CONTRIBUTION TO THE THEORY OF THE DYNAMICS OF INJECTION LASERS

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Possible modes of operation of injection lasers with inhomogeneous excitation are considered theoretically. The conditions for the appearance of hard self-excitation are investigated. A periodic solution is found in the case of unstable stationary operating conditions. The experimental results are discussed.

INTRODUCTION

THE investigation of the dynamics of emission of semiconductor lasers has yielded important results. A hard self-excitation regime was observed,[1] self-synchronization of the axial modes was realized,[2] and a complicated spike structure of the radiation pulse was observed.[3,4] Regular ultrashort light pulses were obtained,[5,6] and synchronization of the light pulses by modulation of the exciting current was realized.[7]

Most experimental results on the dynamics of semiconductor lasers were obtained with injection semiconductor lasers whose excitation was not uniform over the area of the p–n junction. The construction of such a laser is described, for example, in [8]. The semiconductor laser constitutes a double diode, both parts of which, 1 and 2, are electrically insulated from each other and are coupled by a common resonator. By varying the injection current it is possible to vary continuously the gain (or absorption) in each part of the diode. This makes it possible to trace the character of the dynamic processes in a large range of variation of the degree of nonlinearity of the absorbing (amplifying) part of the laser.

Although extensive experimental material on the study of the dynamics of injection lasers with non-uniform excitation has already been accumulated by now, there is still no sufficiently complete theoretical description of the experimental results.

The purpose of the present article is a theoretical investigation of the possible operating regimes of an injection semiconductor laser with non-uniform excitation over the area of the p-n junction.

1. INITIAL EQUATIONS AND ANALYSIS OF STATIONARY SOLUTIONS

A theoretical analysis of the dynamic regimes can be carried out with the aid of rate equations, in which the probability of induced recombination is completely characterized by the position of the Fermi quasilevel. The use of the rate equation is justified up to times on the order of 10^{-12} sec, since the relaxation time of the interband polarization, determined under our conditions by the electron-electron collisions, is of the order of 10^{-13} sec.

The rate equations for a semiconductor laser with non-uniform injection density can be written in the form

$$\dot{S} = [V_1(G_1+\gamma G_2) - 1/\tau_r]S,$$
$$\dot{n}_1 = \frac{J_1}{d} - \frac{n_1}{\tau} - SG_1, \quad \dot{n}_2 = \frac{J_2}{d} - \frac{n_2}{\tau} - SG_2. \tag{1}$$

Here S is the density of the number of photons in the resonator; τ_r is their lifetime; G_i is the rate of induced recombination per unit volume in the part of the diodes; $\gamma = V_2/V_1$ where V_1 and V_2 are the volumes of parts 1 and 2 of the diode; J_i is the density of the injection currents; d is the diffusion length; τ is the time of spontaneous recombination of the carriers. It is convenient to change over to differentiation with respect to the dimensionless time $t' = t/\tau$ and to rewrite the system (1) in the form

$$\dot{\Phi} = \frac{1}{\varepsilon}[g_1+\gamma g_2 - 1]\Phi,$$
$$\dot{n}_1 = j_1 - n_1 - \Phi g_1, \quad \dot{n}_2 = j_2 - n_2 - \Phi g_2, \tag{2}$$
$$\varepsilon = \frac{\tau_r}{\tau}; \quad j_i = \frac{J_i\tau}{d}, \quad g_i = \tau_r V_1 G_i, \quad \Phi = \frac{\tau}{\tau_r V_1}S, \quad i=1,2.$$

It follows from experimental and theoretical investigations of strongly-doped GaAs semiconductor lasers that the radiative recombination is the result of electronic transitions from energy levels due to donor impurities (the so-called "tails" of the density of states) to levels of the acceptor bands. Since the holes, owing to their large effective mass, are concentrated in a narrow energy band, the gain is determined completely by the density of the energy levels near the bottom of the conduction band and by their population. The available experimental data show that the density of states near the bottom of the conduction band depends exponentially on the energy

$$\rho = \rho_0 \exp(E/E_0),$$

where the parameter E_0 depends on the concentration of the donor impurities. If we assume that in the valence band there is a narrow impurity level that can be described by a δ-function, then the gain will have the following form:

$$G_i = A\rho_0\left\{\left[1+\exp\frac{E-F_i}{kT}\right]^{-1} - \frac{1}{2}\right\}\exp\frac{E}{E_0}, \quad i=1,2, \tag{3}$$

where A is a constant that depends on the temperature, F_i is the Fermi quasilevel in the conduction band, and E is the energy of a quantum of the amplified light. The number of electrons is connected with the Fermi quasilevel by the relation

$$n_i = B\rho_0 \exp(F_i/E_0),$$

where B also depends on the temperature.

The gain (3) contains the energy of the light quantum E as a parameter. At specified positions of the Fermi quasilevels, the total gain is a maximum at a frequency determined by the equation

$$\frac{\partial}{\partial E}(g_1 + \gamma g_2) = 0.$$

When the positions of the Fermi quasilevels change, a change takes place in the frequency at which the gain is a maximum. Since in the optical band the Q of the resonator is usually much higher than the Q of the emission line, the generation frequency is determined mainly by the natural frequency of the resonator, and the generation frequency pulling connected with the drift of the top of the spectral radiation line is small. For this reason, E in (3) can be regarded as a quantity independent of F_i.[1)]

The stationary generation amplitudes are determined by the equation

$$K(\Phi) = g_1(\Phi) + \gamma g_2(\Phi) = 1.$$

Depending on the form of the function $K(\Phi)$, one or two stationary states with $\Phi > 0$ are possible. If both parts of the diode amplify the radiation, then saturation causes the total gain to decrease monotonically with increasing field, and consequently the gain is equal to the loss only at one value of the number of photons in the resonator. On the other hand, if one of the parts of the diode amplifies and the other absorbs, then the total gain can have a maximum and two stationary states are possible. Indeed, when the number of photons decreases the Fermi quasilevel drops in the amplifying part of the diode, and rises in the absorbing part. If the drop of the absorption is faster than the increase of the gain when the number of photons increases, then the total gain increases and subsequently, in sufficiently strong fields, the total gain decreases as the result of saturation.[9]

For the last two stationary equations (2) we can find the slope of the $K(\Phi)$ curve at arbitrary Φ:

$$\frac{\partial K}{d\Phi} = -\left(g_1\frac{\partial g_1}{\partial n_1}\tau_1 + \gamma g_2 \frac{\partial g_2}{\partial n_2}\tau_2\right),$$
$$\tau_i^{-1} = 1 + \frac{\partial g_i}{\partial n_i}\Phi_i, \quad i = 1, 2. \tag{4}$$

If the ratio of the currents j_1 and j_2 is such as to satisfy the inequality

$$g_1(j_1)\frac{\partial g_1(j_1)}{\partial n_1} + \gamma g_2 \frac{\partial g_2(j_2)}{\partial n_2} \leqslant 0 \tag{5}$$

and $K(0) \leq 1$, then a hard self-excitation regime exists in the system (2).

Assume that the system has arrived at a state of the type Φ_2, where $dK/d\Phi < 0$. Experimental investigations show that the radiation intensity of a semiconductor laser with non-uniform excitation, at a definite ratio of the injection currents in the parts of the diode, has the form of regular light pulses, i.e., the regime of generation with a stationary value of the amplitude is unstable.

Linearizing the system (2) near the equilibrium position and investigating the condition of these equations, we can obtain the following conditions under which the stationary solution, n_1^0, n_2^0, Φ^0 ($dK/d\Phi|_{\Phi=\Phi'} < 0$) is unstable:

$$\varepsilon\frac{\tau_1+\tau_2}{\tau_1^2\tau_2^2} + \Phi^0\left(g_1\frac{\partial g_1}{\partial n_1}\frac{1}{\tau_1} + \gamma g_2 \frac{\partial g_2}{\partial n_2}\frac{1}{\tau_2}\right) < 0. \tag{6}$$

Here τ_i, the gain g_i, and the derivatives $\partial g_i/\partial n_i$ are taken in the stationary state.

It is seen from (6) that if both parts of the diode amplify the radiation ($g_i > 0$), then the stationary generation regime is always stable (the derivatives $\partial g_i/\partial n_i$ are always positive). The condition (6) can be satisfied only if either $g_1 < 0$ or $g_2 < 0$, i.e., to realize the conditions of the spike regime of laser operation[6, 10] one of the parts of the diode should absorb and the other should amplify the radiation.

2. REGIME OF PERIODIC PULSATIONS OF RADIATION INTENSITY

If the ratio of the currents j_1 and j_2 is such that condition (6) is satisfied, then the stationary solution turns out to be unstable and radiation-intensity oscillations are produced in the system. In this case it is impossible to find an exact analytic solution of the three nonlinear equations. But in the case when $\epsilon \ll 1$ it is possible to obtain sufficiently satisfactory approximate solutions. This is connected with the fact that when $\epsilon \ll 1$ large changes in the number of photons occur already at small deviations of the number of electrons from their stationary values. To describe the regime of radiation-intensity pulsations with sufficiently large depth modulation, the quantities $g_i(n_i)$ can therefore be expanded in powers of $\Delta n_i = n_i - n_i^0$, where Δn_i is the deviation of the number of electrons from its stationary values n_i^0. If the $\partial^k g_i(n_i^0)/\partial n_i^k$ exist and are continuous at the point n_i^0, then

$$g_i(n_i) = g_i^0 + \frac{\partial g_i^0}{\partial n_i}\Delta n_i + \sum_{k=2}^{\infty}\frac{1}{k!}\frac{\partial^k g_i^0}{\partial n_i^k}\Delta n_i^k, \tag{7}$$
$$g_i(n_i^0) = g_i^0, \quad i = 1, 2.$$

All the derivatives in (7) are calculated at the point n_i^0.

Recognizing that

$$g_1(n_1^0) + \gamma g_2(n_2^0) = 1; \quad j_i = n_i + \Phi^0 g_i(n_i^0),$$

we can write the system (2) in equivalent form:

$$\dot{\varphi} = \frac{1}{\varepsilon}\left(\frac{\partial g_1^0}{\partial n_1}\Delta n_1 + \gamma\frac{\partial g_2^0}{\partial n_2^0}\Delta n_2 + \sum_{k=2}^{\infty}\left(\frac{\partial^k g_1^0}{\partial n_1^k}\frac{\Delta n_1^k}{k!} + \gamma\frac{\partial^k g_2^0}{\partial n_2^k}\frac{\Delta n_2^k}{k!}\right)\right)\varphi,$$
$$\Delta\dot{n}_i = g_i^0\Phi^0(1-\varphi) - \Delta n_i - \Phi^0\varphi\sum_{k=1}^{\infty}\frac{\partial^k g_i^0}{\partial n_i^k}\frac{\Delta n_i^k}{k!}, \quad \Phi = \Phi^0\varphi, \quad i = 1, 2. \tag{8}$$

If we go over here to differentiation with respect to the time $t'' = t'/\sqrt{\epsilon}$ and introduce the variables $x = \Delta n_1/\sqrt{\epsilon}$ and $y = \Delta n_2/\sqrt{\epsilon}$, then (8) takes the form

$$\dot{\varphi} = \left(\frac{\partial g_1^0}{\partial n_1}x + \gamma\frac{\partial g_2^0}{\partial n_2}y\right)\varphi + \mu f_1, \quad \mu f_1 = \varphi\sum_{k=2}^{\infty}\frac{\varepsilon^{(k-1)/2}}{k!}\left(\frac{\partial^k g_1^0}{\partial n_1^k}x^k + \gamma\frac{\partial^k g_2^0}{\partial n_2^k}y^k\right);$$

$$\dot{x} = g_1^0\Phi^0(1-\varphi) + \mu f_2, \quad \mu f_2 = -\varepsilon^{1/2}x - \Phi^0\varphi\sum_{k=1}^{\infty}\varepsilon^{k/2}\frac{\partial^k g_1^0}{\partial n_1^k}\frac{x^k}{k!};$$

[1)]At appreciable frequency shifts of the maximum gain, the generation frequency may jump from one mode to another.

$$\dot{y}=g_2{}^0\Phi^0(1-\varphi)+\mu f_3,\quad \mu f_3=-\varepsilon^{1/2}y-\Phi^0\varphi\sum_{k=1}^{\infty}\varepsilon^{k/2}\frac{\partial g_2{}^0}{\partial n_2{}^k}\frac{y^k}{k!}. \quad (9)$$

In the right side of the system (9) we have introduced formally a small parameter μ, in order to visualize the smallness of the functions f_i. Indeed, the dimensions of the injection semiconductor lasers usually amount to 10^{-1}–10^{-2} cm, i.e., $\tau_r = 10^{-11}$–10^{-13} sec, and for gallium-arsenide diodes the lifetime τ is $\sim 10^{11}$ sec and $\epsilon \sim 10^{-2}$–10^{-4}. The functions f_i are proportional to powers of the small quantity ϵ, and one can therefore expect that at sufficiently small ϵ the solution of (9) to be close to the solution of the system

$$\dot{\varphi}=\left(\frac{\partial g_1{}^0}{\partial n_1}x+\gamma\frac{\partial g_2{}^0}{\partial n_2}y\right)\varphi,$$
$$\dot{x}=g_1{}^0\Phi^0(1-\varphi),\qquad \dot{y}=g_2{}^0\Phi^0(1-\varphi), \quad (10)$$

which is obtained from (9) at $\mu = 0$.

The system (10) is conservative and has two single-valued analytic integrals of motion:

$$\frac{1}{2}\left(\frac{\partial g_1{}^0}{\partial n_1}x+\gamma\frac{\partial g_2{}^0}{\partial n_2}y\right)^2-[\omega_0{}^2\ln\varphi-\varphi]=\omega_0{}^2C_1,$$
$$g_2{}^0x-g_1{}^0y=C_2,\quad \omega_0{}^2=\left(\frac{\partial g_1{}^0}{\partial n_1}+\gamma\frac{\partial g_2{}^0}{\partial n_2}g_2{}^0\right)\Phi^0. \quad (11)$$

On the $(\varphi, \dot{\varphi})$ phase plane, Eqs. (10) define closed phase trajectories

$$\tfrac{1}{2}[\dot{\varphi}/\varphi]^2-\omega_0{}^2[\ln\varphi-\varphi]=\omega_0{}^2C_1, \quad (12)$$

which correspond to periodic time variations of the photon numbers $\Phi = \Phi^0\varphi$. The approximation (10) is valid under the condition that $\epsilon C_1 \ll 1$ and it is assumed henceforth that $g_1(0) + \gamma g_2(0) > 1$.[2)]

At small values of μ, the system (2) is close to (10) and we can therefore expect the periodic-radiation-intensity pulsations that appear when the stationary state is unstable to be described by Eqs. (10).

The periodic motions of the system (9) are generated by those closed integral trajectories (11), for which

$$K_1(C_1,C_2)=\int_0^T\left(\frac{\partial C_1}{\partial\varphi}f_1+\frac{\partial C_1}{\partial x}f_2+\frac{\partial C_1}{\partial y}f_3\right)dt''=0,$$
$$K_2(C_1,C_2)=\int_0^T\left(\frac{\partial C_2}{\partial\varphi}f_1+\frac{\partial C_2}{\partial x}f_2+\frac{\partial C_2}{\partial y}f_3\right)dt''=0, \quad (13)$$

where the integral is taken along the curve $\mathscr{L}(C_1, C_2)$ belonging to the two-parameter family of closed curves (11), and $T = T(C_1, C_2)$ denotes the period of the motion in the generating system (10).[11]

The periodic solution of the generating system is

$$\varphi=\varphi(C_1),$$
$$x=\left(g_1{}^0\frac{\partial g_1{}^0}{\partial n_1}+\gamma g_2{}^0\frac{\partial g_2{}^0}{\partial n_2}\right)^{-1}\left(\psi g_1{}^0+\gamma\frac{\partial g_2{}^0}{\partial n_2}C_2\right), \quad (14)$$
$$y=\left(g_1{}^0\frac{\partial g_1{}^0}{\partial n_1}+\gamma g_2{}^0\frac{\partial g_2{}^0}{\partial n_2}\right)^{-1}\left(\psi g_2+\frac{\partial g_1{}^0}{\partial n_1}C_2\right),$$

where $\varphi(C_1)$ is the periodic solution of (12) with the integral of motion C_1, and $\psi = \dot{\varphi}/\varphi$. All the parameters of the light pulses, namely duration, repetition frequency, and depth of modulation, are determined by the quantity C_1.[12] For the most interesting case of pulsations with large depth of modulation, when $C_1 \gg 1$, it is possible to obtain from the simultaneous solution of (13), with the aid of (14),

$$C_1\varepsilon=-\frac{3}{2}\left(g_1{}^0\frac{\partial g_1{}^0}{\partial n_1}\frac{1}{\tau}+\gamma g_2\frac{\partial g_2{}^0}{\partial n_2}\frac{1}{\tau_2}\right)\frac{p^2}{g_1{}^0g_2{}^0},$$
$$p^2=\left(g_1\frac{\partial g_1{}^0}{\partial n_1}+\gamma g_2\frac{\partial g_2{}^0}{\partial n_2}\right)^2\left(g_1\frac{\partial g_1{}^0}{\partial n_1}\tau_1+\gamma g_2\frac{\partial g_2{}^0\tau_2}{\partial n_2}\right)$$
$$\times\left[\Phi^{02}\gamma\frac{\partial g_1{}^0}{\partial n_1}\frac{\partial g_2{}^0}{\partial n_2}\left(g_1{}^0\frac{\partial^2g_1{}^0}{\partial n_1{}^2}-g_2{}^0\frac{\partial^2g_2{}^0}{\partial n_2{}^2}\right)^2\tau_1\tau_2\right]^{-1}. \quad (15)$$

Expression (15) is approximate, since in the calculation of C_1 we have retained in (13) only the first terms in powers of $\epsilon^{1/2}$, leading to the existence of a limit cycle. It is seen from (12) and (15) that a periodic solution for the radiation intensity exists only if the following condition is satisfied

$$g_1{}^0\frac{\partial g_1{}^0}{\partial n_1}\frac{1}{\tau_1}+\gamma g_2{}^0\frac{\partial g_2{}^0}{\partial n_2}\frac{1}{\tau_2}<0. \quad (6a)$$

This condition coincides, accurate to terms $\sim\epsilon$, with the condition for the instability of the stationary solution (6).[3)] An investigation shows that the resultant limit cycle is stable.

The process of generation of light pulses proceeds in the following manner. When $0 \le t \le t_1 = \sqrt{2C_1\tau\tau_r}/\omega_0$, a relatively slow development of generation takes place, in which the active particles accumulate in the amplifying part and bleaching takes place in the absorbing part. The number of photons changes from $\Phi_{min} = \Phi^0 \times \exp -C_1$ to Φ^0 in accordance with the law

$$\Phi(t)=\Phi^0\exp\left(\frac{\omega_0{}^2t^2}{2\tau\tau_r}-C_1\right).$$

Then, after the absorbing part is bleached, a sharp increase takes place in the amplitude, to $\Phi_{max} = \Phi^0C_1$, and the energy stored in the amplifying part is emitted in a narrow pulse. The time behavior of the radiation pulse is described by the following expression:

$$\Phi(t)=\Phi^0C_1\,\mathrm{ch}^{-2}\sqrt{\frac{C_1\omega_0{}^2}{2\tau\tau_r}}(t_2+t_1-t);$$
$$t_1\leqslant t\leqslant t_1+t_2=\sqrt{\frac{2\tau\tau_r}{C_1\omega_0{}^2}}\,\mathrm{Arth}\sqrt{\frac{C_1-1}{C_1}}.$$

The duration of this section, when the number of photons changes from $\Phi = \Phi^0$ to $\Phi_{max} = \Phi^0C_1$ and then again to Φ^0, is $2t_2 \approx \sqrt{2\tau\tau_r/C_1\omega_0^2}\ln(4C_1)$. Consequently, the pulse repetition period is

$$T=\frac{2\sqrt{2C_1\tau\tau_r}}{\omega_0}+\frac{\ln(4C_1)}{\omega_0}\sqrt{\frac{2\tau\tau_r}{C_1}} \quad (16)$$

and the pulse duration at half height is

$$T_0=\frac{2{,}5}{\omega_0}\sqrt{\frac{\tau_r\tau}{C_1}}. \quad (17)$$

Thus, knowing the value of C_1 from (15), we can

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2)The condition $g_1(0) + \gamma g_2(0) > 1$ can be replaced by the less stringent one $g_1(\Phi_{min}) + \gamma g_2(\Phi_{min}) > 1$, where Φ_{min} is the smallest value of Φ in the pulsation regime.

3)When the system (13) is solved with greater accuracy with respect to the powers of $\epsilon^{1/2}$, the condition for the existence of the limit cycle coincides with condition (6).

find all the characteristics of the automodulation process produces when the stationary state is unstable.

It should be noted that when the depth of modulation of the radiation intensity is small, C_1 is small, the pulse repetition period depends little on C_1, and its value, as follows from (12), is

$$T = \frac{2\pi}{\omega_0}\sqrt{\tau\tau_r}.$$

If the pulsations have a large depth of modulation, then it follows from (15), (16), and (17) that the duration of the pulses at half-height is determined by the lifetime τ_r of the photons in the resonator, and the distance between them is determined by the time τ of spontaneous recombination of the carriers, i.e., the Q-switching regime is realized in this limiting case.

3. CALCULATION OF THE GENERATION REGIMES IN A SEMICONDUCTOR LASER FOR THE CASE $\gamma = 1$

We present a concrete calculation for a semiconductor laser with non-uniform excitation in the particular case when $\gamma = 1$. It is convenient to normalize (2) in such a way that n_1, n_2, j_1, and j_2 are equal to the ratios of the corresponding quantities to their threshold values in the case of uniform excitation, so that for example j_2 is the ratio of the current density in the second part of the diode to the threshold current density in the same part in the case of uniform excitation. In this case, the system (2) takes the form

$$\dot{\Phi} = \frac{1}{\varepsilon}\left[\frac{x}{2(\sqrt{2}-1)}\left(\frac{n_1-(\sqrt{2}-1)x}{n_1+(\sqrt{2}-1)x} + \frac{n_2-(\sqrt{2}-1)x}{n_2+(\sqrt{2}-1)x}\right) - 1\right]\Phi,$$
$$\dot{n}_i = j_i - n_i - \frac{x}{2(\sqrt{2}-1)}\,\frac{n_i-(\sqrt{2}-1)x}{n_i+(\sqrt{2}-1)x}\Phi, \quad i = 1, 2, \tag{18}$$

where $x = \exp[(E - \bar{E})/kT]$, E is the frequency at which the total gain is maximal in the case of uniform excitation, and $\bar{E}$ is a constant quantity independent of the injection currents j_i. Since in gallium arsenide diffusion diodes at a donor concentration 10^{18} cm^{-3} the doping parameter is $E_0 \sim 8$ meV, and at nitrogen temperature we have $kT = 6.7$ meV, it was assumed in the derivation of (18) for simplicity that $E_0 \approx kT$. The total gain is maximal for a light quantum with energy

$$E = \bar{E} + {}^1\!/_2 kT \ln n_1 n_2.$$

Since many modes fall inside the emission line, the mode that is excited is the one closest to the frequency of the maximum gain. We shall henceforth assume that the Q of the resonator is sufficiently high compared with the Q of the radiation line, and neglect the pulling of the generation frequency towards the top of the radiation line. Estimates show that, up to appreciable inhomogeneity of the excitation, we have $(E - \bar{E})/kT \ll 1$, and we can therefore put $x = 1$ in (18).

It follows directly from (18) that the threshold curve, i.e., the curve on the (j_1, j_2) plane on the points of which the self-excitation conditions are satisfied, is given by

$$\frac{1}{j_1+(\sqrt{2}-1)} + \frac{1}{j_2+(\sqrt{2}-1)} = \sqrt{2}.$$

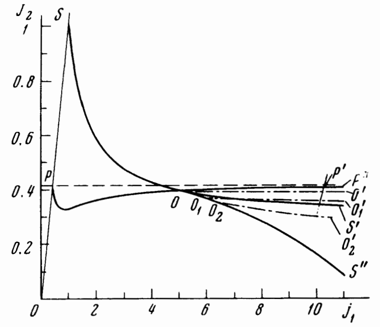

FIG. 1. Results of numerical solution of the system (18): threshold curve SOS', boundary POP'' of the region of existence of the hard regime, OO', $O_1O'_1$, and $O_2O'_2$ – limits of the instability regions at different values of the parameter ϵ for a double diode with $\gamma = 1$ ($\epsilon = 10^{-2}$ for the curve $O_2O'_2$, $\epsilon = 4 \times 10^{-3}$ for $O_1O'_1$, $\epsilon = 0$ for OO'). The axes represent the ratios of the current density in the parts of the diodes to their threshold values in the case of uniform excitation.

In the case of uniform excitation, $j_1 = j_2 = 1$. If the current density in one part of the diode is $j_1 < j_2 = (\sqrt{2} - 1)$, then this part of the diode absorbs the radiation.

Figure 1 shows part of the threshold curve SOS' for $j_1 \geq j_2$. In the region of currents lying below the line PP' ($j_2 = j_0$), part 2 of the diode operates in the regime of nonlinear absorption of the radiation. The solution of (5) is represented by the curve POP''. For all points j_1 or j_2 lying below the curves POP'', the total gain $g_1(\Phi) + g_2(\Phi)$ as a function of the photon number has a maximum. The limiting curve for the existence of the hard regime, POP'', crosses the threshold curve SOS' at the point O, at which $j_1 = 5.01$ and $j_2 = 0.399$. If the point (j_1, j_2) moves along the threshold curve SOS', then the only stationary solution of (18) ahead of the point O is $\Phi = 0$. Beyond the point O, a hard regime of generation is established. Small deviations from the state $\Phi = 0$ increase, and the system goes over into the state Φ^0, where $dK/d\Phi < 0$. With increasing degree of inhomogeneity of the excitation, the slope of $dK/d\Phi|_{\Phi=\Phi'}$ increases, and consequently the stationary value of the generation amplitude Φ^0, into which the system goes over jumpwise from the state $\Phi = 0$, increases. If the point (j_1, j_2) lies below the threshold curve, then the state $\Phi = 0$ is stable against infinitesimally small deviations, since the initial gain is smaller than the loss. Since the absorption decreases more rapidly than the gain at the points (j_1, j_2) lying below the curve POP'', generation can be obtained by introducing into the resonator a definite number of photons from the outside.

The curve OS'' represents the simultaneous solution of the system (18) and of the equation

$$\left.\frac{dK}{d\Phi}\right|_{\Phi=\Phi^0} = 0.$$

It is impossible to obtain a generation regime for all the points (j_1, j_2) lying below the curve OS'', since for these points the maximum value of the total gain as a function of the number of photons is lower than the loss level. Inasmuch as the total gain on the curve OS'' is tangent to the straight line corresponding to the level loss, the distance between the curves OS' and OS'', at a fixed value of the current j_1, determines the width of the hysteresis of the radiation power of the semiconductor laser with non-uniform excitation. An estimate of the width of the hysteresis, obtained from Fig. 1, coincides with the results of [13] where power hysteresis was experimentally observed.

Thus, for currents lying above the curve SOS″, there exists for the system (18) a stationary solution n_1^0, n_2^0, Φ^0 for which $dK/d\Phi|_{\Phi=\Phi'} < 0$.

If condition (6) is satisfied, then the stationary generation regime turns out to be unstable, and pulsations of the radiation intensity appear, with pulse parameters determined by (15), (16), and (17). Figure 1 shows the regions of instability of the stationary generation regime for different values of the parameter ϵ. The limit line OO′ corresponds to the value of the parameter $\epsilon = 0$. All the points (j_1, j_2) lying below OO′ are potentially unstable, since it is always possible to find such a value of ϵ^* for which the stationary state is stable if $\epsilon > \epsilon^*$ and unstable if $\epsilon < \epsilon^*$. The line O_1O_1' corresponds to the value of the parameter $\epsilon = 4 \times 10^{-3}$, and the line O_2O_2' to the value $\epsilon = 10^{-2}$. If, for example, $\epsilon = 4 \times 10^{-3}$ and the point (j_1, j_2) moves along the threshold curve, then the hard generation regime is stable ahead of the intersection of SOS′ with O_1O_1', and a constant generation amplitude is established after the transient process; beyond the intersection point, the stationary regime is unstable and the radiation intensity is modulated in amplitude. If, for example, $\epsilon = 10^{-2}$ and the point (j_1, j_2) is near the threshold curve, then the produced hard generation regime is stable.

The expressions for the light-pulse parameters contain the quantity $\Omega_0 = \omega_0(\tau_r\tau)^{-1/2}$, which is the frequency of pulsations with small depth of modulation. Figure 2 shows a plot of ω_0 against the injection current j_1 for different fixed values of the current j_2 in the absorbing part of the diode. The range of variation of ω_0 is bounded from above, for if a straight line j_2 = const as drawn on Fig. 1, then it intersects the instability boundary of the stationary state and falls into the region of stable generation. The dashed curves in Fig. 2 determine the upper limit of ω_0 for the values of the parameter $\epsilon = 3 \times 10^{-3}$ and 5×10^{-3}, respectively. With decreasing resonator Q, the range of variation of the pulsation frequency increases.

The light-pulse parameters can be determined by calculation with the aid of relations (15)–(17); for example, for $j_2 = 0.362$ and $j_1 = 7.5$, the calculation of (15) using the stationary values of n_1^0, n_2^0, and Φ^0 yields $\epsilon C_1 = 0.02$, i.e., the condition for the applicability of the approximation (10) is satisfied. When $\epsilon < 10^{-2}$, condition (6a) differs little from condition (6), and therefore (15) is a good approximation for the determination of C_1. The radiation-pulse duration at half height is $T_0 = 2 \times 10^{-10}$ sec, the repetition period is $T = 3 \times 10^{-9}$ sec at $\tau = 10^{-9}$ sec and $\tau_r = 2 \times 10^{-12}$ sec. With increasing initial excess over threshold, the repetition period decreases.

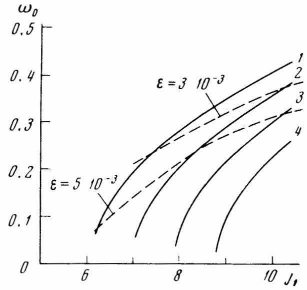

FIG. 2. Dependence of the frequency ω_0 on the current density in the amplifying part of the diode for different values of the current densities in the absorbing parts of the diode. Curves: 1 – $j_2 = 0.374$; 2 – $j_2 = 0.354$; 3 – $j_2 = 0.334$; 4 – $j_2 = 0.314$.

4. DISCUSSION OF EXPERIMENTAL RESULTS

The developed theory is capable of explaining the variety of dynamic regimes of double-diode operation. As seen from Fig. 1, in a wide range of injection currents, the double diode operates in the stationary regime with "soft" onset of generation. If the excitation is highly nonuniform, the generation amplitude is established jumpwise. The hard regime of a double diode occurs as $j_0 = 5j_0$ and $j_2 = 0.4j_0$, where j_0 is the density of the threshold current in the case of uniform excitation.

Figure 3 shows an oscillogram of such a process, photographed from the screen of an electron-optical converter. The transient process is followed by establishment of a stable stationary generation amplitude. This regime corresponds to the section of the threshold curve SOS′ (Fig. 1) prior to its crossing the curve bounding the instability region (for example, O_1O_1'). At a different ratio of the injection currents, a "hard" onset of generation is observed, accompanied by undamped periodic pulsations of the radiation intensity (Fig. 4). Stable periodic pulsations occur almost at the very threshold of generation. The "hard" onset of periodic pulsations corresponds to the points j_1 and j_2 near the threshold curve in the region of instability of the hard stationary generation regime, for example the points near the threshold curve SOS′, lying below the curve O_1O_1'.

If the initial ratio of the injection currents is such that the stationary regime of generation is stable, then the point (j_1, j_2) can fall into the instability region when the injection currents are varied. Such an excitation process corresponds to a "soft" onset of the spikes. Figure 5 shows oscillograms illustrating the soft regime of onset of pulsations. The oscillogram of Fig. 5a corresponds to stable stationary generation regime,

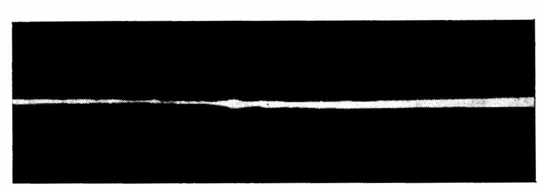

FIG. 3. Transient process of establishment of stable generation in the hard regime of self excitation. Sweep duration 10 nsec.

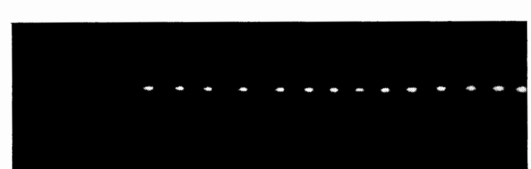

FIG. 4. "Hard" onset of periodic pulsations of radiation intensity. Sweep duration 10 nsec.

FIG. 5. "Soft" onset of pulsations: a – stable regime of generation, b – oscillogram shown "soft" onset of pulsations from the stationary regime (Fig. 5a) when the injection currents change by 10%.

while that of Fig. 5b to the regime of periodic pulsations that appear when the injection currents are changed.

Further refinement of the example considered in Sec. 3, namely a more numerous allowance for the dependences of the rate of induced recombination of the temperature and of the generation frequency on the injection currents, as well as allowance for the bleaching of the absorbing part of the diode by the spontaneous radiation from its amplifying part, will make possible a quantitative comparison of theory with experiment in a wide range variation of the temperature and of the degree of inhomogeneity of the excitation of the double diode.

5. CONCLUSION

Periodic generation of light pulses is possible only in the case of strong inhomogeneity of excitation, when the injection current in one part of the diode is much smaller than the injection current in its other part. In this case, in the amplifying part, all the levels participate in the generation are almost fully populated, and the gain varies slowly when the number of electrons is changed. In the other part of the diode, the states participating in the absorption of the light are little populated, and therefore the rate of change of the absorption with increasing number of electrons greatly exceeds the rate of change of the amplification, i.e., $\partial g_2/\partial n_2 \gg \partial g_1/\partial n_1$. When generation sets in, the absorption decreases as the result of optical pumping more rapidly than the decrease of the gain, i.e., the absorbing part is a readily saturable absorber, and the mechanism of light-pulse generation is analogous in many respects to the case of solid-state lasers with saturable filters.

The advantages of semiconductor lasers, namely their small linear dimensions, low inertia, and high efficiency of conversion of the electric-current energy into coherent radiation, makes it possible to use these lasers for optical information processing. In [14] it was proposed to use semiconductor lasers for ultrahigh-speed logic elements. Different regimes of the semiconductor injection laser with non-uniform excitation make it possible to use this laser either as a timing-frequency generator with a light-pulse repetition frequency from 10^8 to 10^{10} Hz, or else as a memory element with operating speed shorter than 10^{-10} sec, and small changes of the current in the absorbing part make it possible to change over from one regime to another.

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