obtained by the authors in[8]. Summing (4.22) over p or p_1 with (4.9) taken into consideration, we obtain zero. This means that the number of electrons does not fluctuate in volumes whose linear dimensions are large in comparison with κ^{-1}.

5. EVALUATION OF THE CORRELATOR OF THE CURRENTS

Let us solve Eq. (3.1) for $ql_e \ll 1$ and $\omega\tau_e \ll 1$. Having in mind an evaluation of the current correlator (2.10), it is convenient to multiply (3.1) by v_{k1} and sum over p_1, introducing the function $\gamma_k(p)$:

$$\gamma_k=\sum_{p_1} g_q^{\omega}(pp_1)v_{k1}. \quad (5.1)$$

The correlator $[j_i(\tau)j_k]_{\omega q}$ is expressed in terms of γ_k in the following way:

$$[j_i(\tau)j_k]_{\omega q}=\frac{e^2}{\mathscr{V}^2}\sum_p v_i\gamma_k(p). \quad (5.2)$$

From Eq. (3.1) we obtain the following equation for γ_k for $ql_e \ll 1$:

$$(-i\omega+iqv+\mathscr{J}_p)\gamma_k-iqU_q\frac{\partial F_p}{\partial p}\sum_{p'}\gamma_k(p')$$

$$=v_kF_p+\sum_{p'}v_k'(\Phi_q^{(0)}(pp')+\Phi_q^{(1)}(pp')), \quad (5.3)$$

where $\Phi^{(0)}$ and $\Phi^{(1)}$ are given by formulas (4.12) and (4.13). At q = 0 Eq. (5.3) takes the form

$$(-i\omega+\mathscr{J}_p)\gamma_k=(v_k-V_k)N\frac{\partial F_p}{\partial N}-\sum_{p'}(v_k-v_k')\varphi_{pp'}. \quad (5.4)$$

Equation (5.4), which describes the spatially-homogeneous fluctuations, was obtained previously.[8] Its solution as $\omega \to 0$ is given by

$$\gamma_k^0=\mathscr{J}_p^{-1}(v_k-V_k)N\frac{\partial F_p}{\partial N}-\mathscr{J}_p^{-1}\sum_{p'}(v_k-v_k')\varphi_{pp'} \quad (5.5)$$

and in this case the correlator (5.2) is given by

$$[j_i(\tau)j_k]_0=\frac{e^2n_0}{\mathscr{V}^2}(D_{ik}-\Theta_{ik}). \quad (5.6)$$

Thus, for q = ω = 0 the total current correlator (2.6) is given by

$$\langle\delta j_i\delta j_k\rangle_0=\frac{e^2n_0}{2\pi\mathscr{V}^2}(D_{ik}+D_{ki}-\Theta_{ik}-\Theta_{ki}). \quad (5.7)$$

Neglecting pair collisions, $\Theta_{ik} \equiv 0$, and

$$\langle\delta j_i\delta j_k\rangle_0=\frac{e^2n_0}{2\pi\mathscr{V}^2}(D_{ik}+D_{ki}), \quad (5.8)$$

i.e., there is a direct relation between the correlator of the currents and the coefficient of diffusion. This property has already been noted by Lax[10] and Price.[3] However, as is clear from formula (5.6), pair collisions violate this relation.

Now let us evaluate the correlator of the currents for nonvanishing values of ω and q. The iterative procedure used in the previous Section may also be applied to Eq. (5.3), which in this sense is not any more complicated than the kinetic equation considered in the Appendix. Operating in similar fashion, to the first approximation in $\omega\tau_e$ and ql_e we obtain:

$$\gamma_k=\gamma_k^0+C_k\left\{\frac{\partial F_p}{\partial N}-iq_l\mathscr{J}_p^{-1}(v_l-V_l)\frac{\partial F_p}{\partial N}+iq_lU_q\mathscr{J}_p^{-1}\frac{\partial F_p}{\partial p_l}\right\} \quad (5.9)$$

Hence

$$[j_i(\tau)j_k]_{\omega q}=[j_i(\tau)j_k]_0+\frac{e^2}{\mathscr{V}^2}C_k\left(V_i-i\frac{4\pi}{\varepsilon_\parallel q^2}q_l\sigma_{il}-iq_lD_{il}\right). \quad (5.10)$$

The constant $C_k \equiv \sum_p \gamma_k(p)$ is determined from the "equation of continuity," which is obtained by the summation of (5.3) over p:

$$-i\omega C_k+iq_l\sum_p v_l\gamma_k=\frac{q^2N}{q^2+\varkappa^2}\left(1-\frac{\Theta_\parallel}{D_\parallel}\right)$$

$$\times\left(V_k+i\frac{4\pi}{\varepsilon_\parallel q^2}q_l\sigma_{kl}+iq_lD_{kl}\right)-iq_l(D_{kl}-\Theta_{kl})N. \quad (5.11)$$

We used the property (4.6) in order to evaluate $\sum_p \Phi^{(1)}(pp')$. From Eqs. (5.10) and (5.11) we obtain

$$\frac{C_k}{N}=\left\{\frac{q^2}{q^2+\varkappa^2}\left(1-\frac{\Theta_\parallel}{D_\parallel}\right)\left(V_k+i\frac{4\pi}{\varepsilon_\parallel q^2}q_l\sigma_{kl}+iq_lD_{kl}\right)-iq_l(D_{lk}+D_{kl}\right.$$

$$\left.-\Theta_{lk}-\Theta_{kl})\right\}\left\{\frac{4\pi\sigma_\parallel}{\varepsilon_\parallel}+q^2D_\parallel-i(\omega-qV)\right\}^{-1}. \quad (5.12)$$

The total correlator of the currents is given by

$$\langle\delta j_i(\tau)\delta j_k\rangle_{\omega q}=\frac{1}{2\pi}\{[j_i(\tau)j_k]_{\omega q}+[j_k(\tau)j_i]_{-\omega,-q}\}. \quad (5.13)$$

Formulas (5.10), (5.12), and (5.13) solve the posed problem.[12)]

We also present an expression for the spectral density of the fluctuations of the concentration, $\langle\delta n^2\rangle_{\omega q}$, which is simply related to the correlator of the longitudinal (i.e., in the direction of the vector q) currents,

$$q^2\langle\delta j_\parallel^2\rangle_{\omega q} \equiv q_iq_k\langle\delta j_i\delta j_k\rangle_{\omega q}=e^2\omega^2\langle\delta n^2\rangle_{\omega q}, \quad (5.14)$$

$$\langle\delta n^2\rangle_{\omega q}=\frac{1}{\pi\mathscr{V}^2}\frac{q^2n_0(D_\parallel-\Theta_\parallel)}{(4\pi\sigma_\parallel/\varepsilon_\parallel+q^2D_\parallel)^2+(\omega-qV)^2}. \quad (5.15)$$

We recall that, according to Eqs. (4.15) and (4.16), the quantities $D_\parallel$, $\sigma_\parallel$, and $\Theta_\parallel$ depend on the direction of the vector q.

One can give the expression for the correlator of the longitudinal (along q) currents a form which, in the

[12)] We note that expression (5.11) is not directly applicable for q = 0 since expression (3.6) for U_q, which makes sense only for $q \neq 0$, was used in its derivation. At q = 0 it is necessary to assume $q^2U_q = 0$, and in Eq. (5.11) the first term remains.

case of equilibrium, goes over into the fluctuation-dissipation theorem and, by the same token, emphasizes the difference from it in the case of a nonequilibrium state:

$$\langle \delta j_{\parallel}^2 \rangle_{\omega \mathbf{q}} = \frac{T_e}{2\pi \mathscr{V}} \frac{\omega}{\omega - \mathbf{qV}} [s_{\parallel}(\omega, \mathbf{q}) + s_{\parallel}(-\omega, -\mathbf{q})],$$

where $s_{\parallel}$ is defined in Eq. (A.10) and

$$T_e \equiv \frac{\pi \mathscr{V} \langle \delta j_{\parallel}^2 \rangle_0}{\sigma_{\parallel}} = \frac{n_0 e^2 (D_{\parallel} - \Theta_{\parallel})}{\sigma_{\parallel}}.$$

We emphasize that in the nonequilibrium case the quantity $\tilde{T}_e$ depends on the direction of the vector $\mathbf{q}$ and, generally speaking, is not by any means determined by only one average energy of the "hot" electrons.

We note that those very same expressions for the fluctuations of the electron concentration and for the fluctuations of the current density may also be obtained by using the Langevin method if it is postulated that in the system there exist random external currents whose densities $g_i(\mathbf{r}, t)$ satisfy the correlation relation

$$\overline{g_i(\mathbf{r}, t) g_k(\mathbf{r}', t')} = n_0 e^2 (D_{ik} + D_{ki} - \Theta_{ik} - \Theta_{ki}) \delta^3(\mathbf{r} - \mathbf{r}') \delta(t - t'). \quad (5.16)$$

Here the total random current should be expressed in terms of the external current according to the formula

$$\delta j_i = eV_i \delta n - \sigma_{ik} \frac{\partial \varphi}{\partial x_k} - eD_{ik} \frac{\partial \delta n}{\partial x_k} + g_i \quad (5.17)$$

(where $\delta n(\mathbf{r}, t)$ is a random correction to the electron concentration), the potential of the random longitudinal field φ is determined from Poisson's equation

$$\varepsilon_{ik} \frac{\partial^2 \varphi}{\partial x_i \partial x_k} = -4\pi e \delta n \quad (5.18)$$

and the equation of continuity

$$e \frac{\partial \delta n}{\partial t} + \operatorname{div} \delta \mathbf{j} = 0. \quad (5.19)$$

has been taken into consideration.

This system of equations was used in articles by one of the authors[20] for an investigation of the fluctuations in the electron concentration and the acoustic fluctuations associated with them (in[20] the electron-electron collisions were not taken into account, and therefore D appeared in Eqs. (5.16) and (5.15) instead of $D - \Theta$). We see that these equations enable one to determine the dependence on ω and $\mathbf{q}$ (i.e., to take spatial and temporal dispersion into account) of the quantities $\langle \delta n^2 \rangle_{\omega \mathbf{q}}$ and $\langle \delta j_i \delta j_k \rangle_{\omega \mathbf{q}}$ in the limiting case of small ω and small $\mathbf{q}$.[13)]

6. SCATTERING OF LIGHT BY "HOT" ELECTRONS

Let us consider the scattering of light by the conduction electrons in a semiconductor which is placed in a strong electric field.[14)] We shall regard the crystal as optically isotropic, and we assume that the energy spectrum of the conduction electrons has the form

$$\varepsilon_p = p^2 / 2m, \quad (6.1)$$

where m is the effective mass of an electron. For an unpolarized wave the coefficient of extinction, referred to an element of solid angle do and a frequency interval $d\omega$, can be expressed in terms of the spectral density of the fluctuations in the electron concentration:[12]

$$dh \approx \frac{1}{2} \mathscr{V} \left(\frac{e^2}{mc^2} \right)^2 \langle \delta n^2 \rangle_{\omega \mathbf{q}} (1 + \cos^2 \vartheta) \, d\omega \, do. \quad (6.2)$$

Here c is the velocity of light, ω is the change in the frequency of light during the scattering, which we shall assume to be small in comparison with the frequency of the light, $\mathbf{q}$ is the change in the wave vector of the light, and θ is the scattering angle.

Integrating (6.2) over ω, we obtain the total intensity dh_{total} of the scattering in a given direction. Since

$$\int_{-\infty}^{\infty} \langle \delta n^2 \rangle_{\omega \mathbf{q}} d\omega = \sum_{\mathbf{pp}_i} g_{\mathbf{q}}(\mathbf{pp}_i),$$

then in the approximation $q l_e \ll 1$, $\kappa l_e \ll 1$ we have (see Eq. (4.21))

$$dh_{\text{полн}} = \frac{n_0 q^2}{q^2 + \varkappa^2} \left(1 - \frac{\Theta_{\parallel}}{D_{\parallel}} \right) \cdot \frac{1}{2} \left(\frac{e^2}{mc^2} \right)^2 (1 + \cos^2 \vartheta) \, do. \quad (6.3)$$

The quantities κ, $D_{\parallel}$, and $\Theta_{\parallel}$ are defined in Eqs. (4.15) through (4.20). They depend on the external electric field $\mathbf{E}$ and on the direction of the vector $\mathbf{q}$.

Since κ^2 is, roughly speaking, proportional to n_0, with an increase of n_0 the expression $n_0 q^2/(q^2 + \kappa^2)$ ceases (for $\kappa^2 \gg q^2$) to depend on the concentration n_0. The dependence of the quantity $q^2 n_0/\kappa^2$ on the electric field is determined by the dependence of the ratio $D_{\parallel}/\sigma_{\parallel}$ on $\mathbf{E}$. With heating this ratio increases in approximately the same way as the average energy of the electrons, and consequently the light scattering increases. In addition, with an increase of the heating of the electrons, the current in a semiconductor frequently has a tendency to saturate, i.e., the differential conductivity in the direction of the stationary current $\mathbf{J}$ decreases abruptly. If in this connection $\mathbf{q}$ is chosen parallel to $\mathbf{J}$, the ratio $D_{\parallel}/\sigma_{\parallel}$ will increase with the field faster than the average energy of the electrons, and in the indicated direction the scattering of light increases even more.

For $q l_e \ll 1$ and $\kappa l_e \ll 1$ the line shape of the scattered light is determined by expression (5.15). This expression is maximal at $\omega = \mathbf{q} \cdot \mathbf{V}$, and the width of the peak is determined by the quantity $4\pi\sigma_{\parallel}/\epsilon_{\parallel} + q^2 D_{\parallel}$.

In conclusion we wish to thank O. V. Konstantinov and V. I. Perel' for a discussion, and also for indicating the possibility of averaging with the aid of the matrix ρ_0 given by Eq. (2.4) in order to construct a theory of nonequilibrium processes.

APPENDIX

Let us consider a semiconductor in which, along with a strong constant electric field $\mathbf{E}$, there also exists a

[13)] Since $\langle \delta n^2 \rangle_{\omega \mathbf{q}}$, obtained on the basis of Eqs. (5.16) – (5.19), coincides with the result calculated above from first principles, we see that the critical remarks expressed in [21], concerning the reason for using this system for an examination of the fluctuations in the electron concentration and of the acoustic fluctuations associated with them, are groundless. In [21], in connection with an investigation of the fluctuations in a nonequilibrium system, the correlation of the random forces was found with the aid of an expression for the entropy. We hope to examine in a specific article the questions of to what extent a similar "entropy" approach is justified in connection with the fluctuations in a nonequilibrium state.

[14)] The scattering of light in a weakly ionized plasma in an electric field was calculated in [6].

weak variable field of the form

$$\mathscr{E}_k = \mathscr{E}_{0k} \exp(-i\omega t + iqr). \quad (A.1)$$

The variable current generated in this connection is given by

$$\Delta j_i = \frac{e}{\mathscr{V}} \sum_p v_i f_p, \quad (A.2)$$

where f_p is a correction to the distribution function which is proportional to the variable field, and which is determined from the kinetic equation

$$(-i\omega + iqv + \mathscr{I}_p) f_p = -e\mathscr{E}_k \partial F_p / \partial p_k. \quad (A.3)$$

Here the function F_p satisfies Eq. (3.3). Summing (A.3) over p we obtain the equation of continuity

$$-i\omega \sum_p f_p + iq \sum_p v f_p = 0. \quad (A.4)$$

Let us add Eq. (A.3) to the result of multiplying Eq. (A.4) by $-\partial F_p/\partial N$:

$$-i\omega\left(f_p - \frac{\partial F_p}{\partial N}\sum_{p'} f_{p'}\right) + iq\left(vf_p - \frac{\partial F_p}{\partial N}\sum_{p'} v' f_{p'}\right) + \mathscr{I}_p f_p = -e\mathscr{E}_k \frac{\partial F_p}{\partial p_k}. \quad (A.5)$$

Inverting $\mathscr{I}$ we represent (A.5) in a form which is convenient for iteration in terms of the small parameters ql_e and $\omega\tau_e$ (compare with Eq. (4.4)):

$$f_p = B\frac{\partial F_p}{\partial N} - e\mathscr{E}_k \mathscr{I}^{-1}\frac{\partial F_p}{\partial p_k} + i\omega\mathscr{I}^{-1}\left(f_p - \frac{\partial F_p}{\partial N}\sum_{p'} f_{p'}\right) - iq\mathscr{I}^{-1}\left(vf_p - \frac{\partial F_p}{\partial N}\sum_{p'} v' f_{p'}\right), \quad (A.6)$$

where B denotes the following constant: $B = \sum_p f_p \equiv \mathscr{V}\Delta n$, and Δn denotes a variable correction to the electron concentration. Substituting

$$f_p^{(0)} = \mathscr{V}\Delta n \frac{\partial F_p}{\partial N},$$

into the right hand side of Eq. (A.6), we have

$$f_p = \mathscr{V}\Delta n \frac{\partial F_p}{\partial N} - e\mathscr{E}_k \mathscr{I}^{-1}\frac{\partial F_p}{\partial p_k} - i\mathscr{V}\Delta n q_k \mathscr{I}^{-1}(v_k - V_k)\frac{\partial F_p}{\partial N}, \quad (A.7)$$

so that the expression for the variable current (A.2) takes the following form:[15)]

$$\Delta j_i = e\Delta n V_i + \sigma_{ik}\mathscr{E}_k - eD_{ik}\partial\Delta n / \partial x_k, \quad (A.8)$$

where the quantities V_i, σ_{ik}, and D_{ik} are defined by formulas (4.17) through (4.19). Since $V_i = e^{-1}\partial\overline{j}_i/\partial n_0$ it makes sense to call this quantity the differential drift velocity. Further, $\sigma_{ik} = \partial\overline{j}_i/\partial E_k$ is nothing other than the differential conductivity. Finally, we shall call the tensor D_{ik}, which appears in Eq. (A.8) as the coefficient associated with the gradient of the electron concentration, the diffusion tensor. The quantities V and σ_{ik} can be independently determined by experiment; the same assertion also pertains, at least, to the symmetric part of the tensor D_{ik}.

Eliminating the quantity Δn from Eq. (A.8) with the aid of the equation of continuity, we can find the longitudinal part $S_\parallel$ of the tensor S_{ik} which enters into the linear relations $\Delta j_i = S_{ik}\mathscr{E}_k$. We have

$$S_\parallel = \omega\sigma_\parallel / (\omega - qV + iq^2 D_\parallel). \quad (A.9)$$

In such a form this relation appeared in[22].

Having utilized Poisson's equation, one can represent the field $\mathscr{E}$ in the form of a sum of two terms: the external field $E^{(e)}$ and the self-consistent field $E^{(scf)}$ which arises due to the redistribution of the charges in the semiconductor. Then one can introduce the tensor s_{ik} which appears in the linear relations $\Delta j_i = s_{ik}\mathscr{E}_k^{(e)}$. We obtain the following result for the longitudinal part of this tensor ($q \neq 0$):

$$s_\parallel(\omega, q) = \frac{\omega\sigma_\parallel}{\omega - qV + i(4\pi\sigma_\parallel/\varepsilon_\parallel + q^2 D_\parallel)}. \quad (A.10)$$

The conditions for the validity of this expression also include the inequality $\kappa l_e \ll 1$.

[15)] Subsequent iterations of Eq. (A.6) give terms in the expression for the current density which are, generally speaking, small under the assumption that the parameters ql_e and $\omega\tau_e$ are small. An exception may, for example, be the region near a bend in the current-voltage characteristics, when the corresponding component of the tensor σ_{ik} is small and it may be necessary to take account of the terms which are proportional to the spatial derivative of the component of the variable electric field $\mathscr{E}$.

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