CONTRIBUTION TO THE THEORY OF SATURATION OF EXCHANGE NARROWED MAGNETIC RESONANCE LINES

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A theory of magnetic-resonance saturation in a solid body is developed for the case when exchange interaction exists between the spins. The analysis is based on the three-reservoir model in which an exchange reservoir is introduced along with the lattice and the Zeeman system. The calculations are based on the method of setting up the non-equilibrium density matrix. Expressions are obtained for the imaginary part of the complex magnetic susceptibility and for the line width. The results obtained can be applied to NMR in solid He^3 and to EPR in a nonmetallic crystal containing magnetic atoms.

1. The theory of saturation of magnetic resonance in a solid with allowance for the dipole-dipole (d-d) interaction between the spins was given in the papers of Redfield[1] and Provotorov[2]. It is assumed in these papers that only d-d interaction occurs between the spins.

Ol'khov and Provotorov[3] considered the saturation of resonance in a paramagnet with exchange interaction greatly exceeding the Zeeman energy (which is realized above the Curie point in the case of a ferromagnet). Equations of the Bloch type with equal times of longitudinal and transverse relaxation are obtained for the magnetization. This is connected with the fact that the system is actually isotropic, in view of the smallness of the Zeeman energy.

There are encountered, however, important cases (NMR in solid He^3, EPR in a nonmetallic crystal having a large concentration of magnetic atoms) in which the exchange interaction greatly exceeds the d-d interaction, but is smaller than or of the order of the Zeeman energy.

Many experiments were performed on the solid He^3 for the purpose of determining the influence of the exchange interaction on the longitudinal and transverse relaxation of the nuclei (see, for example,[4]). To explain the experimental data, one uses the so-called three-reservoir model, in which an exchange reservoir connected with the exchange degrees of freedom is introduced besides the lattice and the nuclear Zeeman system.

Studies were also made of EPR in nonmetallic crystals with large magnetic-ion concentration. It is well known that the exchange interaction (between identical spins) leads to a narrowing of the resonance line[5].

We note the following limitation. For solid He^3, the exchange reservoir can be introduced only when the condition[4] $J\tau_c > 1$ (J—exchange constant for the nearest neighbors, τ_c—correlation time for atomic diffusion) is satisfied. A similar limitation holds also in the case of a crystal with a large concentration of magnetic systems.

The present paper is aimed at an investigation of the influence of exchange interaction on the saturation of magnetic resonance. We consider the case when the exchange interaction is smaller than or of the order of the Zeeman energy. For concreteness, we shall speak of the case of a system of nuclear spins.

2. We start from the premise that the presence of a strong exchange interaction in the system gives rise to a time scale τ which is much smaller than the other characteristic times of the system (the time connected with the d-d interaction, or the time which is reciprocal to the transition probability due to the alternating field). This circumstance allows us to perform the analysis on the basis of the method of constructing a non-equilibrium density matrix, developed by Zubarev[6].

Since this method is valid for times larger than τ, we can obtain equations of motion for the statistical mean values of the operators that commute with the interaction responsible for the small time scale. In our problem, this condition is satisfied, generally speaking, by the total spin operator $I^\alpha = \sum_i I_i^\alpha$ (α = x, y, z, and I_i is the α-th projection of the spin operator of the nucleus located in the i-th site of the lattice) and by the exchange-interaction operator.

The total Hamiltonian of the system is written in the form

$$\mathcal{H} = \mathcal{H}_z + \mathcal{H}_E + \mathcal{H}_d + \mathcal{H}_{zf}, \tag{1}$$

where $\mathcal{H}_Z$ is the Zeeman energy of the nuclei, $\mathcal{H}_E$ is the exchange Hamiltonian, $\mathcal{H}_d$ is the Hamiltonian of the d-d interaction, and $\mathcal{H}_{Zf}$ is the energy of the interaction of the nuclear spins with the alternating field. We have left out the interactions of the lattice with the nuclear spins and with the exchange degrees of freedom, since these will be taken into account phenomenologically.

We introduce the notation $\omega_0 = \gamma H$ and $\omega_1 = \gamma H_1$ (γ—gyromagnetic ratio, H—main field, directed along the z axis, $2H_1$—amplitude of the alternating field with frequency ω, applied perpendicular to H). We introduce further the average quanta of the exchange and d-d interactions, respectively, by means of the following formulas (we assume $\hbar = 1$):

$$\omega_E^2 = \frac{\mathrm{Sp}\,\mathcal{H}_E^2}{\mathrm{Sp}\,(I^z)^2}, \quad \omega_d^2 = \frac{\mathrm{Sp}\,\mathcal{H}_d^2}{\mathrm{Sp}\,(I^z)^2} \tag{2}$$

(in the formula for ω_d we take $\mathcal{H}_d$ to mean the secular part of the d-d interaction). We choose $\mathcal{H}_E$ in the form

$$\mathcal{H}_E = \frac{1}{2}\sum_{ij}{}' J_{ij}\mathbf{I}_i\mathbf{I}_j, \tag{3}$$

where J_{ij} is the constant of exchange interaction between the nuclei located in the i-th and j-th lattice sites. Simple calculation yields

$$\omega_E^2 = \frac{1}{2}I(I+1)\sum_j{}' J_{ij}^2. \tag{4}$$

We further have $\omega_d \sim \gamma H_d$, where H_d is the local field due to the d-d interaction. (An exact expression for ω_d can be found, for example, in[7].)

Since the operator $\mathcal{H}_{zf}$ commutes with $\mathcal{H}_E$, the picture of resonance saturation depends strongly on the ratio ω_1/ω_d. The saturation picture depends strongly also on the ratio ω_0/ω_E. If this ratio is sufficiently large, we can neglect the cross relaxation transitions due to the nonsecular terms of the d-d interaction.

We confine ourselves throughout to the high-temperature approximation for the Zeeman and exchange energies. The specific heat of the lattice is assumed to be sufficiently large, and the change of the lattice temperature is neglected.

3. We consider first the case $\omega_1 \ll \omega_d$. Let, in addition, ω_0/ω_E be so large that the nonsecular part of $\mathcal{H}_d$ can be neglected. We change over to a rotating system of coordinates, with the x axis along the rotating field H_1. The secular part of the d-d interaction is combined with the exchange interaction, and the corresponding reservoir is called the exchange reservoir (E). We introduce further, as separate subsystems, the lattice reservoir (L) and the nuclear Zeeman reservoir (Z). The energy of interaction of the spins with the alternating field does not commute with the energies of the Zeeman and exchange reservoirs, so that the alternating field tends to bring these two systems into equilibrium with each other.

We denote by β_L, β_Z, and β_E the reciprocal temperatures (in energy units) of the reservoirs L, Z, and E, respectively. Without first considering the lattice, we compile for our system the nonequilibrium density matrix, and we find $d\beta_Z/dt$ and $d\beta_E/dt$ (in analogy with the procedure used in[8,9]). Then, following[2,10], we add phenomenologically the terms due to the interaction with the lattice, and obtain

$$\frac{d\beta_z}{dt} = -2W(\omega-\omega_0)(\beta_z-\beta_E) - \frac{1}{T_{zL}}\left(\beta_z - \frac{\omega_0-\omega}{\omega_0}\beta_L\right), \tag{5}$$
$$\frac{d\beta_E}{dt} = 2\alpha W(\omega-\omega_0)(\beta_z-\beta_E) - \frac{\beta_E-\beta_L}{T_{EL}},$$

where T_{zL} and T_{EL} are the relaxation times of the Z and E parts with the lattice, $\alpha = (\omega-\omega_0)^2/\omega_E^2$, $W(\omega-\omega_0)$ in the probability of the transition (per unit time) due to the alternating field:

$$W(\omega-\omega_0) = \frac{\pi}{2}\omega_1^2\varphi(\omega-\omega_0), \quad \varphi(\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\langle I^+I^-(t)\rangle}{\langle|I^+|^2\rangle}e^{i\omega t}\,dt, \tag{6}$$

$\varphi(\omega-\omega_0)$ is the normalized line-shape function, $I^\pm = I^X \pm iI^Y$, and $\langle A\rangle = \mathrm{Tr}\,A/\mathrm{Tr}\,1$.

It is easy to obtain the stationary solution of the system (5)[1]:

$$\beta_z = \beta_L\frac{\omega_0}{\omega_0-\omega}\frac{2\alpha WT_{EL}+1}{2WT_{zL}+2\alpha WT_{EL}+1},$$
$$\beta_E = \beta_L\frac{\omega_0}{\omega_0-\omega}\frac{2\alpha WT_{EL}}{2WT_{zL}+2\alpha WT_{EL}+1} \tag{7}$$

For the imaginary part of the complex susceptibility we can use the expression

$$\chi''(\omega) = \frac{\pi}{2}\chi_0(\omega_0-\omega)\varphi(\omega-\omega_0)\frac{\beta_z-\beta_E}{\beta_L} \tag{8}$$

(χ_0—statistical susceptibility). We obtain

$$\chi''(\omega) = \frac{\pi}{2}\chi_0\omega_0\varphi(\omega-\omega_0)\frac{k\omega_E^2}{2WT_{zL}[(\omega-\omega_0)^2+k\omega_E^2]+k\omega_E^2}, \tag{9}$$

where $k = T_{zL}/T_{EL}$.

In the case of strong exchange narrowing[7]

$$W(\omega-\omega_0) = W_0\frac{\Gamma^2}{(\omega-\omega_0)^2+\Gamma^2} \qquad W_0 = \frac{\omega_1^2}{2\Gamma} \qquad \Gamma = \frac{\omega_d^2}{\omega_E} \tag{10}$$

Since $\omega_d \ll \omega_E$, we find that $\Gamma \ll \omega_d$. Further, we can assume that $k\omega_E^2 \gg \Gamma^2$.

Taking (10) into account, we can transform (9) into

$$\chi''(\omega) = \frac{\pi}{2}\chi_0\omega_0\left[\frac{k\omega_E^2}{\Gamma^2(2W_0T_{zL}+1)(2W_0T_{zL}+k\omega_E^2/\Gamma^2)}\right]^{1/2}\varphi^*(\omega-\omega_0), \tag{11}$$

where

$$\varphi^*(\omega-\omega_0) = \frac{1}{\pi}\frac{\Gamma^*}{(\omega-\omega_0)^2+\Gamma^{*2}}. \tag{12}$$

The saturation line width Γ^* is given by the formula

$$\Gamma^{*2} = k\omega_E^2\frac{2W_0T_{zL}+1}{2W_0T_{zL}+k\omega_E^2/\Gamma^2}. \tag{13}$$

According to (13), $\Gamma \le \Gamma^* \le (k\omega_E^2)^{1/2}$. Let us consider particular cases.

In the absence of saturation, i.e., when $2W_0T_{zL} \ll 1$, $\Gamma^* = \Gamma$, $\chi''(\omega) = \frac{1}{2}\pi\chi_0\omega_0\varphi(\omega-\omega_0)$.

When the condition $1 \ll 2W_0T_{zL} \ll k\omega_E^2/\Gamma^2$ is satisfied, we get

$$\Gamma^{*2} = 2W_0T_{zL}\Gamma^2 = \omega_1^2T_{zL}\Gamma, \quad \chi''(\omega) = \frac{\pi}{2}\chi_0\omega_0\varphi^*(\omega-\omega_0)\frac{1}{(2W_0T_{zL})^{1/2}}. \tag{14}$$

On the other hand, if $2W_0T_{zL} \gg k\omega_E^2/\Gamma^2$ (i.e., $\omega_1^2 \gg \omega_E^2/\Gamma T_{EL}$)[2], then

$$\Gamma^{*2} = k\omega_E^2, \quad \chi''(\omega) = \frac{\pi}{2}\chi_0\omega_0\varphi^*(\omega-\omega_0)\left(\frac{k\omega_E^2}{\Gamma^2}\right)^{1/2}\frac{1}{2W_0T_{zL}}. \tag{15}$$

Thus, the dependence of the line width Γ^* on H_1 should have the following form: when H_1 increases, the line width Γ^* first remains constant, then increases, and ultimately reaches saturation[3].

4. We shall assume that ω_0/ω_E is sufficiently large, but consider now the case $\omega_1 \gg \omega_d$. We change over again to a rotating system of coordinates with the x axis along the rotating field H_1. (The operators of the spin components in this coordinate system will be designated by I^X, I^Y, and I^Z). As the subsystems we consider the lattice (L), the exchange reservoir (E), and the nuclear Zeeman system in the summary effec-

[1] In [11], the expression for β_E was used to explain certain features of dipole polarization of nuclei.

[2] Consideration of this particular case is valid if the condition $\Gamma T_{EL} \gg (\omega_E/\omega_d)^2$ is satisfied.

[3] When $2W_0T_{zL} = k\omega_E^2/\Gamma^2$, the quantity χ'' becomes smaller than its unsaturated value by a factor $(\Gamma^*/\Gamma^2)(2k\omega_E^2)^{1/2}$. It is difficult to observe the region of width saturation at large values of this quantity.

tive field (Z). The secular part of $\mathcal{H}_d$ will now play the role of the perturbation energy. Discarding again the terms connected with the lattice, we obtain

$$\mathcal{H} = (\omega_0 - \omega) I^z + \omega_1 I^x + \mathcal{H}_E + \mathcal{H}_d. \quad (16)$$

We proceed to an effective coordinate system with z axis along the effective field[7] (we denote the spin components in this system by I_x, I_y, I_z). The transition is effected by means of the transformation

$$I_z = I^z \cos\vartheta - I^x \sin\vartheta, \quad I_x = I^z \sin\vartheta + I^x \cos\vartheta, \quad I_y = I^y, \quad (17)$$

where

$$\cos\vartheta = \frac{\omega_0 - \omega}{\Omega}, \quad \sin\vartheta = -\frac{\omega_1}{\Omega}, \quad \Omega^2 = (\omega_0 - \omega)^2 + \omega_1^2 \quad (18)$$

the Hamiltonian takes on the form

$$\bar{\mathcal{H}} = \Omega I_z + \mathcal{H}_E + \mathcal{H}_d. \quad (19)$$

As already noted, the small time scale τ is due to the exchange interaction. The spin-lattice and d-d interactions give rise to a slow variation of the thermodynamic quantities. Since the operators $I_\pm$ do not commute with the Zeeman energy, the mean values of these operators change within a time of the order of $1/\omega$, which can turn out to be smaller than τ. We shall therefore write out the generalized integrals not for the operators $I_\pm(t)$ in the Heisenberg representation, but for the following operators

$$I_\pm^*(i) = e^{i\mathcal{H}t} e^{-i\mathcal{H}_z t} I_\pm e^{i\mathcal{H}_z t} e^{-i\mathcal{H}t} \quad (20)$$

($\mathcal{H}_Z = \Omega I_Z$), for which the rapid change due to the main field is excluded.

Using Zubarev's method[6], we write down the equation of motion

$$\frac{dI_\pm^*(t)}{dt} = K_\pm(t), \quad \frac{d\mathcal{H}_z(t)}{dt} = K_z(t), \quad \frac{d\mathcal{H}_E(t)}{dt} = K_E(t) \approx -K_z(t), \quad (21)$$

where

$$K_\pm(t) = -ie^{\mp i\Omega t} e^{i\mathcal{H}t} [I_\pm, \mathcal{H}_d] e^{-i\mathcal{H}t}, \quad K_z(t) = -ie^{i\mathcal{H}t} [\mathcal{H}_z, \mathcal{H}_d] e^{-i\mathcal{H}t}. \quad (22)$$

In the high-temperature approximation, the density matrix takes the form ($\epsilon \to +0$):

$$\rho = (\mathrm{Sp}\,1)^{-1} \Big\{ 1 - h_- I_+ - h_+ I_- - \beta_z \mathcal{H}_z - \beta_E \mathcal{H}_E + \int_{-\infty}^{0} e^{\varepsilon t}\, dt\, [h_- K_+(t) + h_+ K_-(t) + (\beta_z - \beta_E) K_z(t)] \Big\}. \quad (23)$$

The quantities $h_\pm$, β_Z, and β_E can be expressed in terms of the mean values of the operators $I_\pm^*$, $\mathcal{H}_Z$, and $\mathcal{H}_E$. It is easy to find that ($\overline{A}$ denotes the mean value of the operator A)

$$\overline{I_\pm^*} = -h_\pm \langle |I_+|^2 \rangle, \quad \overline{\mathcal{H}_z} = -\beta_z \langle \mathcal{H}_z^2 \rangle, \quad \overline{\mathcal{H}_E} = -\beta_E \langle \mathcal{H}_E^2 \rangle.$$

It is also easy to calculate the mean values of the operators $K_\pm$, K_Z, and K_E. We average further equations (21) and express the time derivatives of $h_\pm$, β_Z, and β_E in terms of $h_\pm$, β_Z, and β_E. Further, we take into account phenomenologically the interaction between the exchange reservoir and the lattice. On the other hand, the direct relaxation of the nuclear Zeeman system with the lattice in a rotating system of coordinates is neglected (see below). We thus obtain

$$\frac{dh_\pm}{dt} = -\frac{h_\pm}{T_1} + \frac{h_\mp}{T_2} + \frac{\Omega(\beta_E - \beta_z)}{T_z},$$

$$\frac{d\beta_z}{dt} = -\frac{\beta_z - \beta_E}{T_{zE}} + \frac{2(h_+ + h_-)}{\Omega T_z}, \quad (24)$$

$$\frac{d\beta_E}{dt} = -\frac{\Omega^2}{\omega_E^2} \frac{d\beta_z}{dt} - \frac{\beta_E - \beta_L}{T_{EL}}.$$

The kinetic equations are given by the following formulas (concrete expressions are given for the case $\Omega \ll \omega_E$)

$$\frac{1}{T_1} = \frac{1}{\langle |I_+|^2 \rangle} \int_{-\infty}^{0} e^{\varepsilon t} \langle K_\pm K_\mp(t) \rangle\, dt \approx \frac{1}{2T_x}(1 + \cos^2\vartheta), \quad (25a)$$

$$\frac{1}{T_2} = -\frac{1}{\langle |I_+|^2 \rangle} \int_{-\infty}^{0} e^{\varepsilon t}\, dt\, \langle K_\pm K_\pm(t) \rangle \approx \frac{1}{2T_x} \sin^2\vartheta, \quad (25b)$$

$$\frac{1}{T_z} = -\frac{1}{\langle |I_+|^2 \rangle} \frac{1}{\Omega} \int_{-\infty}^{0} e^{\varepsilon t}\, dt \langle K_\pm K_z(t) \rangle \approx \frac{1}{2T_x} \sin\vartheta \cos\vartheta, \quad (25c)$$

$$\frac{1}{T_{zE}} = \frac{1}{\langle \mathcal{H}_z^2 \rangle} \int_{-\infty}^{0} e^{\varepsilon t}\, dt\, \langle K_z K_z(t) \rangle = \frac{1}{2T_x} \sin^2\vartheta, \quad (25d)$$

$$\frac{1}{T_x} = -\frac{1}{\langle I_x^2 \rangle} \int_{-\infty}^{0} e^{\varepsilon t}\, dt\, \langle [I_x, \mathcal{H}_d]\, e^{i\mathcal{H}_E t} [I_x, \mathcal{H}_d]\, e^{-i\mathcal{H}_E t} \rangle. \quad (26)$$

(expression (26) was obtained and investigated in[12,13]).

Let us examine the stationary solution of (24). We obtain $\overline{I_X} = 0$, $\beta_Z = \beta_E = \beta_L$, and $\overline{I^X} = I_Z \sin\vartheta$. We further have

$$\chi' = \frac{N\gamma \overline{I^x}}{2H_1} = \frac{1}{2} N\gamma \frac{\overline{I_z} \sin\vartheta}{H_1}$$

(N is the spin concentration). Calculation yields $\overline{I_Z} = -\Omega \langle I_Z^2 \rangle \beta_L$. The quantity χ'' can be determined by using the energy balance[7]

$$2\omega\chi'' H_1^2 = \frac{M_0 - 2H_1\chi' \operatorname{ctg}\vartheta}{T^{zL}} H,$$

where M_0 is the equilibrium magnetization, and T^{ZL} is the relaxation time of I^Z. Finally we obtain

$$\chi' = -\frac{1}{6} N\gamma^2 I(I+1)\beta_L, \quad \chi'' = -\frac{N\gamma^2}{2T^{zL}\omega_1^2} \beta_L. \quad (27)$$

Thus, in the case when $\omega_1 \gg \omega_d$, we find that χ' does not depend on H_1, and χ'' is proportional to H_1^{-2}.

Neglect of the direct relaxation of the system Z with the lattice in the calculation of $\overline{I_Z}$ and inclusion of this relaxation in the calculation of χ'' do not contradict each other. We have carried out a calculation in which we added to the second equation of the system (24) a term describing the relaxation of the system Z with the lattice. The result reduces to (27) if the condition $\Omega^2 \ll k\omega_E^2$ is satisfied. This condition can always be regarded as satisfied if the volume interaction is sufficiently large.

5. Let us consider now a case when the ratio ω_0/ω_E is not so large as to be able to neglect the cross relaxation transitions due to the nonsecular terms of the d-d interaction[4]. If the nonsecular terms are retained in $\mathcal{H}_d$, the changeover to a rotating system of coordinates does not lead to a Hamiltonian that is independent of the time. If, however, $\omega_1 < \omega_d$, then the alternating field can be regarded as a quantum subsystem. Then, following[9], we can obtain a system of equations for β_Z and β_E in the laboratory frame (terms connected with the interaction with the lattice are again taken into account phenomenologically):

[4] On the other hand, we assume that ω/ω_E is not small enough not to be able to separate the d-d interaction into secular and non-secular parts. If $\omega_0 \ll \omega_E$, the analysis given in [3] is valid.

$$\frac{a\beta_z}{dt}=-2W(\omega-\omega_0)\Big(\beta_z-\frac{\omega_0-\omega}{\omega_0}\beta_E\Big)-\frac{\beta_z-\beta_E}{T_{zE}}-\frac{\beta_z-\beta_L}{T_{zL}},$$

$$\frac{d\beta_E}{dt}=2W(\omega-\omega_0)\frac{\omega_0(\omega_0-\omega)}{\omega_E^2}\Big(\beta_z-\frac{\omega_0-\omega}{\omega_0}\beta_E\Big)-\frac{\beta_E-\beta_z}{T_{Ez}}-\frac{\beta_E-\beta_L}{T_{EL}}, \quad (28)$$

$$\frac{1}{T_{zE}}=\frac{2\pi}{[\mathrm{Sp}(I^z)^2]^2}\sum_m m^2\,\mathrm{Sp}(\mathcal{H}_d^{-m}\mathcal{H}_d^m)f(m\omega_0), \qquad T_{Ez}=T_{zE}\Big(\frac{\omega_E}{\omega_0}\Big)^2, \quad (29)$$

$$f(m\omega)=\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\mathrm{Sp}\,\mathcal{H}_d^m\mathcal{H}_d^{-m}(t)}{\mathrm{Sp}\,\mathcal{H}_d^m\mathcal{H}_d^{-m}}e^{im\omega t}\,dt, \quad (30)$$

$\mathcal{H}_d^m$ is that part of the d-d interaction which changes the projection of the total spin by m. Expression (29) for T_{zE} was obtained earlier in[13,14]. In the limit when the cross relaxation transitions can be neglected, the system (28) reduces to a system of equations obtained by transforming the system (5) to the laboratory frame.

Let us consider the particular case when the conditions $T_{EL} \gg T_{Ev}$ and $T_{zL} \gg T_{zE}$ are satisfied. It is easy to obtain

$$\chi''(\omega)=\frac{{}^1/_2\pi\chi_0\omega_0\omega_E\varkappa\varphi^*(\omega-\omega_0)}{(2W_0T_{Ez}\Gamma_1^2+\omega_E^2\varkappa)^{1/2}(2W_0T_{zL}+\varkappa)^{1/2}}, \quad (31)$$

$$\varkappa=1+k\omega_E^2/\omega_0^2.$$

The function $\varphi^*(\omega-\omega_0)$ is again given by formula (12), but for the width we obtain

$$\Gamma^{*2}=\Gamma_1^2\,\omega_E^2\frac{2W_0T_{zL}+\varkappa}{2W_0T_{Ez}\Gamma_1^2+\omega_E^2\varkappa}, \qquad \Gamma_1=\frac{10}{3}\Gamma. \quad (32)$$

If the condition

$$2W_0T_{Ez}(\Gamma_1/\omega_E)^2\ll\varkappa\ll 2W_0T_{zL} \quad (33)$$

is satisfied then formulas (31) and (32) yield

$$\chi''(\omega)=\frac{\pi}{2}\chi_0\omega_0\Big(\frac{\varkappa}{2W_0T_{zL}}\Big)^{1/2}\varphi^*(\omega-\omega_0), \qquad \Gamma^{*2}=\Gamma_1^2\frac{2W_0T_{zL}}{\varkappa}. \quad (34)$$

The result obtained for Γ^* can be easily understood[5]. The condition (33) denotes that the probability of the transition due to the alternating field is smaller than the probability of the cross relaxation transitions (produced by the nonsecular part of $\mathcal{H}_d$), but is larger than the probability of the transitions due to the lattice. Therefore, during the process of absorption, the Z and E systems will be in equilibrium with each other, i.e., they will be characterized by a single temperature. In this connection, we can assume that there is a single system Z + E, for which we can introduce an effective time T_{eff} of relaxation with the lattice, and an effective probability W_{eff} of the transition due to the alternating field (it must be recognized here that the alternating field acts directly only on the system Z):

$$\frac{1}{T_{eff}}=\Big(\frac{\omega_0^2}{T_{zL}}+\frac{\omega_E^2}{T_{EL}}\Big)\frac{1}{\omega_0^2+\omega_E^2}=\frac{1}{T_{zL}}\frac{\omega_0^2}{\omega_0^2+\omega_E^2}\varkappa, \quad (35)$$

$$W_{eff}=W\frac{\omega_0^2}{\omega_0^2+\omega_E^2}. \quad (36)$$

Thus, we obtain $W_{eff}T_{eff}=WT_{zL}/\kappa$. We can then rewrite formula (34) for Γ^* in the form:

$$\Gamma^{*2}=2W_{0,\,eff}\,T_{eff}\,\Gamma_1^2, \quad (37)$$

which agrees with the formula for the line width in the case of saturation, obtained in accordance with the simple theory of[16].

On the other hand, if

$$2W_0T_{Ez}\gg(\omega_E/\Gamma_1)^2\varkappa,$$

i.e., the probability of the transition due to the alternating field exceeds the probability of the cross relaxation transitions, the resonance line will not be observed, owing to its excessive broadening.

6. Quite a few experiments have been devoted to an investigation of the relaxation of nuclei in solid He^3 after application of a saturating field (see, for example,[4,12]). So far, however, no experiments have been reported in which a picture of the stationary saturation of NMR in solid He^3 was investigated.

Many experiments were performed on crystals containing magnetic ions, for the purpose of determining the exchange effects. The effect of exchange narrowing of the EPR line was fully confirmed. These experiments, however, were performed under conditions of weak saturation of the resonance.

Thus, we are unable at present to compare the theoretical results obtained by us with the experimental data.

In the case of a crystal containing magnetic ions, the summation in formula (4) is not over all the lattice sites, but only over the sites j occupied by the magnetic ions. At relatively low concentration of the magnetic ions, and when the ions are randomly distributed, this sum reduces to a sum over all the lattice sites, multiplied by the relative concentration f of the magnetic ions. Thus, $w_E^2 \propto f$. Depending on the values of f and H_1, the situations considered in Secs. 3, 4, and 5 can be realized in the experiments.

In the case of solid He^3, greatest interest attaches to the case when the systems Z and E are strongly coupled with each other, i.e., the situation referred to in Sec. 5.

[5] A similar situation takes place when the Zeeman system and the d-d reservoir are considered with allowance for cross-relaxation transitions between them [15].

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