GENERATION OF LONGITUDINAL WAVES AT COMBINATION FREQUENCIES IN A BEAM-PLASMA SYSTEM

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It is shown that the second harmonic of the plasma frequency can be generated in a beam-plasma system; this effect is due to the nonlinear interaction between the harmonic plasma wave that is excited as a result of the two-stream instability and the higher frequency longitudinal waves that are in synchronism with it. The net effect is a limitation on the amplitude of the excited primary wave, that is to say, dynamic stabilization of the two-stream instability. Both two-wave and three-wave processes are considered for various relative densities of the beam and plasma. It is found that stability can be achieved in the corresponding equilibrium processes in a plasma if the collision frequency is high enough. The analogous problem in nonlinear optics is discussed. The analysis is carried out for a single-velocity beam and a cold, highly magnetized plasma in the hydrodynamic approximation. Estimates show that it should be possible to observe the effect under laboratory conditions.

IT is well known[1] that longitudinal waves with frequencies $\omega \lesssim \omega_0$ (ω_0 is the electron plasma frequency) can be excited in a beam-plasma system by the two-stream instability. In the linear approximation, waves at frequencies much higher than ω_0 are damped (when collisions are taken into account). It is the purpose of the present work to show that nonlinear processes in a beam-plasma system can lead to the generation of longitudinal waves at frequencies much higher than ω_0, including waves at a frequency $\omega \sim 2\omega_0$. It is found that by virtue of the nonlinear interaction of the growing wave with other waves, for which the dissipation is positive, a limitation appears on the amplitude of the growing wave; this leads to the termination of the smearing of the beam, i.e., this is an effective method of dynamic stabilization of the two-stream instability. The mechanism involved in this mode of stabilization differs from other well-known mechanisms.[2,3]

Estimates carried out below show the conversion of energy of longitudinal waves at frequency ω_0 into higher-frequency waves is always effective and can be realized under laboratory conditions. The mechanisms being analyzed here may also be pertinent to explanations of a number of effects observed in astrophysical plasmas.

The basic features of these nonlinear effects appear at relatively low amplitudes of the density and velocity oscillations of the electrons in the beam and in the plasma. In particular, this feature allows us to regard the nonlinear terms in the original equations that describe the system as small, while the nonlinear processes are regarded as the result of an interaction between individual harmonic waves.

1. BASIC EQUATIONS. SYNCHRONISM CONDITIONS

We will investigate the interaction of a single-velocity beam with a cold plasma making use of the hydrodynamic equations[1)]

$$\frac{\partial E}{\partial x} - 4\pi e(\rho + \rho_s) = 0,$$

$$\frac{\partial v}{\partial t} - \frac{e}{m}E = \left\{-v\frac{\partial v}{\partial x} - \nu_{\text{eff}}\, v\right\}, \quad \frac{\partial \rho}{\partial t} + N\frac{\partial v}{\partial x} = -\frac{\partial}{\partial x}(\rho v),$$

$$\frac{\partial v_s}{\partial t} + V_0\frac{\partial v_s}{\partial x} - \frac{e}{m}E = -v_s\frac{\partial v_s}{\partial x}, \quad \frac{\partial \rho_s}{\partial t} + N_s\frac{\partial v_s}{\partial x} + V_0\frac{\partial \rho_s}{\partial x} = -\frac{\partial}{\partial x}(\rho_s v_s). \tag{1}$$

Here, E is the electric field; e/m is the specific charge of the electron; ρ, ρ_s, ν, and ν_s are respectively the deviations from the equilibrium values N, N_s, 0, and V_0 of the densities and velocities of the electrons in the plasma and the beam, while ν_{eff} is the effective collision frequency. Since all deviations from equilibrium are assumed to be small, the right sides of the equations in (1), which contain quadratic terms, essentially appear in the equation with a small parameter.[2)] Neglecting collisions, in the linear approximation we can write the following dispersion equation for the beam-plasma system:

$$1 = \frac{\omega_0^2}{\omega^2} + \frac{\omega_{0s}^2}{(\omega - kV_0)^2}, \quad \omega_0^2 = \frac{4\pi Ne^2}{m}, \quad \omega_{0s}^2 = \frac{4\pi N_s e^2}{m}. \tag{2}$$

Perturbations at frequencies $\omega \lesssim \omega_0$ in the system in (1) grow with a growth rate $\gamma \leq 2^{-4/3}\sqrt{3}\,(N_s/N)^{1/3}\omega_0$ (cf. [1]) so long as nonlinear processes do not come into play. In the hydrodynamic approximation, i.e., when beam smearing can be neglected, such a process would be the interaction of the growing wave with higher frequency waves. Since Eq. (1) only contains quadratic nonlinearities the elementary nonlinear process in the system is a three-frequency interaction, which is possi-

1) It is assumed that the system is in a strong magnetic field $\omega_H \gg \omega_0$ (ω_H is the electron gyrofrequency); hence the investigation is limited to one-dimensional waves, i.e., solutions that depend only on x and t.

2) The frictional force ν_{eff} is also assumed to be small.

ble if the following synchronism condition is satisfied:

$$\omega_1+\omega_2=\omega_3,\quad k_1+k_2=k_3. \tag{3}$$

If one of the waves ω_i grows by virtue of this combination interaction, it is also possible for the system to generate waves with frequencies $\omega_{j\neq i}$. If the stream is weak, $\beta^2 = N_s/N \ll 1$, the growth rate γ ($\gamma \ll \omega_0$) falls off rapidly as the frequency is reduced and is a maximum near ω_0. In view of this feature we can regard the growing wave as being quasiharmonic. These considerations apply even more strongly if there is a linear damping mechanism in the plasma which leads to the compression of the excitation band.[3)]

Assuming that the highest growth rate γ corresponds to a wave characterized by $k_1 = \omega_0/V_0$, and using Eqs. (2) and (3), we can determine the parameters of the other two waves that satisfy the synchronism condition. The satisfaction of the synchronism condition is sensitive to the ratio of the frequencies ω_0 and ω_{0s}, i.e., it is sensitive to the electron density in the beam and in the plasma. If the plasma is dense, $\beta > \beta_{cr}$ ($\beta_{cr}^{2/3} \approx 0.5$), the synchronism condition can only be satisfied for one wave mode (lying on one branch of the dispersion curve) with parameters

$$k_1=\omega_0/V_0,\quad \omega_1=\omega_0-\delta\quad (\omega_1=\operatorname{Re}\omega(k_1),\ \operatorname{Im}\omega(k_1)=\gamma);$$

$$k_2=\frac{\omega_0+\Delta_0}{V_0}-\frac{\beta\omega_0}{V_0(1-\omega_0^2/(\omega_0+\Delta_0)^2)^{1/2}},\quad \omega_2=\omega_0+\Delta_0; \tag{4}$$

$$k_3=\frac{2\omega_0+\Delta_0-\delta}{V_0}-\frac{\beta\omega_0}{V_0(1-\omega_0^2/(2\omega_0+\Delta_0-\delta)^2)^{1/2}},\quad \omega_3=2\omega_0+\Delta_0-\delta;$$

here $\delta = 2^{-4/3}\omega_0\beta^{2/3}$ while

$$\frac{\Delta_0}{\omega_0}=\sqrt{\frac{\alpha_0^2}{\alpha_0^2-1}}-1,\qquad \alpha_0^2=\frac{(-\delta\pm2\beta/\sqrt{3})^2}{\beta^2}.$$

If the beam is weak, $\beta < \beta_{cr}$, in addition to the waves k_2 and k_3, which are in synchronism with k_1, we also find synchronism for waves with parameters

$$k_2'=\frac{\omega_0+\Delta_0'}{V_0}-\frac{\beta\omega_0}{V_0(1-\omega_0^2/(\omega_0+\Delta_0')^2)^{1/2}},\quad \omega_2=\omega_0+\Delta_0';$$
$$k_3'=\frac{2\omega_0+\Delta_0'-\delta}{V_0}+\frac{\beta\omega_0}{V_0(1-\omega_0^2/(2\omega_0+\Delta_0'-\delta)^2)^{1/2}},\quad \omega_3=2\omega_0+\Delta_0'-\delta, \tag{5}$$

which pertain to different branches of the dispersion equation.

If the beam is weak, in addition to the three-wave interactions, which satisfy the synchronism condition rigorously, it is also necessary to consider two-wave interactions that satisfy (3) approximately. Taking $k_2 = k_1$, we find from Eqs. (2) and (3)

$$k_3=2k_1,\qquad \omega_3=2\omega_1+\Delta\omega_1, \tag{6}$$

where $\Delta\omega > 0$ is the deviation from synchronism.

2. SECOND-HARMONIC GENERATION. STABILIZATION OF THE INSTABILITY

For the processes that are being considered here, it will be shown below by estimates that the lifetime for the phases of the individual waves $\tau_{\varphi_i} \sim 1/\Delta\omega_i$ is much greater than the nonlinear interaction time τ_n. Hence, the phases can be regarded as regular functions of the coordinates and time and treated in the analysis of the nonlinear interactions by a dynamic description.[4]

We shall investigate the three-wave interactions using the method of averaging over two variables.[5,6] The solution of Eq. (1) is written in the form of three-waves which satisfy the synchronism condition (3):

$$E=\sum_{j=1}^{3}\Psi_1^jA_j(x,t)\exp\{i[\omega_jt-k_j^mx+\varphi_j(x,t)]\}+\text{c.c.}\qquad \Psi_1^j=1;$$

$$v=\sum_{j=1}^{3}\Psi_2^jA_j(x,t)\exp\{i[\omega_jt-k_j^mx+\varphi_j(x,t)]\}+\text{c.c.}\qquad \Psi_2^j=\frac{e}{im\omega_j};$$

$$\rho=\sum_{j=1}\Psi_3^jA_j(x,t)\exp\{i[\omega_jt-k_j^mx+\varphi_j(x,t)]\}+\text{c.c.}\qquad \Psi_3^j=\frac{ek_jN}{im\omega_j^2};$$

$$v_s=\sum_{j=1}^{3}\Psi_4^jA_j(x,t)\exp\{i[\omega_jt-k_j^mx+\varphi_j(x,t)]\}+\text{c.c.}$$
$$\Psi_4^j=\frac{e}{im(\omega_j-k_j^mV_0)}\cdot \tag{7}$$

$$\rho_s=\sum_{j=1}^{3}\Psi_5^jA_j(x,t)\exp\{i[\omega_jt-k_j^mx+\varphi_j(x,t)]\}+\text{c.c.}$$
$$\Psi_5^j=\frac{ek_j^mN_s}{im(\omega_j-k_j^mV_0)^2};$$

where Ψ_i^j is the distribution coefficient, $A_j(x, t)$ and $\varphi_j(x, t)$ are functions of the coordinates and time and vary slowly compared with $\exp i(\omega t - kx)$, while ω and k are related by the dispersion equation of the linear problem (2). We recall that one of the interacting waves, specifically the wave k_1, is a growing wave, that is to say, it is characterized by a complex frequency $\omega = \omega_1 - i\gamma$ where $\omega_1 = \omega_0 - \delta$. Assuming that γ is small ($\gamma \ll \omega_0$) in what follows, ω will be assumed to be real in all expressions apart from the exponentials: $\omega = \omega_1$.

Equations for the slow variables A_j and φ_j are obtained by averaging over x and t and are written in the form[5,6]

$$\frac{\partial A_j^m}{\partial t}+v_{\text{rp}}^j\frac{\partial A_j^m}{\partial x}=\operatorname{Re}\frac{a_m\sum_{l=1}^{n}\zeta_{lm}f_l^{m,j}}{D_p'}+\gamma_i\delta_{ij}A_j, \tag{8}$$

$$\frac{\partial\varphi_j}{\partial t}+v_{\text{rp}}^j\frac{\partial\varphi_j^m}{\partial x}=\operatorname{Im}\frac{a_m\sum_{l=1}^{n}\zeta_{lm}f_l^{m,j}}{D_p'A_j^m},\qquad p=i\omega,\quad \varkappa=-ik, \tag{9}$$

where $D(p, \kappa) = 0$ is the dispersion equation for the system; A_j^m and φ_j^m are the amplitude and phase of the waves at frequency ω_j (m is the subscript for the wave type, the branch of the dispersion equation) and v_{gr}^j is the group velocity at this frequency; ξ_{lm} is the characteristic function of the conjugate algebraic system to (1) ($\partial/\partial t \to p$, $\partial/\partial x \to \kappa$, the right sides equal to zero), a_m = const, and

$$f_l^{m,j}=\frac{1}{T\lambda_0}\int_0^T\int_0^{\lambda_0}f_l(x,t)\exp\{-i[\omega_jt-k_j^mx+\varphi_j(x,t)]\}\,dx\,dt, \tag{10}$$

where $f_l(x, t)$ represents the right side of Eq. (1) when the unknown functions are expressed in terms of (7). In the present case, taking account of (3) we obtain the following averaged equations:

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3) This single-mode approximation is obviously valid if the width of the instability region Δk satisfies the relation [cf. (11), (17)]

$$k_3\gg\Delta k\sim\frac{\sigma A}{v_{\text{gr}}A}\sim\frac{A'}{A}\sim\frac{A}{v_{\text{gr}}A},\qquad (')\equiv\frac{\partial}{\partial x},\qquad (\dot{})\equiv\frac{\partial}{\partial t}.$$

$$\dot{A}_1 + v_{\mathrm{gr}_1} A_1' = \sigma_1 A_2 A_3 \cos\Phi + \gamma(A_j^2) A_1,$$
$$\dot{A}_2 + v_{\mathrm{gr}_2} A_2' = \sigma_2 A_1 A_3 \cos\Phi - \nu_2 A_2,$$
$$\dot{A}_3 + v_{\mathrm{gr}_3} A_3' = \sigma_3 A_1 A_2 \cos\Phi - \nu_3 A_3; \tag{11}$$

$$\dot{\varphi}_1 + v_{\mathrm{gr}_1}\varphi_1' = -\frac{A_2 A_3}{A_1}\sigma_1 \sin\Phi, \qquad \dot{\varphi}_2 + v_{\mathrm{gr}_2}\varphi_2' = -\frac{A_1 A_3}{A_2}\sigma_2 \sin\Phi,$$
$$\dot{\varphi}_3 + v_{\mathrm{gr}_3}\varphi_3' = \frac{A_1 A_2}{A_3}\sigma_3 \sin\Phi, \tag{12}$$
$$v_{\mathrm{gr}} = V_0[1 + \omega_0^2\omega_{0s} / \omega^3(1 - \omega_0^2/\omega^2)^{3/2}]^{-1}.$$

Here

$$\sigma_1 = \frac{1}{2D_1}\left\{\frac{e\omega_0^2 k_1(\omega_1 - k_1V_0)^2}{m\omega_2\omega_3} + \frac{e\omega_1(\omega_1 - k_1V_0)^2\omega_0^2}{m\omega_2\omega_3}\left(\frac{k_2}{\omega_2} + \frac{k_3}{\omega_3}\right)\right.$$
$$+ \frac{\omega_1^2 e k_1 \omega_{0s}^2}{m(\omega_2 - k_2V_0)(\omega_3 - k_3V_0)} + \frac{e\omega_1^2(\omega_1 - k_1V_0)\omega_{0s}^2}{m(\omega_2 - k_2V_0)(\omega_3 - k_3V_0)}\cdot$$
$$\left.\times\left(\frac{k_2}{\omega_2 - k_2V_0} + \frac{k_3}{\omega_3 - k_3V_0}\right)\right\},$$

$$\sigma_2 = \frac{1}{2D_2}\left\{\frac{e\omega_0^2 k_2(\omega_2 - k_2V_0)^2}{m\omega_1\omega_3} + \frac{e\omega_2(\omega_2 - k_2V_0)^2\omega_0^2}{m\omega_1\omega_3}\left(\frac{k_1}{\omega_1} + \frac{k_3}{\omega_3}\right)\right.$$
$$+ \frac{e\omega_2^2 k_2 \omega_{0s}^2}{m(\omega_1 - k_1V_0)(\omega_3 - k_3V_0)} + \frac{e\omega_2(\omega_2 - k_2V_0)\omega_{0s}^2}{m(\omega_1 - k_1V_0)(\omega_3 - k_3V_0)}\cdot$$
$$\left.\times\left(\frac{k_1}{\omega_1 - k_1V_0} + \frac{k_3}{\omega_3 - k_3V_0}\right)\right\}, \tag{13}$$

$$\sigma_3 = \frac{-1}{2D_3}\left\{\frac{e\omega_0^2 k_3(\omega_3 - k_3V_0)^2}{m\omega_1\omega_2} + \frac{e\omega_3(\omega_3 - k_3V_0)^2\omega_0^2}{m\omega_1\omega_2}\left(\frac{k_1}{\omega_1} + \frac{k_2}{\omega_2}\right)\right.$$
$$+ \frac{e\omega_3^2 \omega_{0s}^2 k_3}{m(\omega_1 - k_1V_0)(\omega_2 - k_2V_0)} + \frac{e\omega_3^2(\omega_3 - k_3V_0)\omega_{0s}^2}{m(\omega_1 - k_1V_0)(\omega_2 - k_2V_0)}$$
$$\left.\times\left(\frac{k_1}{\omega_1 - k_1V_0} + \frac{k_2}{\omega_2 - k_2V_0}\right)\right\};$$

$$\Phi = \varphi_1 + \varphi_2 - \varphi_3,$$
$$D_{i=1,2,3} = [\omega_{0s}^2\omega_i - \omega_i(\omega_i - k_iV_0)^2 + (\omega_0^2 - \omega_i^2)(\omega_i - k_iV_0)].$$

It is interesting to note that these equations are the same as the equations that describe the nondegenerate parametric interaction of electromagnetic waves in a transparent medium with a quadratic nonlinearity and small losses.[7, 8] However, the corresponding problem in nonlinear optics is actually quite different from the one being considered here. This difference arises primarily from the fact that in nonlinear optics one usually considers the interaction of weakly damped waves: the parameters ω and k lie in the region of transparency of the medium. The coefficients σ_j, which determine the efficiency of the nonlinear interaction, satisfy the well-known Manley–Rowe relations in this case;[9] the nonlinear process itself is the transformation of the energy of a wave which appears from outside (a pumping wave) into the energy of waves at other frequencies.

In the present problem the generation or amplification of the waves k_2 and k_3 that lie in the transparency band is realized by virtue of the acquisition of energy of the other characteristic wave, which is a growing wave in the linear approximation. In this case the Manley–Rowe relations are obviously not satisfied. We note that the analogous problem can also be formulated in nonlinear optics. It arises in the analysis of the combination interaction of waves in an oscillator. An oscillator of this kind might be realized experimentally if, for example, a population inversion is produced in a medium with a quadratic nonlinearity for one of the frequencies that satisfies the condition in (3).

We shall consider the three-wave process in an infinite system, assuming that the solution is uniform in space. For this purpose we find the equilibrium state of the system (11)–(13) and analyze the stability of this state with respect to perturbations in amplitude and phase. The existence of stable regimes will show the beam-plasma system can support stationary generation of the waves k_1, k_2 and k_3, which implies stabilization of the two-stream instability. It is interesting to note that if dissipation effects are neglected for the non-growing waves (waves 2 and 3) i.e., if collisions in the plasma are neglected, then in the approximation being treated here an equilibrium three-frequency regime is not possible.

In the general case, taking account of the feedback effect of the generated waves on the single-velocity beam we find that the growth rate of the wave falls off with increasing energy of waves 1, 2, and 3; this feature appears as a result of the heating of the beam and the plasma, i.e., the smearing of the electron velocity distribution function.[2, 3] At small energies of the interacting waves this effect can be treated phenomenologically by taking

$$\gamma(A_j^2) = \gamma_0 - \sum_{j=1}^{3}\gamma_j A_j^2 \qquad (\gamma_j A_j^2 \ll \gamma_0), \tag{14}$$

where $\gamma_0 = \gamma - \bar{\nu}$.

Setting $\dot{A}$, A', $\dot{\varphi}$ and φ' equal to zero in Eqs. (11) and (12) we find the steady-state generation parameters:[4)]

$$A_{01}^2 = \frac{\nu_2\nu_3}{\sigma_2\sigma_3}, \qquad A_{02}^2 = \frac{\gamma_0\nu_2}{\sigma_1\sigma_3}\alpha, \qquad A_{03}^2 = \frac{\gamma_0\nu_2}{\sigma_1\sigma_2}\alpha,$$
$$\alpha = \frac{1 - \gamma_2\nu_2\nu_3/\gamma_0\sigma_2\sigma_3}{1 + \gamma_1\nu_2/\sigma_1\sigma_2 + \gamma_3\nu_3/\sigma_1\sigma_3} \sim 1, \qquad \Phi = \pm\pi, 0. \tag{15}$$

Limiting ourselves to spatially uniform perturbations, it is easy to show that the stationary regime (15), which corresponds to $\Phi = 0$, is stable with respect to such perturbations in the parameter region

$$\nu_2 > \gamma_0, \qquad \frac{\gamma_0}{\gamma_2 A_{02}^2}\left(\frac{N_s}{N}\right)^{1/2} < 1. \tag{16}$$

A similar result is obtained for convective instabilities which depend only on $\zeta = ut - x$ (u = const).

A three-frequency stationary mode of operation is possible only if the electron beam is relatively dense ($\beta \gtrsim \beta_{cr}$). If the electron density in the beam is small ($\beta < \beta_{cr}$) the basic process is the two-frequency process, for which the synchronism condition in (6) is satisfied approximately. Writing A_2, $\varphi_2 \equiv A_1$, φ_1, from (6), (11)–(13) we obtain the averaged equations which describe generation of the second harmonic $\sim 2\omega_0$:

$$\dot{A}_1 + v_{\mathrm{gr}_1} A_1' = \sigma_1 A_1 A_3 \cos\Phi + \gamma_0 A_1,$$
$$\dot{A}_3 + v_{\mathrm{rp}_3} A_3' = \sigma A_1^2 \cos\Phi - \nu A_3; \tag{17}$$
$$\dot{\varphi}_1 + v_{\mathrm{rp}_1}\varphi_1' = -\sigma_1 A_2 \sin\Phi,$$

4) The steady-state amplitudes [cf. (15) and (20)], as can be seen from a comparison with the results of [2], are found to be much smaller in this parameter region than the amplitudes corresponding to the beginning of the beam-smearing process (i.e., the quasilinear relaxation of the distribution function). Thus, the analysis given here is valid.

$$\dot{\varphi}_3 + v_{\text{гр}_3}\varphi_3' = \sigma_3 \frac{A_1^2}{A} \sin \Phi, \quad \Phi = 2\varphi_1 - \varphi_3 - \Delta\omega t; \tag{18}$$

where

$$\sigma_1 \approx -\frac{2^{2/3} e}{m V_0 \beta^{4/3}}, \quad \sigma_3 \approx \frac{2^{5/3} e}{m V_0 \sqrt{3} \beta^{1/3}}, \quad \gamma_0 = \gamma - \nu_1,$$
$$\nu_1 = \frac{\nu_{\text{эфф}}}{6}, \quad \nu = \nu_{\text{эфф}} \frac{\beta}{6\sqrt{3}}. \tag{19}$$

The steady-state generation of the second harmonic is characterized by the parameters

$$A_{03} = \frac{\gamma_0}{|\sigma_1| \cos \Phi_0}, \quad A_{01}^2 = \frac{\gamma_0 \nu}{|\sigma_3 \sigma_1| \cos^2 \Phi_0}, \quad \operatorname{tg} \Phi_0 = \frac{\Delta\omega}{2\gamma_0 - \nu}. \tag{20}$$

Linearizing Eqs. (17) and (18) we can show easily that this regime is stable if

$$\nu > 2\gamma_0. \tag{21}$$

Thus, if the instability is to be stabilized the damping of the second harmonic must be rather large.

3. DISCUSSION OF RESULTS

We now wish to evaluate the possibility of observing the effects considered here in a laboratory plasma. First, however, we note that under laboratory conditions interest also attaches to the problem of conversion of energy in a growing wave at frequency ω_0 into energy at the second harmonic in a bounded sytem. If a perturbation at frequency $2\omega_0$ is excited at the boundary of the beam-plasma system it will be amplified in the propagation process and, in a system of adequate length, will reach the limiting value $A_{03} = \gamma_0/|\sigma_1| \times \cos \Phi_0$. The stationary processes in this amplifier are described by Eqs. (17) and (18) with $\dot{A}_j = \dot{\varphi}_j = 0$ and $\Phi = 2\varphi_1 - \varphi_3 - \Delta kx$ where $\Delta k \approx 2\delta/V_0$ is the deviation from spatial synchronism. Although we shall not here actually consider the solution of a concrete boundary value problem or an unbounded problem, we can make estimates that are useful for equilibrium regimes.

Three-wave process. The parameters of the beam and plasma will be the following: $N \approx 1.25 \times 10^{11}$ cm^{-3}, $\omega_0 \approx 2 \times 10^{10}$ sec^{-1}, $N_S \approx 1.6 \times 10^{10}$ cm^{-3}, $\beta \gtrsim 0.5$, the thermal velocity of the electrons $v_T^2 \approx 5 \times 10^{13}$ cm^2/sec^2, the effective electron-neutral collision frequency[11] is $\nu_{\text{eff}} \approx 5 \times 10^9$ sec^{-1}. The energy of the beam electrons is $W \approx 1$ keV, whence $V_0 \approx 1.7 \times 10^9$ cm/sec while the current density is $j = eN_S V_0 \approx 4$ A/cm^2. With these parameters, the beam-plasma system will generate plasma waves with wavelength $\lambda_1 \approx 1$ cm and $\lambda_3 \approx 5.35$ mm. In the stationary regime the power of these waves, in accordance with Eq. (15), is respectively $P_1 \approx 10^{-2}$ W/cm^2 and $P_3 = 27$ W/cm^2. These quantities are much smaller than the beam power $W = N_S m V_0^3/2 \approx 4 \times 10^3$ W/cm^2, so that the approximation used here is valid.

We note that to achieve one-dimensional motion in the system it is necessary to apply a longitudinal magnetic field $H_0 \approx 5 \times 10^3$ g; in this case $\omega_H \approx 10^{11}$ sec^{-1} $> \omega_0$.

Two-frequency process. We shall retain most of the parameters of the plasma given above, merely changing the velocity and density of the beam: $N_S \approx 10^9$ cm^{-3}, $\beta^{2/3} \approx 0.2$, $V_0 \approx 1.7 \times 10^9$ cm/sec, $W \approx 20$ W/cm^2, $j \approx 0.03$ A/cm^2 and $\nu_{\text{eff}} \approx 10^9$ sec^{-1}. With these values of the parameters the power in the generated waves, as follows from (20) is $P_1 \approx 0.1$ W/cm^2 and $P_3 \approx 0.3$ W/cm^2. As before, the condition $P_{1,2} \ll W$ is satisfied.

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