RECOMBINATION RADIATION IN SEMICONDUCTORS DURING THE PINCH EFFECT UNDER CONDITIONS OF STRONG DEGENERACY OF THE ELECTRON GAS

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Recombination radiation in a semiconductor under pinch-effect conditions is considered. The spectral distribution of the radiation and of the effective temperature as a function of the current passing through the crystal are calculated for a situation characteristic of a semiconductor of the InSb type at low temperatures (the light current carriers are degenerate and the heavy ones are not).

WHEN a strong electric current flows through a semiconductor with bipolar conductivity, the magnetic field produced by the current presses the carriers towards the center of the crystal, thus producing a pinch effect in the electron-hole plasma of the semiconductor.

The development of the pinch effect is hindered by the volume recombination of the nonequilibrium carriers. One can indicate cases, however, in which the role of this recombination in the formation of the nonequilibrium spatial distribution of the carriers is noticeably limited. This includes a case of injection from contacts, when the distance between the current electrodes is smaller than the diffusion length "stretched" by the field, and also the case of a long crystal, in which at least one of the transverse dimensions is comparable in magnitude with the bipolar diffusion length.

In a number of semiconductors, a noticeable fraction of the electron-hole pair recombination is radiative recombination. An investigation of the radiation emerging from the crystal makes it possible to establish the existence of a pinch effect and to estimate the characteristics of the plasma pinch—the radius, the average density, and the carrier temperature[1]. The most information concerning these characteristics can be extracted from a detailed analysis of the spectral composition of the radiation as a function of the current, with allowance for the singularities of the absorption. In the analysis of recombination radiation under pinch-effect conditions, it is necessary to distinguish between the case of the intrinsic conductivity and the case in which the bipolar conductivity is produced by injection of nonequilibrium carriers and greatly exceeds the conductivity in weak currents. In the latter case (to which we confine ourselves here), the recombination occurs practically in the entire volume of the crystal.

It has been shown earlier[1] that the pinch effect in indium antimonide with degenerate carriers is accompanied by intense recombination radiation in the region $\lambda \sim 5\,\mu$. It is shown there also that when the current flowing through the crystal is increased, the integral intensity of the radiation decreases, and the maximum of the spectral distribution of the radiation shifts towards the short-wave side. The shift of the maximum is attributed to the growth of the Fermi level in the pinch channel, and the decrease of the intensity is attributed to the broadening of the region of absorption of the light as the plasma pinch becomes narrower[1,2].

In the present paper we use the data of[3,4] on the distribution of the carriers in the stationary pinch effect to obtain quantitative estimates of the recombination radiation in semiconductors of the InSb type. The electron gas is assumed to be strongly degenerate. In view of the large difference between the effective masses of the electrons and the holes, the degree of degeneracy of the holes does not exert a strong influence on the spatial distribution of the carrier[3]. We shall henceforth assume that the holes are described by Boltzmann's statistics, and that their Fermi quasilevel μ_p is much lower in absolute magnitude than the Fermi quasilevel μ_n for the electrons. All the electron and hole energies are reckoned from the tops of the corresponding bands. The analysis is carried out for a crystal having a cylindrical shape: $0 \leq r \leq d$.

If the following condition is satisfied (see for example,[5])

$$h\nu - E_g < \mu_n(r') + \mu_p(r') \tag{1}$$

(where ν is the frequency of the light and E_g the width of the forbidden band), the optical transition occur in the region $0 \leq r \leq r'$ between inversely-populated states, and this is a region of negative absorption for the recombination radiation. Under these conditions, if r' is sufficiently large, the integral intensity may greatly exceed the intensity of radiation produced in the pinch effect in the absence of carrier degeneracy.

In the case of direct allowed interband transitions, the energy radiated per unit time by a unit volume in the spectral interval $d\nu$ is equal to[6]

$$W(r, \nu)\,d\nu = 8\pi h\nu^3 q^2 \varkappa(\nu) c^{-2} f_n(r) f_p(r)\,d\nu. \tag{2}$$

Here

$$f_{n,p}(r) = \left\{ \exp\left[\frac{E_{n,p} - \mu_{n,p}(r)}{kT}\right] + 1\right\}^{-1};$$

the absorption coefficient is $\kappa(\nu) = \kappa_0\sqrt{(h\nu - E_g)E_g}/h\nu$; q is the refractive index (in indium antimonide, $q \approx 4$ and $\kappa_0 \approx 7 \times 10^3\ \text{cm}^{-1}$).

Using the foregoing assumptions concerning the degree of degeneracy of the carriers, we obtain for the transitions from the states $E_n < \mu_n(r)$

$$W_{E_n<\mu_n}(r,\nu) = A_1\sqrt{h\nu - E_g}\,\nu^2 \exp\left(\frac{m_n}{m_p}\,\frac{E_g - h\nu}{kT}\right) p(r). \qquad (3)$$

Here

$$A_1 = \sqrt{\frac{2E_g}{\pi}}\,\frac{\varkappa_0 q^2 h^3}{c^2 (m_p kT)^{3/2}};$$

p(r) is the local concentration of electrons and holes.

For transitions from the states $E_n > \mu_n$ (the electron distribution function takes here a classical form) we obtain

$$W_{E_n>\mu_n}(r,\nu) = A_2\sqrt{h\nu - E_g}\,\nu^2 \exp\left(\frac{E_g - h\nu}{kT}\right) p^2(r), \qquad (4)$$

where

$$A_2 = \frac{h^6 \varkappa_0 \sqrt{E_g}\, q^2}{4\pi^2 c^2 (kT)^3 (m_n m_p)^{3/2}}$$

According to the data of[4], in the assumed model of a strongly degenerate electron gas, the carriers occupy the entire volume of the crystal only when the total current J does not exceed a certain critical value J_0. When $J \geq J_0$, the plasma breaks away from the crystal walls and fills a cylinder of radius $R \leq d$, and in this case $p(r \geq R) = 0$. A typical profile of the spatial distribution of the Fermi quasi level of the electrons $\mu_n(r) \equiv \mu_n[p(r)]$, is shown in Fig. 1. Actually, of course, there is no complete detachment of the plasma from the walls of the crystal, for in the case of nonzero temperature the region $d \geq r \geq R$ contains a certain quantity of carriers whose concentration does not satisfy the strong-degeneracy criterion. The contribution of these currents to the current and to the recombination radiation is insignificant, and we shall henceforth assume that $p(r \geq R) = 0$.

If we introduce the dimensionless current $\Theta = (J/J_0)^{1/4}$, then we get under conditions when the drift velocity of the electrons saturates with the field

$$R = \Theta^{-1} d, \quad p_R = p_d \Theta^6, \qquad (5)$$

where p_d and p_R are the carrier densities averaged over the crystal and over the cylinder with radius R, respectively. The carrier distribution is of the form

$$p(r) \approx p(0) \exp(-\Lambda r^2) \qquad (6)$$

in the central part of the plasma pinch and

$$p(r) \approx p_R (1 - r^2/R^2)^{3/2} \qquad (7)$$

near the boundary of the pinch. The foregoing expressions approximate well the true distribution of the carriers in the case when the rates of volume and surface recombination are low. Numerical calculations performed for the case when the total number of carriers

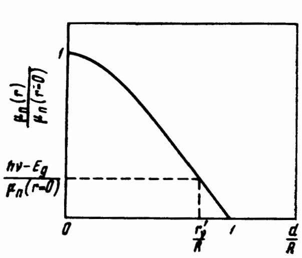

FIG. 1. Typical spatial distribution of the electron Fermi quasilevel in the pinch effect in a degenerate electron-pole plasma.

in the crystal is conserved show that $p(0) \approx 3p_R$ and $\Lambda \approx 3R^{-2}$.

If the radiative recombination time is large or comparable with the carrier recombination time, then we can neglect in the first approximation the reaction of the light on the spatial distribution of the carriers. In this case (to which we confine ourselves here) the recombination radiation is determined by the magnitude (and by the sign) of the light-absorption coefficient and by the path lengths of the individual rays in the regions of positive and negative absorption. The rays whose angles with the normal to the surface of the cylinder exceeds $\sin^{-1} q^{-1}$ experience total internal reflection. We have excluded from consideration the reflected rays. The absorption of light by free carriers at the concentrations assumed in the estimates (see below) is negligible.

Using the assumption made above, namely that the Fermi quasilevels of the electrons and holes are different, we define r_ν, the boundary of the inversion region for radiation with frequency ν, by the condition

$$h\nu - E_g = \mu_n(r_\nu). \qquad (8)$$

From (7) and (8) we obtain

$$r_\nu = R(1 - \varepsilon_\nu), \quad \varepsilon_\nu = \frac{h\nu - E_g}{2\mu_n(p_R)}. \qquad (9)$$

Further calculations confirm the fact that when $\nu \approx \bar{\nu}$, where $\bar{\nu}$ is the light frequency corresponding to the maximum spectral distribution, we have $\epsilon_\nu \ll 1$.

It follows from geometrical considerations that when account is taken of the positive and negative absorptions of the radiation when $r_\nu > d/3$, the main contribution to the radiation of light of frequency ν from the surface of the crystal is made by the nearest vicinity of the rays connecting the points with cylindrical coordinates

$$\mathbf{r}_i = (r_\nu, \varphi, z) \quad \text{и} \quad \mathbf{r}_f = \left(d, \varphi + \pi, z \pm \frac{r_\nu + d}{\sqrt{q^2 - 1}}\right),$$

since such rays traverse the largest path in the region of the inverted population. When $r_\nu < d/3$, the main contribution is made by the nearest vicinity of the radial rays passing through the center of the crystal. Since $r_\nu \approx R$ when $\nu \approx \bar{\nu}$, if $\Theta < 3$, the path of the ray $\mathbf{r}_f - \mathbf{r}_i$ in the region of negative absorption exceeds the path of this ray in the region of positive absorption. When $\Theta > 3$, the path in the region of positive absorption is larger everywhere than the path in the region of negative absorption. We can thus expect the characteristics of the spectral radiation to reveal a different dependence in the current when $\Theta < 3$ and $\Theta > 3$; this will be confirmed by the calculation that follows.

The rays emerging from the point r and making the main contribution to the radiation from the surface of the crystal propagate in a solid angle η whose magnitude when $\kappa(\nu) r \gg 1$ is estimated by the expression

$$\eta \approx \frac{4\sqrt{2}}{q\sqrt{\varkappa(\nu)\, r}\,(1 + r/r_\nu)}.$$

The spectral density of the radiation from a unit crystal length is of the form

$$I_{E_n<\mu_n}(\nu) = \frac{A_1 \nu^2 \sqrt{h\nu - E_g}}{q\sqrt{\varkappa(\nu)}}\, 2\sqrt{2} \exp\left[\frac{m_n}{m_p}\,\frac{E_g - h\nu}{kT}\right.$$

$$+\varkappa(\nu)(2r_\nu - d)\Big]\int_0^{r_\nu} p(r)\,e^{\varkappa(\nu)r}\sqrt{\frac{r}{1+r/r_\nu}}\,dr. \qquad (10)$$

Substituting here (5), (7), and (9) and putting $\mu_n(r) \ll E_g$, we obtain the spectral distribution, which depends on the current Θ, of the intensity of the recombination radiation from the surface of the crystal (when $\kappa r_\nu \gg 1$, the main contribution to the integral (10) is made by $r \approx r_\nu$):

$$I_{E_n<\mu_n}(\Theta,\Omega) = A_3\Theta^{-1/2}\left(1-\frac{\Omega}{2\Theta^4}\right)^{1/2}\Theta^{5/4}$$
$$\times\exp\left[-\varkappa\sqrt{\Omega}\left(\frac{\Theta-3}{\Theta}+\frac{3\Omega}{2\Theta^5}\right)-\zeta\Omega\right]. \qquad (11)$$

Here

$$\Omega = \frac{h\nu - E_g}{\mu_n(p_d)} = 2\varepsilon_\nu\Theta^4, \qquad \chi = \varkappa_0 d\sqrt{\frac{\mu_n(p_d)}{E_g}},$$
$$\zeta = \frac{m_n}{m_p}\,\frac{\mu_n(p_d)}{kT}, \qquad A_3 = \frac{4qhE_g{}^3p_d d}{\sqrt{2\pi}\chi\, c^2(m_p kT)^{3/2}}.$$

Figure 2 illustrates the spectral distribution of recombination radiation. It shows the dependence of the function $Y = A_3^{-1}\Theta^{1/2}I_{E_n<\mu_n}(\Theta,\Omega)$ on the dimensionless frequency Ω for different values of the current Θ. The numerical calculations (here and below) were made for a crystal with parameters of InSb, assuming $p_d = 10^{16}\ \mathrm{cm}^{-3}$, $\kappa_0 d = 50$, $T = 10^\circ\mathrm{K}$, $m_n = 0.013\,m_0$, $m_p = 0.5\,m_0$, and $q = 4$.

We note that in the region $\Theta < 3$ and $\chi \gg 1$ all the singularities of the spectral distribution are contained mainly in the factor

$$\exp\left[-\chi\sqrt{\Omega}\left(\frac{\Theta-3}{\Theta}+\frac{3\Omega}{2\Theta^5}\right)\right]. \qquad (12)$$

Using (12), we obtain the following expression for the frequency of the maximum of the spectral distribution:

$$\tilde{\Omega} = \Omega(\tilde{\nu}) = {}^2/_9\Theta^4(3-\Theta). \qquad (13)$$

Estimating by means of (12) the half-width of the spectral curve, we arrive at the equation

$$y^3 - 3y + 2 = \frac{9\ln 2}{\sqrt{2}\chi\Theta(3-\Theta)^{3/2}}. \qquad (14)$$

Here $y = \sqrt{\Omega'/\tilde{\Omega}}$, where Ω' is the frequency at which the intensity amounts to half the maximum value. If the right side of (14) is small compared with unity, the half-width Δ at $\Theta < 3$ is equal to

$$\Delta \approx 0.9\chi^{-1/2}\Theta^{7/2}(3-\Theta)^{1/4}. \qquad (15)$$

Figure 3 shows the dependences of the frequency $\tilde{\Omega}$ and of the half-width Δ and the dimensionless current Θ. The solid lines show the results of calculations by means of formula (11), and the dashed lines correspond to formulas (13) and (15). It is seen from the figure that when $\Theta < 2.4$ the spectral maximum shifts with increasing current towards the short-wave side in accordance with the result of[1]. When $\Theta > 2.4$, the situation is reversed: with increasing current, the spectral maximum shifts towards the long-wave side, since the broadening of the region of positive absorption, located between the plasma pinch and the surface of the crystal, comes into play. A similar picture is observed also in the dependence of the half-width Δ of the spectral distribution on the current Θ: the half-width increases with increasing current when $\Theta < 2.8$ and decreases when $\Theta > 2.8$.

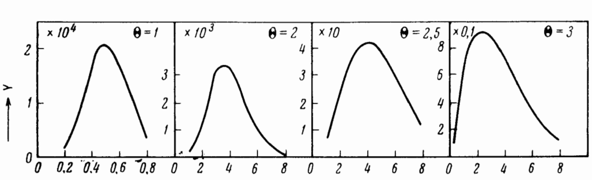

FIG. 2. Spectral distribution of the recombination radiation at different values of the current Θ.

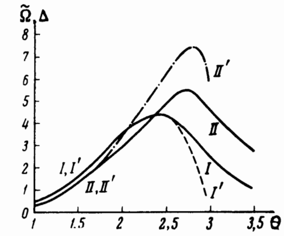

FIG. 3

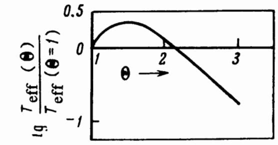

FIG. 4

FIG. 3. Dependence of the frequency $\tilde{\Omega}$ of the maximum and of the half-width Δ of the spectral distribution of the recombination radiation in the current Θ: I, I′ – $\tilde{\Omega}(\Theta)$, II, II′ – $\Delta(\Theta)$.

FIG. 4. Dependence of the effective temperature of the recombination radiation on the current Θ.

The integral intensity of the radiation from a unit length of the crystal

$$Q(\Theta) = \int_{E_g/h}^{\infty} I(\nu,\Theta)\,d\nu$$

can be estimated by means of the quantity

$$\frac{\mu_n(p_d)}{h}I(\tilde{\Omega},\Theta)\Delta. \qquad (16)$$

When $\Theta < 3$ we get from (11), (13), (15), and (16)

$$Q(\Theta) \approx A_4\Theta^8(3-\Theta)^{3/2}$$
$$\times\exp\{{}^2/_9\Theta(3-\Theta)[\chi\sqrt{2(3-\Theta)} - \zeta\Theta^3]\}, \qquad (17)$$

where

$$A_4 = 0.2\frac{qE_g{}^3p_d\mu_n(p_d)\,d}{\chi c^2(m_p kT)^{3/2}}.$$

The result of the investigation of the dependence of the integral intensity on the current Θ is shown in Fig. 4. The nature of the intensity employed here is the effective temperature

$$(2\pi d)^{-1}Q(\Theta) = \sigma T_{\mathrm{eff}}^4(\Theta),$$

where σ is the Stefan-Boltzmann constant. At the semiconductor parameter constants indicated above, we have $T_{\mathrm{eff}}(\Theta = 1) \approx 5\times 10^{3\circ}\mathrm{K}$. The radiation temperature also depends on the current non-monotonically. It increases with increasing current when $\Theta < 1.4$ and decreases when $\Theta > 1.4$.

So far we have confined ourselves to allowance for transitions from inversely-populated metals. Estimates performed using (6) and (4) show that in the presence of stimulated radiation the contribution of the classical radiation is small. The latter must be taken into consideration in practice only at the frequencies

$$h\nu - S_g \geqslant \mu_n(r=0) \approx 2\mu_n(p_d)\Theta^4,$$

when there is no stimulated radiation. In this frequency region, the spectral distribution of the radiation has the usual classical form

$$I(\nu) \approx \exp\left(\frac{E_g - h\nu}{kT}\right).$$

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