POLARIZATION EFFECTS IN COMPTON SCATTERING BY RELATIVISTIC ELECTRONS

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The polarization properties of γ rays produced in Compton scattering of laser photons by relativistic electrons are discussed. Expressions are also obtained for the polarization parameters of photons emitted in scattering of an intense wave by electrons.

1. In recent years there has been considerable interest in obtaining energetic γ rays by scattering of laser light by relativistic electrons.[1-5] The first experimental results have been obtained in this area (see for example refs. 6 and 7), and similar experiments are being prepared in a number of electron accelerators. Arutyunyan et al.[2] have discussed the polarization features of these γ rays. The same authors have converted the expressions for the Compton effect cross sections with polarized photons from the rest system (RS) of the initial electron to a system in which a head-on collision (HC) occurs. Here it turns out that the Stokes parameters of the initial and final photons in the unit vectors attached to the scattering plane remain unchanged in the transition from RS to HC. However, the expression for the circular polarization of the final photon, which is used in refs. 1 and 2, and which was taken from Akhiezer and Berestetskiĭ,[8] is in error. The correct expression can be obtained from the results of Lipps and Tolhoek[9] (see also Berestetskiĭ et al.[10]) and has the form

$$\xi_2^{(2)} = F^{-1}\left(\frac{\varkappa_1}{\varkappa_2}+\frac{\varkappa_2}{\varkappa_1}\right)\cos\vartheta'\xi_2^{(1)}, \tag{1}$$

here

$$F = \frac{\varkappa_1}{\varkappa_2}+\frac{\varkappa_2}{\varkappa_1}+(\xi_3^{(1)}-1)\sin^2\vartheta'.$$

$$\cos\vartheta' = 1-\frac{\varkappa_1-\varkappa_2}{\varkappa_1\varkappa_2}m^2$$

(ϑ' is the photon scattering angle in the RS), $k_1(\omega_1, \mathbf{k}_1)$, $k_2(\omega_2, \mathbf{k}_2)$ are the 4-momenta of the initial and final photons, $\kappa_1 = (k_1p)$, $\kappa_2 = (k_2p)$, $p(E, \mathbf{p})$ is the 4-momentum of the initial electron.

From Eq. (1) it follows that in scattering by an angle $0(\pi)$ the degree of circular polarization is preserved (changes sign). This fact is a consequence of the conservation of the projection of angular momentum on the direction of motion for a head-on collision.

Interest is presented also by those photons which in HC are moving at small angles ($\vartheta \ll 1$) to the direction of motion of the initial electrons (see for example ref. 1). This is true for the entire range of frequencies ω_2, except for $\omega_2 \sim \omega_1$. Under these conditions the energy of the final photon is

$$\omega_2 = 2\Lambda E/(1+(\vartheta\gamma)^2+2\Lambda), \quad \Lambda = 2\omega_1 E/m^2, \tag{2}$$

and the quantities which determine the Stokes parameters of the final photon are

$$\frac{\varkappa_1}{\varkappa_2} = \frac{1+(\vartheta\gamma)^2+2\Lambda}{1+(\vartheta\gamma)^2}, \quad \cos\vartheta' = \frac{(\vartheta\gamma)^2-1}{(\vartheta\gamma)^2+1}. \tag{3}$$

The expressions for the polarization parameters of final photons, scattered at definite azimuthal angles φ and fixed unit vectors,[1,2] have the form

$$\begin{gathered}
\xi_1^{(2)} = (F')^{-1}\{{}^1/{}_2\xi_3^{(1)}\sin 4\varphi(1+\cos\vartheta')^2 \\
-\xi_1^{(1)}[\sin^2 2\varphi(1+\cos\vartheta')^2-2\cos\vartheta']+\sin 2\varphi\sin^2\vartheta'\}, \\
\xi_2^{(2)} = (F')^{-1}\left(\frac{\varkappa_2}{\varkappa_1}+\frac{\varkappa_1}{\varkappa_2}\right)\cos\vartheta'\xi_2^{(1)}, \\
\xi_3^{(2)} = (F')^{-1}\{\xi_3^{(1)}[\cos^2 2\varphi(1+\cos\vartheta')^2-2\cos\vartheta'] \\
-{}^1/{}_2\,\xi_1^{(1)}\sin 4\varphi(1+\cos\vartheta')^2+\cos 2\varphi\sin^2\vartheta'\}, \\
F' = \frac{\varkappa_1}{\varkappa_2}+\frac{\varkappa_2}{\varkappa_1}+(\xi_3^{(1)}\cos 2\varphi-\xi_1^{(1)}\sin 2\varphi-1)\sin^2\vartheta'.
\end{gathered} \tag{4}$$

These equations (4) are valid for all photons scattered in HC at small angles ($\vartheta \ll 1$) to the direction of motion of the initial electrons.[1)]

After averaging $\xi_i^{(2)}$ over the angle φ with a weight proportional to the scattering cross section, the degree of linear polarization is multiplied by an attenuation factor $P_t = \bar{\xi}_3^{(2)}/\xi_3^{(1)}$, and the degree of circular polarization by a factor $P_c = \bar{\xi}_2^{(2)}/\xi_2^{(1)}$. The expressions for P_t and P_c have the following form:

$$P_t = (1-\cos\vartheta')^2/2\left(\frac{\varkappa_1}{\varkappa_2}+\frac{\varkappa_2}{\varkappa_1}-\sin^2\vartheta'\right), \tag{5}$$

$$P_c = \left(\frac{\varkappa_1}{\varkappa_2}+\frac{\varkappa_2}{\varkappa_1}\right)\cos\vartheta' \Big/ \left(\frac{\varkappa_1}{\varkappa_2}+\frac{\varkappa_2}{\varkappa_1}-\sin^2\vartheta'\right).$$

Figures 1 and 2 illustrate the behavior of P_c and P_t for electron energies of 6 and 40 BeV and photon frequencies $\omega_1 = 1.78$ eV (ruby laser) and $\omega_1 = 3.56$ eV.

The results presented ((1), (4), and (5) and Figs. 1 and 2) correct the corresponding formulas and figures of refs. 1 and 2.

From Eqs. (5) it follows that for $\omega_2 \sim \omega_{2\max}$ it is possible to obtain beams of energetic γ rays with a high degree of polarization. We note that the value of the maximum achievable degree of linear polarization of the γ rays falls with increasing energy of the initial electrons and photons, and that the maximum achievable value of circular polarization does not depend on the energy (and is equal to -1).

2. For very high photon beam intensity it is neces-

[1)]Note that introduction of the reversed sign in $\cos\vartheta'$ in the similar formulas for $\xi_1^{(2)}$ and $\xi_3^{(2)}$ given in ref. 1 has meaning only for photons with $\omega_2 \sim \omega_1$, and the rule for using the second sign in ref. 1 is in error.

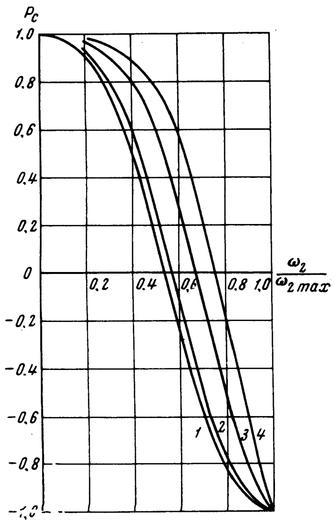

FIG. 1

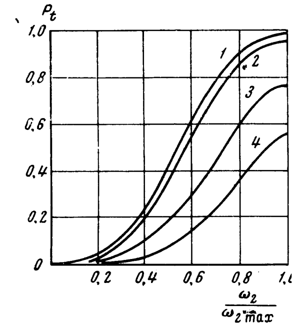

FIG. 2

FIG. 1. Values of P_c for scattered photons as a function of their energy. Curves 1 – E = 6 BeV, ω_1 = 1.78 eV; 2 – E = 6 BeV, ω_1 = 3.56 eV; 3 – E = 40 BeV, ω_1 = 1.78 eV; 4 – E = 40 BeV, ω_1 = 3.56 eV.

FIG. 2. Values of P_t for scattered photons as a function of their energy. Curves 1 – E = 6 BeV, ω_1 = 1.78 eV; 2 – E = 6 BeV, ω_1 = 3.56 eV; 3 – E = 40 BeV, ω_1 = 1.78 eV; 4 – E = 40 BeV, ω_1 = 3.56 eV.

sary to keep in mind the possibility of simultaneous absorption of several (n) photons with subsequent emission of one more energetic photon.[11] In the case of circular polarization of the primary photons, the Stokes parameters of the final photon in the unit vectors attached to the scattering plane[11] have the form[2)]

$$\xi_1 = 0,$$
$$\xi_2 = 2J_nJ_n'\left(\frac{\rho}{\xi}-\frac{n}{s}\right)\left(\frac{\lambda}{\lambda'}+\frac{\lambda'}{\lambda}\right)G^{-1}, \qquad (6)$$
$$\xi_3 = 2\left[J_n'^2-\left(\frac{\rho}{\xi}-\frac{n}{s}\right)^2J_n^2\right]G^{-1},$$
$$G=\left(\frac{\lambda}{\lambda'}+\frac{\lambda'}{\lambda}\right)\left[J_n'^2+J_n^2\left(\frac{n^2}{s^2}-1\right)\right]-2\xi^{-2}J_n^2.$$

We note that, as before, for $\vartheta = 0(\pi)$ for any values of ξ, the parameter $\xi_2 = 1(-1)$. After averaging over the azimuthal angle in the case being discussed, only the circular polarization of the final photon remains.

For existing lasers $\xi \ll 1$ and in this case it is sufficient to take the main terms in the Bessel function expansion. The result obtained in this case agrees with the usual expression for P_c, Eq. (5), in which the frequency of the initial photons ω_1 is replaced by $n\omega_1$. The cross section for this process is smaller by roughly a factor $\xi^{-2(n-1)}$ than the cross section for the main single-photon process.[11] However, the frequency of the final photon in this case is higher than for the case of the single-photon process:

$$\frac{\omega_2^{(n)}}{\omega_2^{(1)}} = 1+\frac{(n-1)(1+(\vartheta\gamma)^2)}{1+(\vartheta\gamma)^2+2n\Lambda}. \qquad (7)$$

[2)]Here we have used the designations of ref. 11.

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