TURBULENT RELAXATION OF A PLASMA FAR FROM THE STABILITY THRESHOLD

A. A. GALEEV

Nuclear Physics Institute, Siberian Division, USSR Academy of Sciences

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We consider an anisotropic plasma far from the stability threshold, when the appearance of collective oscillations affects the motion of the plasma particles. It is shown that the motion of the particles in turbulent electric fields can be represented as the result of nonlinear phase shifts in the "wave + particle" system. This nonlinear effect does not prevent instability, but merely decreases its increment. The instability can therefore be regarded as weak in the nonlinear stage of its development and the relaxation of the anisotropic ion distribution is therefore studied on the basis of the quasilinear theory developed for our case. It is shown that the instability disappears before relaxation to a completely stable state occurs. This is the reason why the instability consists of consecutive oscillation spikes, each of which contribute to relaxation of the plasma.

THIS paper is devoted to an investigation of turbulent plasma in which the oscillation level is already high enough to appreciably perturb the particle motion, but not high enough to cause quasilinear relaxation of the plasma distribution to the stable state. It then turns out that the effect of particle motion in stochastic electric and magnetic fields can be described in terms of the stochastic variation of the phase of the interaction of the given oscillations with the plasma particles.

The stochastic collapse of the oscillation phase, as is well known, does not stop the course of the energy-exchange process, and only slows it down. Therefore the instability of the plasma, which is the result of such an interaction, only slows down its rate of development, but does not stop completely.

The equations for the oscillation amplitudes and for the particle distribution function, after averaging over the stochastic phase wandering, contain as a time-varying parameter certain integral characteristics of the turbulence spectrum, which serve as a measure of the time of phase correlation between the resonant particle and the oscillation. Principal attention will be paid to the investigation of the properties of the equations obtained in this manner.

1. DERIVATION OF FUNDAMENTAL EQUATIONS

Before we proceed to a derivation of the equations, it is useful to make a few remarks concerning the relation between the nonlinear phase collapse considered by us and the quasilinear relaxation of the plasma distribution. In a plasma without a magnetic field, both these phenomena become manifest simultaneously in a turbulent electric field $U_\sim$ at particle velocities exceeding the phase velocity of the wave (the latter is assumed to be large or comparable with the thermal velocity of the particle v):

$$U_\sim^2 = \sum_q \frac{e^2q^2|\Phi_q|^2}{m^2\omega_q^2} > \frac{\omega_k^2}{k^2} \gg v^2, \tag{1}$$

where $|\Phi_q|$ is the amplitude of the oscillations with wave vector q and frequency ω_q, while e and m are the charge and mass of the particle.

Moreover, at such high amplitudes, the rate of exchange of energy between the different oscillation modes turns out to be comparable with the rate of energy supply to each mode, owing to the instability. All this is the reason why the phase collapse is not the main effect under the given conditions, and therefore it is impossible to simplify the equations for the oscillation amplitudes.

The foregoing considerations remain valid also for turbulent motion of the particles along the magnetic field. However, the effects of turbulent motion of particles across a strong magnetic field can be appreciable even at small oscillation amplitudes, when the reaction of the oscillations on the particle distributions can be neglected.

Indeed, let us represent the turbulent electric field in the form[1)]

$$\mathbf{E} = -\nabla\Phi, \quad \Phi = \sum_{\mathbf{k},n} |\Phi_\mathbf{k}^n| \exp\{-i\omega_n t + i\mathbf{kr} + i\psi_\mathbf{k}^n\}. \tag{2}$$

It is then easy to calculate the change of the transverse energy of the particles, averaged over the time of the Larmor rotation. It will be small compared with the total kinetic energy of the Larmor rotation of the particles under the condition

$$\sum_{\mathbf{k},n} \frac{e^2|\Phi_\mathbf{k}^n|^2}{m^2v_\perp^4} \frac{n^2\omega_c^2}{(\omega_n - n\omega_c)^2} J_n^2\left(\frac{k_\perp v_\perp}{\omega_c}\right) \ll 1, \tag{3}$$

where $\omega_c = eH_0/mc$ is the Larmor frequency, $v_\perp$ is the velocity of the particle across the field H_0, J_n is a Bessel function of order n, and the oscillation frequencies are assumed to be multiples of the Larmor frequency of the particles ($\omega_n \approx n\omega_c$).

On the other hand, the phenomenon of nonlinear phase collapse takes place when the phase shift of the resonant particle as it moves in the turbulent electric field exceeds the linear change of the phase of the wave:

$$k^2U_{\perp\sim}^2 = \sum_{\mathbf{q},p} c^2[\mathbf{k}_\perp\mathbf{q}_\perp]^2|\Phi_\mathbf{q}^p|^2 J_n^2\left(\frac{q_\perp v_\perp}{\omega_c}\right)\Big/H_0^2 \gg (\omega_n - n\omega_c)^2 \tag{4*}$$

[1)]As usual, we assume in (3) that the normalization volume is equal to unity.

*$[\mathbf{k}_\perp\mathbf{q}_\perp] \equiv \mathbf{k}_\perp \times \mathbf{q}_\perp$.

The inequalities (3) and (4) can be satisfied simultaneously only if the maximum of the spectral density of the electric field fluctuations occurs in the region of wavelengths shorter than the Larmor radius of the particles interacting with the oscillations. In other words, the wavelength of the most developed oscillations with frequency $\omega_n \approx n\omega_c$ should obey the condition

$$n^2/\lambda^2 \ll 1. \tag{5}$$

Here $\lambda = k_\perp^2 r_c^2$ and $r_c = v_\perp/\omega_c$ is the Larmor radius.

Bearing in mind all the foregoing, we consider as a concrete example the nonlinear stage of development of a short-wave ion-cyclotron turbulence and the relaxation of the ion distribution in an anisotropic plasma ($T_{\perp i} \gg T_{\| i}$) placed in a strong magnetic field. The effects of nonlinearity of motion of the electrons, and also the quasilinear relaxation of the distribution of particles of both sorts will be less important than the nonlinear phase collapse of the ions when the following inequalities are satisfied:

$$\frac{k_\perp^2 U_{\perp i}^2}{\gamma_k^2} \gg 1 \gg \max\left[\frac{k_\perp^2 U_{\perp i}^2}{\gamma_k^2}\frac{n^2}{\lambda_i^2};\ \frac{k_\|^2 U_{\| i}^2}{\gamma_k^2},\ \frac{k_\|^2 U_{\| e}^2}{n^2\omega_{ci}^2}\right], \tag{6}$$

where γ_k is the increment of the ion-cyclotron instability, and $U_{\perp j}$ and $U_{\| j}$ are the mean-square pulsation velocities of the particles of type j across and along the magnetic field, respectively.

The effect of interest to us, the nonlinear phase collapse, is due to the motion of particles in a turbulent electric field. Under conditions (6), it suffices for us to consider the motion of the ions across the magnetic field. Averaging the equations of motion over the Larmor rotation of the ions, we reduce this equation to the "generalized" drift equation:

$$\mathbf{r}_\perp(t) = \mathbf{r}_0 - \frac{[\mathbf{v}_\perp(t)\mathbf{h}]}{\omega_{ci}} + \delta\mathbf{r}_\perp(t), \tag{7}$$

$$\frac{d\delta\mathbf{r}_\perp}{dt} = -\sum_{\mathbf{q},p} ic[\mathbf{qh}]|\Phi_\mathbf{q}{}^p|J_p\left(\frac{q_\perp v_\perp}{\omega_{ci}}\right)H_0^{-1}$$
$$\times\exp\left\{i\frac{[\mathbf{qv}]\mathbf{h}}{\omega_{ci}} + ip\left(\frac{\pi}{2}-\theta_\mathbf{q}\right) - i(\omega_p - p\omega_{ci})t + i\mathbf{q}\delta\mathbf{r}_\perp + i\psi_\mathbf{q}{}^p\right\} \tag{8}$$

Here h is a unit vector along the constant magnetic field H_0, θ_q is the initial phase of rotation of the ions around the force line, reckoned from the direction of the wave-vector component q transverse to the magnetic field. The second term in (7) is responsible for the Larmor rotation of the ions, and the third is due to the drift in the turbulent electric field.

In solving Eq. (8), we make the following assumptions:

a) The phases of the oscillations are random, i.e.,

$$\langle\exp[i\psi_\mathbf{k}{}^n]\rangle = 0,\quad \langle\exp[i(\psi_\mathbf{k}{}^n + \psi_{\mathbf{k}'}{}^{n'})]\rangle = \delta_{\mathbf{k}',-\mathbf{k}}\delta_{n',-n}. \tag{9}$$

The angle brackets denote averaging over the statistical ensemble.

b) The amplitude of the deviation of the ions from the trajectory of free motion in the field of an individual mode of oscillations is much smaller than the wavelength

$$\langle[\mathbf{k}\delta\mathbf{r}_\mathbf{q}(t)]^2\rangle \ll 1. \tag{10}$$

c) To the contrary, the mean-square deviation of the ions in the turbulent electric field is comparable or larger than the wavelength:

$$\sum_\mathbf{q}\langle[\mathbf{k}\delta\mathbf{r}_\mathbf{q}(t)]^2\rangle \gg 1. \tag{11}$$

Under these assumptions, it is reasonable to expect the motion of the ions to be described by the random function with independent increments $\delta\mathbf{r}_\perp(t)$, having the following properties:

$$\langle\delta r_\alpha(t)\rangle = 0,\quad \langle[\delta r_\alpha(t+\tau) - \delta r_\alpha(t)]\delta r_\beta(t)\rangle = 0,$$
$$\langle\delta r_\alpha(t)\delta r_\beta(t)\rangle = \delta_{\alpha\beta}Dt. \tag{12}$$

These properties can be proved by the induction method. In other words, we assume that they are satisfied, and then calculate the change of the ion trajectory under the influence of a small group of "trial" waves and verify that the trajectory, with allowance for these changes, has the properties (12) as before. Calculations performed in this manner lead to the following formula for the coefficient D (see the appendix):

$$D = \left\{\sum_{\mathbf{q},p} c^2|\Phi_\mathbf{q}{}^p|^2 J_p{}^2\left(\frac{q_\perp v_\perp}{\omega_{ci}}\right)H_0^{-2}\right\}^{1/2}. \tag{13}$$

Knowledge of the statistical properties of the particle trajectory in a turbulent electric field makes it possible to obtain a simple equation for the oscillations of a turbulent plasma.

In the approximation that is a natural generalization of the linear theory of small oscillations in a quiet plasma, it is possible to write down the solution of the kinetic equation for the plasma ions in the form of an integral over the trajectories of the ions with properties (12)

$$f_\mathbf{k}{}^i = \frac{ie}{m_i}\int_{-\infty}\Phi_\mathbf{k}\mathbf{k}\frac{\partial}{\partial\mathbf{v}}\left[f_{0i} + \sum_\mathbf{q} f_\mathbf{q}{}^i(\mathbf{r},\mathbf{v},t')\right]dt'. \tag{14}$$

The second term in the square brackets takes into account here the change of the distribution of the particles owing to the presence of "background" oscillations of finite amplitude. Since the exact trajectory of the ions is unknown to us, we average this expression over the phases of the "background" oscillations and by the same token express it in terms of the average characteristics of the trajectory. In the first term of (14), the averaging over the phases of the oscillations reduces to a calculation of the mean value of the mixed functional of the particle trajectory:

$$\langle\Phi_\mathbf{k}(\mathbf{r}(t),t)\rangle \sim \langle\exp[i\mathbf{k}\delta\mathbf{r}_\perp(t)]\rangle = \exp\{-k_\perp^2 Dt'\}. \tag{15}$$

When averaging the second term in (14), it must be borne in mind that the correction to the distribution function of the particles turns out to be proportional to the amplitude of the oscillations and can be written in the form (henceforth, to simplify the notation, we omit the index n or p in the expressions for f_k and D_k, assuming that they are encountered and summed only in pairs (**k**, n) or (q, p)):

$$f_\mathbf{q}(\mathbf{r},\mathbf{v},t) = \tilde{f}_\mathbf{q}(\mathbf{v})|\Phi_\mathbf{q}{}^p|J_p\left(\frac{q_\perp v_\perp}{\omega_{ci}}\right)$$
$$\times\exp\left\{-i(\omega_p - p\omega_{ci})t + i\mathbf{qr} + i\frac{[\mathbf{qv}]\mathbf{h}}{\omega_{ci}} + ip\left(\frac{\pi}{2}-\theta_\mathbf{q}\right) + i\psi_\mathbf{q}{}^p\right\} \tag{16}$$

In the case of short-wave oscillations considered by us, the gradients of the particle distribution in velocity space are determined by the phase factor $\exp\{i[\mathbf{k}\times\mathbf{v}]\cdot\mathbf{h}/\omega_{ci}\}$, so that we can carry out the differentiation over the velocities and rewrite the second term in expression (14) in the following form:

$$-\sum_{\mathbf{q}} f_{\mathbf{q}}(\mathbf{v}) \int_{-\infty}^{t} dt' \Phi_{\mathbf{k}}(\mathbf{r},t') \frac{d}{dt'} [i\mathbf{k}\delta\mathbf{r}_{\mathbf{q}}(t')].$$

Since the displacements of the ions in the fields of the individual oscillation modes are statistically independent, the averaging over the phases of the "background" oscillations can be easily carried out here, and as a result we obtain

$$2\sum_{\mathbf{q}} f_{\mathbf{q}}(\mathbf{v}) ([\mathbf{kq}]\mathbf{h})^2 q_\perp^{-2} D_{\mathbf{q}} \langle \Phi_{\mathbf{k}}(\mathbf{r},t')\rangle, \tag{17}$$

where

$$D_{\mathbf{q}} \approx c^2 H_0^{-2} |\Phi_{\mathbf{q}}{}^p|^2 J_p^2\left(\frac{q_\perp v_\perp}{\omega_{ci}}\right) \Big/ D.$$

To solve (14) with respect to the distribution function of the particles in the oscillation field, we divide it by the quantity

$$J_n\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right) \exp\left\{i\frac{[\mathbf{kv}]\mathbf{h}}{\omega_{ci}} + in\left(\frac{\pi}{2}-\theta_{\mathbf{k}}\right)\right\} |\Phi_{\mathbf{k}}{}^n|$$
$$\times \exp\{-i(\omega_n - n\omega_{ci})t + i\mathbf{kr} + i\psi_{\mathbf{k}}{}^n\},$$

and then average over the space of the wave numbers with weight $D_{\mathbf{k}}$. As a result, we obtain an equation for the quantity $\Sigma f_{\mathbf{k}} D_{\mathbf{k}}$:

$$\sum_{\mathbf{k}} f_{\mathbf{k}}(\mathbf{v}) D_{\mathbf{k}} = i\frac{e}{m_i}\sum_{\mathbf{k}} D_{\mathbf{k}} \int_{-\infty}^{t} dt' \left[k_\parallel \frac{\partial f_{0i}}{\partial v_\parallel} + n\omega_{ci}\frac{\partial f_{0i}}{v_\perp \partial v_\perp}\right]$$
$$\times \exp\{-i[\omega_n - n\omega_{ci} - k_\parallel v_\parallel + ik_\perp^2 D](t'-t)\}$$
$$+ \left[\sum_{\mathbf{k}} f_{\mathbf{k}} D_{\mathbf{k}}\right] \sum_{\mathbf{k}} \int_{-\infty}^{t} dt' k_\perp^2 D_{\mathbf{k}}\,. \tag{18}$$

Combining (14), (17), and (18) we arrive at the final result

$$f_{\mathbf{k}}{}^i = \frac{e}{m_i}\sum_{n=-\infty}^{+\infty} \int_{-\infty}^{t} dt' \left\{ i\left[k_\parallel \frac{\partial f_{0i}}{\partial v_\parallel} + n\omega_{ci}\frac{\partial f_{0i}}{v_\perp \partial v_\perp}\right]\right.$$
$$-\sum_{\mathbf{q}} D_{\mathbf{q}} \int_{-\infty}^{0} d\tau \left[q_\parallel \frac{\partial f_{0i}}{\partial v_\parallel} + p\omega_{ci}\frac{\partial f_{0i}}{v_\perp \partial v_\perp}\right] \mathscr{E}(p,q,\tau)$$
$$\times \left(\sum_{\mathbf{q}} D_{\mathbf{q}} \int_{-\infty}^{0} d\tau (\omega_p - p\omega_{ci} - q_\parallel v_\parallel) \mathscr{E}(p,q,\tau)\right)^{-1}$$
$$\left.\times \left[\frac{\partial}{\partial t} + i(\omega_n - n\omega_{ci} - k_\parallel v_\parallel)\right]\right\} |\Phi_{\mathbf{k}}{}^n| J_n\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)$$
$$\times \exp\left\{in\left(\frac{\pi}{2}-\theta_{\mathbf{k}}\right) + i\frac{[\mathbf{kv}]\mathbf{h}}{\omega_{ci}} + i\psi_{\mathbf{q}}{}^p\right\} \mathscr{E}(n,k,t-t'),$$
$$\mathscr{E}(p,q,\tau) = \exp\{-[i(\omega_p - p\omega_{ci} - q_\parallel v_\parallel) - q^2 D]\tau\}. \tag{19}$$

This equation was first obtained by the author by integrating the kinetic equation along the exact trajectories of the particles in a random oscillation field, followed by averaging of the results over the phases of the oscillation.[2)] [1] The first published paper in which the wandering of the particles in turbulent fields was taken into account is that of Dupree,[2] and concerns turbulence of a plasma without a magnetic field. To be sure, his equation is incorrect, since he does not take into account the change of the particle distribution in the presence of background oscillations (see the second term in Eq. (14)). Later, Orszag and Kraichnan[3] again reviewed Dupree's problem and presented a correct description of the turbulence for certain statistical models, whose energy, momentum, and other characteristic integrals coincided exactly with their values for the real plasma. The latter circumstance has made it possible to hope that turbulence in such a model will have much in common with the true plasma turbulence.

Orszag and Kraichnan, comparing their results with those of Dupree, noted that Dupree's equation does not satisfy the initial conditions, and therefore proposed to add to it a term describing the evolution of the distortion of the particle distribution in the turbulent field existing at the initial instant of time. The obtained equation (7.5) of [3] still does not take into account the correlation, reflected in our Eq. (14), between this initial distortion of the distribution and the turbulent field, and is therefore likewise inaccurate.

The presence of a second term in (19) is of fundamental importance for a correct description of the evolution of the instability. We shall show later that in our model the nonlinear collapse of the phases of the interacting waves leads only to a decrease of the instability increment with decreasing phase-correlation time, and the total stabilization of the instability occurs as a result of the quasilinear relaxation of the plasma distribution to the stable state. At the same time, the appearance of the phase-collapse effect in the Dupree model leads immediately to stabilization of the instability.

There are objections also against the applicability of the developed theory for a description of turbulence in a plasma without a magnetic field. The point is that the trajectory of the ions in turbulent fields is described by a random function with independent increments only in the limit of large oscillation amplitudes (see formula (A.2) of the Appendix). The latter does not take place in a plasma without a magnetic field, where the quasilinear relaxation of the plasma distribution stops the growth of the oscillation amplitude at a low level of the order of $kU_\sim = \omega_k$. For particles whose memory of the past path does not vanish, equations of the type (19) are already approximate and cannot describe the evolution of the turbulence correctly. Thus, in particular, they do not take into account the processes of resonant interactions in the system of plasma waves, which in the presence of a strong magnetic field turn out to be small,[4] and can here already make a contribution comparable with the effect of nonlinear phase collapse.[3)]

2. ION-CYCLOTRON INSTABILITY OF AN ANISOTROPIC PLASMA

Before we proceed to an investigation of the nonlinear stage of development of the instability in an

[2)] In [1] we considered the drift-cyclotron instability of a plasma in the presence of a weak long-wave turbulence. Because the square of the deviation of the ions in a weak magnetic field is proportional to the square of the time, the integration with respect to time along the average trajectory turned out in [1] to be equivalent of a thermal scatter of the ion velocities.

[3)] Such processes in themselves conserve the total momentum and the oscillation energy, and consequently, do not change the energy and momentum balance equation in the "wave plus particle" system investigated in [6].

anisotropic plasma, we recall the fundamental results of the linear theory.

Our problem has the simplest form under the following assumptions:

1. The electrons are cold, $T_e = 0$.

2. Only a small fraction of the plasma ions remains hot or, expressing ourselves more concretely, the densities of the hot and cold ionic components satisfy the condition:

$$\omega_{ph}^2 \ll \omega_{pc}^2 \left(\frac{m_e}{m_i}\right)^{1/2} (\omega/\omega_{ci})^{2/3}, \tag{20}$$

where $\omega_{ph,c}$ is the plasma frequency, determined from the density of the hot and cold components of the ions, respectively.

3. The thermal velocity of the hot ions is much lower than the phase velocity of the oscillations, and the distribution over the longitudinal velocities is Maxwellian.

The first of these assumptions makes it possible, in describing the electrons, to use the drift approximation and to neglect the thermal motion of the electrons. The second is a condition which when satisfied enables us to describe the electrons with the aid of linear equations even in the nonlinear stage of the instability development.

The dispersion equation for the ion-cyclotron oscillations in such a plasma is well known (see, for example, [4,5])

$$1 - \frac{\omega_{pe}^2}{\omega^2}\frac{k_\parallel^2}{k^2} - \frac{\omega_{ph}^2}{(\omega - n\omega_{ci})^2}\frac{k_\parallel^2}{k^2}\Gamma_n(\lambda_i)\left(1 + \frac{3}{4}\frac{k_\parallel^2 v_{\perp i}^2 T}{(\omega - n\omega_{ci})^2}\right) + \frac{\omega_{ph}^2}{\omega_{ci}^2\lambda_i}\Delta_n(\lambda_i)\frac{n\omega_{ci}}{\omega - n\omega_{ci}} - \frac{\omega_{pc}^2}{\omega_{ci}(\omega - n\omega_{ci})} = 0, \tag{21}$$

where

$$\Gamma_n(\lambda_i) = \int J_n^2\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right) f_{0i}\, d^3\mathbf{v},$$

$$\Delta_n(\lambda_i) = v_{\perp i}^2 \int J_n^2\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)\frac{\partial f_{0i}}{v_\perp \partial v_\perp} d^3\mathbf{v},$$

$T = v_{\parallel i}^2 / v_{\perp i}^2$ is the degree of the anisotropy of the ion distribution.

In this section we are not interested in effects of instability of the distributions with a cut-out "loss cone," so that we confine ourselves to the approximation in which only the first two terms of (21) are fundamental. The third term is found to be small if

$$\frac{k_\parallel^2}{k^2} > \frac{\gamma_\mathbf{k}}{\omega_{ci}}\frac{n}{\lambda_i}.$$

It turns out that the inequality coincides with the condition under which the quasilinear relaxation of the distribution with respect to the longitudinal velocities of the ions is more rapid than with respect to the transverse ones (compare the first two terms in the right side of the inequality (6)). Supplementing this inequality with the condition that the nonlinear phase collapse due to the longitudinal motion of the ions be small, we ultimately rewrite (6) in the form

$$\frac{\gamma_\mathbf{k}}{\omega_{ci}} > \frac{k_\parallel^2}{k^2} > \frac{\gamma_\mathbf{k}}{\omega_{ci}}\frac{n}{\lambda_i}. \tag{22}$$

When a magnetic trap is filled with a quiescent plasma, the most favorable conditions of development occur for the oscillations that have a maximum increment. Such are the electronic Langmuir oscillations, which are in resonant with one of the harmonics of the ion cyclotron frequency

$$\mathrm{Re}\,\omega = \pm\,\omega_{pe}\frac{k_\parallel}{k} \approx n\omega_{ci}. \tag{23}$$

The maximum growth increment of these oscillations and the limitation of the anisotropy under which the development is possible, is obtained from (21):

$$\delta\omega_\mathbf{k}^L + i\gamma_\mathbf{k}^L = n\omega_{ci}(0{,}5\varepsilon\Gamma_n)^{1/3}\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right), \tag{24}$$

$$T \leqslant \frac{\omega_{ph}^2\Gamma_n^{2/3}}{\omega_{ci}^2\lambda_i\varepsilon^{1/3}}; \qquad \varepsilon = \frac{\omega_{ph}^2}{\omega_{pe}^2}. \tag{25}$$

In this paper, for simplicity, we confine ourselves to a plasma with a filled "loss cone." Then the third term in (21) does not introduce any new unstable roots in the dispersion equation. To the contrary, it even exerts a stabilizing effect on the instability due to the anisotropy. In the entire region of plasma-parameter values of interest to us, this stabilizing influence can be neglected. The limits of the region of applicability of our theory (shown shaded in Fig. 1) is obtained by substituting in (22) the increment and the wavelength of the oscillations (24) and (25):

$$x^{-2} < \tau < x^{-1/4}, \tag{26}$$

where

$$\tau = T/\varepsilon^{1/3}, \quad x = \omega_{ph}/\varepsilon^{1/3}\omega_{ci}.$$

In this region of parameters, the dispersion equation (19) is valid. This equation becomes much simpler if account is taken of the fact that in the initially quiescent plasma the oscillation that increases most rapidly and to the largest amplitude is the one having the instability increment that is maximal attainable at the given plasma parameters. In view of the fact that the growth increment of the oscillations depends only on the modulus of the wave number ($\mathbf{k}_\perp$), the spectrum of the resultant fluctuations turns out to be axially symmetrical[4)] and has a clearly pronounced maximum in the vicinity of the maximal increment. Specifying the thermodynamic-equilibrium noise level at the initial instant of time, we can estimate for the width of the turbulence spectrum by using the linear growth of these fluctuations:

$$|\Phi_\mathbf{k}|^2 = |\Phi_{\mathbf{k}th}|^2 \exp\left\{2\Lambda - 2\gamma_\mathbf{k}^{-1}\frac{\partial^2\gamma_\mathbf{k}}{\partial k_\perp^2}\Lambda(k_\perp - k_\perp^{(0)})^2\right\}. \tag{27}$$

Here $\Lambda/\gamma_\mathbf{k}$ coincides in order of magnitude with the time of development of the instability, and the coefficient itself is determined by the logarithm of the ratio of the turbulence level to the thermal background and

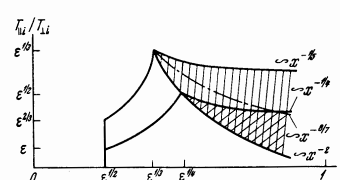


FIG. 1

4)We have already used this in the derivation of Eq. (19).

is of the order of magnitude of the "Coulomb" logarithm, i.e., $\Lambda \approx 10$.

By virtue of the foregoing, averaging over the spectrum of the turbulence with wave D_q in (19) becomes elementary, and the equation takes the form

$$1-\frac{\omega_{pe}^2}{\omega^2}\frac{k_\parallel^2}{k^2}-\omega_{ph}^2\frac{k_\parallel^2}{k^2}\int\frac{v_\parallel(\partial f_{0i}/\partial v_\parallel)J_n^2\,d^3\mathbf{v}}{\gamma_{\mathbf{k}}k^2D+\delta\omega_{\mathbf{k}}^2+k_\parallel^2v_\parallel^2}\times\left[1+\frac{i\delta\omega_{\mathbf{k}}}{k^2D}\right]+\frac{\omega_{ph}^2}{2\omega_{ci}^2\lambda_i}\frac{n\omega_{ci}}{k^2\tilde{D}}\tilde{\Delta}_n(\lambda_i)=0, \tag{28}$$

where $\delta\omega = \omega_n - n\omega_{ci}$ is the frequency shift and γ_k is the nonlinear increment, while the coefficients $\tilde{\Delta}_n$ and $\tilde{D}$ are modified somewhat:

$$\tilde{\Delta}_n = v_{\perp i}^2\int\frac{\partial f_{0i}}{v_\perp\partial v_\perp}\left|J_n\left(\frac{k_\perp v_\perp}{\omega_{ci}}\right)\right|\,d^3\mathbf{v},$$

$$\tilde{D}=\left\{\sum_{\mathbf{k}}c^2|\Phi_{\mathbf{k}}|^2/H_0^2\right\}^{1/2}.$$

Under the same assumptions concerning the plasma turbulence spectrum, we can supplement the equation for the amplitudes of the oscillations (28) by the quasilinear equation for the ion distribution function. The latter is obtained by the usual procedure of averaging over the phases of the oscillations of the nonlinear term in the Boltzmann equation (see [6,7])

$$\frac{\partial f_{0i}}{\partial t}=-\left\langle\frac{ie}{m}\sum_{\mathbf{k}}\mathbf{k}\Phi_{\mathbf{k}}^*\frac{\partial f_k^i}{\partial\mathbf{v}}\right\rangle$$
$$=\omega_{ci}^2\frac{\partial}{\partial v_\parallel}\frac{\gamma_{\mathbf{k}}k_\parallel^2D^2}{\gamma_{\mathbf{k}}k^2D+\delta\omega_{\mathbf{k}}^2+k_\parallel^2v_\parallel^2}\frac{\partial f_{0i}}{\partial v_\parallel}$$
$$+\omega_{ci}^4\frac{\partial}{\partial_\perp\partial v_\perp}\frac{\gamma_{\mathbf{k}}n^2D^2}{\gamma_{\mathbf{k}}k^2D+\delta\omega_{\mathbf{k}}^2+k_\parallel^2v_\parallel^2}\frac{\partial f_{0i}}{\partial_\perp\partial v_\perp}$$
$$+\omega_{ci}^4\left[v_\parallel\frac{\partial}{v_\perp\partial v_\perp}-\left(1-\frac{\omega}{n\omega_{ci}}\right)\frac{\partial}{\partial v_\parallel}\right]$$
$$\times\frac{n^2k_\parallel^2D/k^2}{\gamma_{\mathbf{k}}k^2D+\delta\omega_{\mathbf{k}}^2+k_\parallel^2v_\parallel^2}\left[v_\parallel\frac{\partial}{v_\perp\partial v_\perp}-\left(1-\frac{\omega}{n\omega_{ci}}\right)\frac{\partial}{\partial v_\parallel}\right]f_{0i}. \tag{29}$$

Let us subdivide the process of development of the instability into several stages in accordance with the investigated effect. In the linear regime, the oscillations that increase exponentially have a wavelength determined by the requirement that the Landau damping (25) by the ions be small:

$$n\approx(x^2/\tau_i)^{9/22}. \tag{30}$$

At an oscillation-amplitude level exceeding the value $D > \gamma_k^L/k^2$, the motion of the ions in the turbulent field takes them constantly out of resonance with the wave. At the same time, the quasilinear relaxation of the ions can still be neglected if the oscillation amplitude is not very large, namely, if it lies in the interval

$$1<\frac{k^2D}{\gamma_{\mathbf{k}}^L}\lesssim x\tau_i^{1/2}. \tag{31}$$

As follows from (28), a constant phase collapse leads to a decrease of the increment of the instability and increases somewhat the frequency shift compared with the linear increment

$$\delta\omega_{\mathbf{k}}=\sqrt{2}[\Delta_0+\Delta_1]\delta\omega_{\mathbf{k}}^L,\qquad \gamma_{\mathbf{k}}=\frac{2[\Delta_1^2-\Delta_0^2]}{3k^2D}\gamma_{\mathbf{k}}^{L^2}, \tag{32}$$

where

$$\Delta_0\approx\left(\pm\frac{k_\parallel}{k}\omega_{pe}-n\omega_{ci}\right)/\sqrt{2}\delta\omega_{\mathbf{k}}^L,$$

$$\Delta_1=2^{-1/3}\left\{\left(1+\sqrt{1-\frac{4}{27}\Delta_0^6}\right)^{1/3}+\left(1-\sqrt{1-\frac{4}{27}\Delta_0^6}\right)^{1/3}\right\}.$$

The expression for the instability increment coincides in structure with the formula obtained in the problem of the decay instability of a wave with a random phase [8]

$$\gamma_{\mathbf{k}}\sim\gamma_{\mathbf{k}}^{L^2}\tau_c, \tag{33}$$

where γ_k^L is the increment of the instability of the wave in the same amplitude with fixed phase, and τ_c is the time of the mixing of the phase of the wave. Comparing formulas (37) and (36), we arrive at the conclusion that the effective phase mixing time in a turbulent field decreases with increasing amplitude $\tau_c^{-1} \sim {}^3\!/\!_2 k^2D$, and therefore the instability increment decreases. However, the growth of the amplitude does not stop (see Fig. 2), and an instant is arrived at which the right-hand inequality of (31) is violated, and we should already take into consideration the decrease of the anisotropy of the ion temperature due to the quasilinear broadening of the ion distribution with respect to the longitudinal velocities.

Using (28), we can easily show that this effect is even incapable of stopping the instability. To be sure, with increasing amplitude the instability increment decreases much more rapidly than before:

$$\gamma_{\mathbf{k}}=\frac{\varepsilon^2n^6\omega_{ci}^6}{4k^2\tilde{D}k_\parallel^4}\int\frac{\partial f_{0i}}{v_\parallel\partial v_\parallel}|J_n|^2\,d^3\mathbf{v}\int\frac{\partial f_{0i}}{v_\parallel\partial v_\parallel}|J_n|\,d^3\mathbf{v}-\frac{\omega_{ph}^2n^2\omega_{ci}^2\tilde{\Delta}_n}{2\omega_{ci}^2\lambda_ik^2\tilde{D}} \tag{34}$$

$$\delta\omega_{\mathbf{k}}=0{,}5\varepsilon n^3\omega_{ci}^3k_\parallel^{-2}\int\frac{\partial f_{0i}}{v_\parallel\partial v_\parallel}|J_n|^2\,d^3\mathbf{v}. \tag{35}$$

As is easily seen from the quasilinear equation (29), the stationary state is reached by establishing a two-dimensional plateau in velocity space at the instant when the instability increment vanishes. In other words, in the stationary state there take place the relations:

$$\left(\frac{\partial}{v_\perp\partial v_\perp}+\frac{\delta\omega_{\mathbf{k}}}{n\omega_{ci}}\frac{\partial}{v_\parallel\partial v_\parallel}\right)\cdot f_{0i}(v_\parallel,v_\perp,t=\infty)=0, \tag{36}$$

$$\gamma_{\mathbf{k}}\{D,f_{0i}\}=0. \tag{37}$$

From the last relation we can estimate the degree of anisotropy of the plasma in the final state τ_f at a specified initial anisotropy τ_i

$$x\tau_f^5=[x\tau_i^5]^{6/11}\gg 1. \tag{38}$$

It follows therefore that even after relaxation, the plasma parameters remain in the instability region shown shaded in Fig. 1.

As follows from (29) and (34), the amplitude of the oscillations increases in proportion to $\sim t^{1/3}$ in the in-

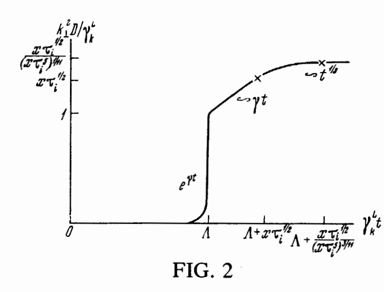


FIG. 2

terval where these equations are valid, namely at

$$x\tau_i^{1/2} < \frac{k^2 D}{\gamma_k^L} < x\tau_i^{1/2} / (x\tau_i^5)^{1/11}. \tag{39}$$

The described picture of development of the instability takes place only in an unbounded plasma. In a bounded plasma, the drift of the oscillations may terminate the instability much earlier than the natural quasilinear relaxation of the plasma distribution. The length of the plasma column in which this takes place lies in the following interval (see Fig. 2):

$$\Lambda < \frac{\gamma_k^L L}{\partial\omega/\partial k_\parallel} < \Lambda + \frac{x\tau_i^{1/2}}{(x\tau_i^5)^{3/11}}$$

Substituting here the parameters of the buildup oscillations from (24) and (25), we rewrite this in explicit form:

$$\Lambda \lesssim [x^2/\tau_i]^{8/11} L/\varepsilon^{-1/2} r_{ci} x \leqslant \Lambda + \frac{x\tau_i^{1/2}}{(x\tau_i^5)^{3/11}}. \tag{40}$$

Knowing the length of the plasma column, we are able to determine the maximum level of the turbulence on the basis of Fig. 2 from the maximum time of the oscillation drift.

In systems that are even shorter than allowed for by inequality (40)), the convective instability does not develop at all. In this case the plasma may turn out to be unstable with respect to perturbations with a smaller phase velocity.

The instability increment and the wavelength of the growing oscillations is determined from the dispersion equation (21)

$$\gamma_k^L = (\varepsilon\Gamma_n)^{1/2} n\omega_{ci}, \tag{41}$$

$$\frac{k_\parallel}{k_\perp} \approx \frac{n\omega_{ci}}{\omega_{pe}} \sqrt{\frac{\omega_{ph}^2 \Gamma_n}{\lambda_i T \omega_{ci}^2}} \geqslant \frac{n\omega_{ci}}{\omega_{pe}}. \tag{42}$$

The latter inequality guarantees here the possibility of neglecting the thermal motion of the ions and consequently the Landau damping due to the resonance with these ions. Following the previously described scheme, we estimate the wavelength of the most stable oscillation from Eq. (42)

$$n \approx [x_*^2/\tau_{*i}]^{1/8}, \tag{43}$$

where

$$x_* = \omega_{ph}/\varepsilon^{1/4}\omega_{ci}; \quad \tau_{*i} = T/\sqrt{\varepsilon}.$$

Substituting the values of γ_k, $k_\parallel$, and $k_\perp$ obtained by us from (41) and (42) into the condition (23), we find that the theory developed here is valid in the following plasma-parameter interval:

$$x_*^{-2} < \tau_{*i} < x_*^{-1/4}. \tag{44}$$

This region lies entirely within the previously obtained region (26), where our theory can describe the development of convective instability, and is shown doubly hatched in Fig. 1.

The nonlinear stage of development of an aperiodic instability with increment (41) turns out to be much shorter, and is described in the following equation:

$$1 + \varepsilon n^2 \omega_{ci}^2 \int \frac{v_\parallel \frac{\partial f_{0i}}{\partial v_\parallel} |J_n|^2 d^3v}{\gamma_k k^2 D + k_\parallel^2 v_\parallel^2} \approx 0. \tag{45}$$

We see that the nonlinear phase collapse of the oscillations leads to a decrease of the increment in accordance with the law (33):

$$\gamma_k \approx \frac{\gamma_k^{L^2}}{k^2 D} \Gamma_n^{-1} \int f_{0i} |J_n| d^3v. \tag{46}$$

The relaxation of the ion distribution is described as before by Eq. (29). Retaining in it the principal terms, we have

$$\frac{\partial f_{0i}}{\partial t} = \omega_{ci}^2 \frac{\partial}{\partial v_\parallel} \frac{\gamma_k k_\parallel^2 D^2}{\gamma_k k^2 D + k_\parallel^2 v_\parallel^2} \frac{\partial f_{0i}}{\partial v_\parallel}. \tag{47}$$

We see that the growth of the oscillation amplitude and the quasilinear relaxation stops simultaneously already following a small change of the plasma anisotropy $\Delta\tau \sim \tau_i$. Therefore, unlike the preceding case, the only relatively large interval of the amplitudes is the one in which the nonlinear phase collapse takes place:

$$1 < \frac{k^2 D}{\gamma_k^L} < x_* \tau_{*i}^{1/2}. \tag{48}$$

In short systems, the drift of the oscillations may stop the instability earlier than the quasilinear relaxation of the ion temperature, if the length lies in the interval

$$\Lambda < [x_*^2/\tau_{*i}]^{3/4} L/\varepsilon^{-1/4} r_{ci} x_* < \Lambda + x_* \tau_{*i}^{1/4}. \tag{49}$$

The quasilinear relaxation of the electrons is in this case immaterial under a condition that is somewhat less stringent than (20)

$$\omega_{pc}^2 > \omega_{ph}^2 [x_*^2/\tau_{*i}]^{1/4}. \tag{50}$$

CONCLUSION

It should be borne in mind that a number of limitations imposed on the plasma parameters in the present paper can be lifted without loss of rigor of the solution. For example, the requirement that particles be present in the ion "loss cone" is not obligatory. In long systems, the temperature anisotropy leads to a stronger instability than the cone instability in an appreciable part of the plasma parameter region considered above (this part of the region lies in Fig. 1 below the dash-dot curve). In short systems, the cone instability, being convective, is also less dangerous than the aperiodic instability, owing to the anisotropy of the temperatures, which can develop at a shorter length. It is therefore possible to hope that the obtained results will be useful in the analysis of experiment with the Phoenix II[9] and Alice[10] installations.

In experiments on cyclotron heating of ions in Stellarator C,[11] our assumption concerning the presence of particles in the loss cone is satisfied, but the electrons can no longer be regarded as cold. The latter imposes a limitation on the wavelengths and by the same token indirectly on the amplitudes of the growing oscillations. The contribution of the electron to the nonlinear interaction of the oscillations, which was neglected by us, can be taken into account within the framework of the theory of a weakly turbulent plasma (see, for example, [12]), for the instability becomes weak because of the nonlinear phase collapse in the "wave plus particle" system. The equation of weakly-turbulent plasma was already used earlier to estimate the amplitude of the electronic oscillations of a magnetized plasma with anisotropic distribution of the hot electron.[13] The same equations are applicable also to our case after substitution in them of the nonlinear increment of instability.

The most serious limitation of our theory follows from the instability of the narrow turbulence spectrum obtained by us. Indeed, according to Eq. (14), short-wave oscillations always have a tendency to increasing in scale, since during the nonlinear stage the increment of the long-wave oscillations γ_q exceed the increment for the short-wave oscillations γ_k

$$\gamma_{\mathbf{q}} \sim \gamma_{\mathbf{k}}\left(\frac{k^2}{q^2}\right)^3\left(\frac{\omega_{\mathbf{q}}}{\omega_{\mathbf{k}}}\right)^2 \quad \text{for} \quad \delta\omega_{\mathbf{q}} \sim \delta\omega_{\mathbf{k}}\frac{k^2}{q^2}\frac{\omega_{\mathbf{q}}}{\omega_{\mathbf{k}}} \ll q^2 D.$$

or using the last inequality for the most rapidly growing oscillation, we have

$$\gamma_{\mathbf{q}} \sim \gamma_{\mathbf{k}}\left(\frac{k^2 D}{\delta\omega_{\mathbf{k}}}\right)^{4/3}. \tag{51}$$

From this we obtain with the aid of (32) the time of destruction of the short-wave spectrum of the oscillations

$$\Delta t_1 \approx \Lambda^{3/4} / \gamma_{\mathbf{k}}{}^L.$$

After this time has elapsed, the oscillations either stop or else the narrow turbulent spectrum becomes essentially converted into a broad one.

In a number of modern traps[9,10] the short length of the plasma column causes a rapid drift of the oscillations from the system. This causes the instability to become manifest in the form of successive bursts of oscillations, each of which is incapable of ensuring relaxation of the states of the plasma to the fully stable one. The observed decrease of the oscillation frequencies during the process of the bursts of the instability turns out to be comparable in order of magnitude with the theoretically predicted shift of the frequency in the nonlinear stage of the instability (see (32)).

Our analysis shows that the development of the instability is accompanied by a decrease of the plasma anisotropy, but is not always capable of making an appreciable contribution to the particle loss.

APPENDIX

PROPERTIES OF PARTICLE TRAJECTORIES IN RANDOM FIELDS

The purpose of our appendix is to prove the properties (12) for the trajectories of the particles in a random electric field of the form (1). To this end, we break up all the plasma oscillations into two parts—a small number of trial waves, and the main "background" of the random oscillations. The properties (12) will be assumed proved for the process of wandering of the ions among the waves of the "background," so that our problem reduces to proving that the ion-trajectory distortion due to the presence of trial waves does not violate the properties (12).

The trajectory of the particles, as a functional of the fields of the trial oscillations, will be represented in the form of a series in powers of the amplitudes of these fields. Then a nonzero contribution to the average quantities of interest to us is made only by those terms of the series, which contain the amplitudes of the oscillations in the form of pairs of complex-conjugate amplitudes. Each of these terms of the series will be represented graphically in the form of a time axis with vertices arranged in it in chronological order, and corresponding trial-wave amplitudes. The sequence of the choice of the amplitudes will be represented with the aid of a wavy line joining the corresponding vertices.

By way of illustration, Fig. 3 shows the first nonvanishing term in the expansion of the average displacement in a series in the oscillation amplitudes. The corresponding term of the series is

$$\langle ik_\perp \delta r_\perp\rangle \sim -\sum_{\mathbf{q},\mathbf{p}}[\mathbf{k}_\perp\mathbf{q}_\perp][\mathbf{q}_\perp\mathbf{p}_\perp][\mathbf{p}_\perp\mathbf{q}_\perp][\mathbf{q}_\perp\mathbf{p}_\perp]\Gamma_{\mathbf{q}}\Gamma_{\mathbf{p}}$$

$$\times\left\langle \int_0^t e^{i\mathbf{q}\delta\mathbf{r}(t_1)}\,dt_1 \int_0^{t_1} e^{i\mathbf{p}\delta\mathbf{r}(t_2)}\,dt_2 \int_0^{t_2} e^{-i\mathbf{q}\delta\mathbf{r}(t_3)}\,dt_3 \int_0^{t_3} e^{-i\mathbf{p}\delta\mathbf{r}(t_4)}\,dt_4 \right. \tag{A.1}$$

where

$$\Gamma_{\mathbf{q}} = \frac{c^2}{H^2}|\Phi_{\mathbf{q}}{}^n|^2 J_n{}^2\left(\frac{q_\perp v_\perp}{\omega_{ci}}\right).$$

Thus, to each segment up to the extreme right vertex there corresponds its own vector product and its own integral with respect to the time.[5)]

It is obvious from Fig. 3 that among the internal momenta there is always one that is encountered an odd number of times in the product. It corresponds to the extreme right vertex (p in Fig. 3). The momenta of all the wavy lines terminating at internal points of the axis are encountered an even number of times. As a result, summation over the extreme right momentum causes the entire expression to vanish, thus proving the first of the properties in (12).

Let us calculate further the mean-square displacements of the ions in random fields. The general form of the possible diagrams of interest to us is shown in Fig. 4.

On the two last diagrams there are wavy lines joining points on the same time axis. Therefore such diagrams make no contribution to the necessary process, owing to the antisymmetry of the final expression with respect to the momentum of that wavy line, which goes over from some point of a given time axis to its extreme right point.

As to the second diagram of Fig. 4, it either represents a function that is odd with respect to the input

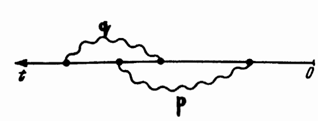
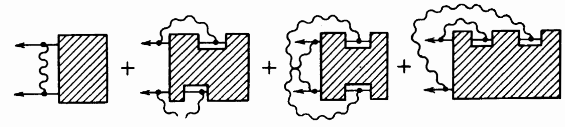


FIG. 3



[5)]If an entire bundle of wavy lines converges to a single vortex, it is necessary to correct the corresponding expression by a factor that results from the series expansion of the exponential.

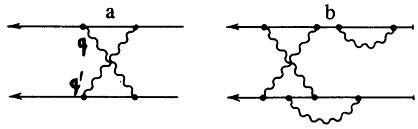


FIG. 5

momentum q (or q′) (Fig. 5a), or else contains an internal wavy line terminating in the extreme right point (Fig. 5b), and therefore the contribution from it vanishes after the summation. The contribution from the first diagram of Fig. 4 leads to the dependence of (12) on the time.

In perfect analogy we can obtain

$$\langle [\mathbf{k}, \delta\mathbf{r}(t+\tau) - \delta\mathbf{r}(t)]\,\mathbf{k}\delta\mathbf{r}(t)\rangle = \sum_{\mathbf{q},\,p} \frac{c^2[\mathbf{k}_\perp \mathbf{q}_\perp]^2 |\Phi_{\mathbf{q}}{}^p|^2}{H^2(q_\perp{}^2 D)^2} J_p{}^2\left(\frac{q_\perp v_\perp}{\omega_{ci}}\right). \qquad \text{(A.2)}$$

We see that the distortion of the trajectory due to the trial waves does not violate the property (12), and only changes the coefficient D.

All possible diagrams of the type shown in Fig. 4 correct quantitatively the integral in Eq. (A.2), owing to the effect of the diffusion of the ion trajectories in the trial fields. With allowance for the latter, expression (A.2) takes the form

$$\langle(\mathbf{k}\delta\mathbf{r})^2\rangle_{test} \approx \sum_{\mathbf{q}} \frac{2c^2[\mathbf{k}_\perp \mathbf{q}_\perp]^2 |\Phi_{\mathbf{q}}{}^p|^2 J_p{}^2 t^2}{H_0{}^2(q^2 D t + \langle(\mathbf{q}\delta\mathbf{r})^2\rangle_{test})}. \qquad \text{(A.3)}$$

Going to the limit, when all the waves are trial waves, we obtain the required expression for the coefficient D:

$$D \approx \left[\sum_{\mathbf{q},\,p} c^2|\Phi_{\mathbf{q}}{}^p|^2 H_0{}^{-2} J_p{}^2\left(\frac{q_\perp v_\perp}{\omega_{ci}}\right)\right]^{1/2}. \qquad \text{(A.4)}$$

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