TURBULENT HEATING OF IONS IN A PLASMA WITH A STRONGLY NONUNIFORM MAGNETIC FIELD

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High frequency ($\omega_{Hi} \ll \omega \ll \omega_{He}$) drift instabilities of a plasma with a strongly nonuniform magnetic field are considered. The ions are assumed to be hot ($T_i \gg T_e$) and unmagnetized in the oscillations under consideration. It is shown that the oscillation spectrum is continuous under these conditions. Spectral spreading of the oscillations is due to the thermal spread of the centrifugal and diamagnetic drift velocities. Anomalous ion heating and the anomalous resistance caused by small-scale turbulence in installations of the "Zeta" type are discussed.

In the present article, which is a continuation of [1–4], we discuss ion instability and ion heating produced by the equilibrium Larmor current in a plasma with a strongly inhomogeneous magnetic field. Such a situation arises in strong-current toroidal discharges.[5–7] Rudakov[3] called attention to the possibility of heating multiply-charged impurity ions under such conditions as the result of the development of a small-scale turbulence.

A discharge with a moderate longitudinal magnetic field was used to obtain a high-temperature dense plasma first with the Zeta setup, and then with a number of similar installations.[6,7] The "force-free" character of the discharge was noticed already in the first experiments. It was established that the plasma in such systems is highly unstable against a great variety of disturbance types, and has anomalously low conductivity[8–10] and anomalous diffusion across the containing magnetic field.[11,12] The temperature of the bulk of the ions exceeded the temperature of the electrons, $T_i > T_e$, and the thermal energy of the impurity ions turned out to be proportional to their charge number. This question was investigated systematically with the Alpha apparatus.[6,7,13]

We shall analyze below possible types of high-frequency drift instabilities under such conditions, establish their importance in the processes of heating of heavy-impurity ions, present and investigate a quasilinear equation determining the rate of heating of multiply charged impurities, and estimate their limiting temperature. In conclusion we analyze the collective-friction mechanisms that lead to an increase of the electric resistance of a plasma in a direction perpendicular to the magnetic field; we present a discussion of the experiment and obtain good agreement between theory and results of investigations with the Alpha apparatus.

1. MICROSCOPIC INSTABILITY OF AN INHOMOGENEOUS CURRENT PLASMA

In the analysis of the instabilities, we shall start throughout from a simple equilibrium configuration, that of a plasma cylinder. We assume that in the equilibrium state all the macroscopic quantities (charged-particle density n, ion temperature T_i, magnetic-field intensity H) depend on the radius. They can be determined from the following equilibrium conditions:

$$-\nabla p + \frac{1}{c}[\mathbf{jH}] = 0, \qquad (1)^*$$

$$-en\mathbf{E} + \frac{1}{c}[\mathbf{jH}] + \frac{en}{\sigma}\mathbf{j} = 0. \qquad (2)$$

Here $p = n(T_i + T_e)$ is the total gas-kinetic pressure in the plasma, σ is the conductivity, **E** is the constant electric field, and **j** is the density of the total current; the transverse component of this current, due to the Larmor and magnetic drifts, is

$$\mathbf{j}_\perp = \frac{c}{H^2}[\mathbf{H}, \nabla p]. \qquad (3)$$

We note that in the case when the gas-kinetic pressure is much smaller than the magnetic pressure, i.e., $\beta = 8\pi p/H^2 \ll 1$, it follows from (1) that $\nabla H^2/8\pi = (H^2/4\pi R)\mathbf{n}$, where R is the radius of curvature of the force lines and **n** is the unit vector of the principal normal to the force line. This means that, accurate to $\beta \ll 1$, the electric current flows along the force lines of the magnetic field ("force-free discharge"). Taking all these circumstances into account, let us investigate near the quasi-equilibrium state (1) and (2), by the perturbation method, the electrostatic oscillations of an inhomogeneous plasma $\mathbf{E} = \nabla\varphi$ with frequencies $\omega > \omega_{Hi}$ and wavelengths smaller than the Larmor radius ρ_{Hi} of the ions, but larger than the Larmor radius ρ_{He} of the electrons. We assume that the inhomogeneity of the magnetic field is of the same order as the inhomogeneity of the plasma.

In accordance with [4,14], for the case of hot ions $T_i \gg T_e$ and with allowance for the Coulomb collisions, we obtain the following dispersion equation:

$$\varepsilon = 1 + \frac{\omega_{pe}^2 k_\perp^2}{\omega_{He}^2 k^2}\left(1 + i\frac{\nu}{\omega}\right) - \frac{k_\parallel^2 \omega_{pe}^2}{k^2\omega(\omega + i\nu)} - \frac{1}{k^2\omega}\operatorname{div}\left(\frac{\omega_{pe}^2}{\omega_{He}}[\mathbf{k}, \mathbf{h}]\right)$$
$$+ \frac{\omega_{pi}^2}{k^2 v_{Ti}^2}\left\{1 + \sum_{n=-\infty}^{\infty}\int J_n^2\left(\frac{k_\perp v_\perp}{\omega_{Hi}}\right)\left[\frac{\omega}{v_\perp}\frac{\partial f_{0i}}{\partial v_\perp} - \frac{k_\perp}{M\omega_{Hi}}\frac{\partial f_{0i}}{\partial r}\right]\right.$$
$$\left.\times\frac{dv_\parallel v_\perp\, dv_\perp}{(\omega - n\omega_{Hi} - k_\parallel v_\parallel - \mathbf{ku}_1 - \mathbf{ku}_2)}\right\} = 0, \qquad (4)$$

in the derivation of which it was assumed that

$$\frac{v_{Te}}{\omega_{He}} \ll |\mathbf{k}|^{-1} \ll \frac{c}{\omega_{pe}}, \quad \frac{\omega}{k_\parallel} \gg v_{Te}, \quad \nu \ll |\omega|,$$

*$[\mathbf{jH}] \equiv \mathbf{j} \times \mathbf{H}$.

(the magnetic field is helical). Here

$$\omega_{p\alpha}^2=\frac{4\pi n e_\alpha^2}{m_\alpha},\quad \omega_{H\alpha}=\frac{e_\alpha H}{m_\alpha c},\quad v_{T\alpha}=\left(\frac{T_\alpha}{m_\alpha}\right)^{1/2},\quad \mathbf{h}=\frac{\mathbf{H}}{H},\quad \alpha=e,i$$

(we henceforth put throughout $m_i = M$ and $m_e = m$); $k_\perp$ and $k_\parallel$ are the projections of the wave vector on directions perpendicular and parallel to the magnetic field, respectively; u_1 and u_2 are the velocities of the diamagnetic and centrifugal drifts, respectively:

$$\mathbf{u}_1=\frac{v_\perp^2}{2\omega_{Hi}}\frac{\mathbf{n}}{R},\quad \mathbf{u}_2=\frac{v_\parallel^2}{\omega_{Hi}}\frac{\mathbf{n}}{R}$$

We shall consider only the case when $\beta \ll 1$.

In the dispersion equation (4) we can sum the contributions of several of the first cyclotron harmonics when one of the "non-magnetization conditions" is satisfied:

$$\operatorname{Im}\omega \gg \omega_{Hi},\quad k_\parallel v_{Ti}\gg\omega_{Hi},\quad \frac{k_\perp v_{Ti}^2}{2\omega_{Hi}}\frac{|\nabla H|}{H}\gg\omega_{Hi}. \tag{5}$$

The first two inequalities, generally speaking, may not be satisfied. For installations of the Zeta type, where $|\nabla H|/H \sim |\nabla n|/n$, the motion of the ions in oscillations of any frequency can be regarded as non-magnetized even if β is small, provided that the following relatively easy criterion is satisfied:

$$k_\perp\rho_{Hi}\gg\frac{1}{\rho_{Hi}}\frac{H}{|\nabla H|}. \tag{6}$$

This is connected with the fact that the overlap of the cyclotron resonances can result from the thermal spread of the velocities of the centrifugal and diamagnetic drifts. An investigation of this question is beyond the scope of the present article. It was carried out by the author in [15].

In this case, when $\omega < kv_{Ti}$, Eq. (4) takes the form

$$\varepsilon=1+\frac{\omega_{pe}^2k_\perp^2}{\omega_{He}^2k^2}\left(1+i\frac{\nu}{\omega}\right)-\frac{k_\parallel^2\omega_{pe}^2}{k^2\omega(\omega+i\nu)}-\frac{1}{k^2\omega}\operatorname{div}\left(\frac{\omega_{pe}^2}{\omega_{He}}[\mathbf{k},\mathbf{h}]\right)$$
$$+\frac{\omega_{pe}^2}{k^2v_{Ti}^2}\left[1+i\sqrt{\pi}\frac{\omega-\omega^*(1+\eta)}{kv_{Ti}}\left(1+\frac{\omega_{Hi}}{\omega_1^*}\exp\left\{-\left|\frac{\omega}{\omega_1^*}\right|\right\}\right)\right]=0, \tag{7}$$

where

$$\eta=\frac{d\ln T_i}{d\ln n},\quad \omega^*=\frac{cT_i}{eH}k_\perp\varkappa,\quad \omega_1^*=\frac{cT_i}{eH}k_\perp\varkappa_H,\quad \varkappa=\frac{d\ln n}{dr},$$
$$\varkappa_H=\frac{d\ln H}{dr}.$$

The obtained equation describes the high-frequency drift oscillations in a strong inhomogeneous magnetic field $|\nabla H|/H \sim |\nabla n|/n$. It should be noted that the instability of the plasma in a strongly inhomogeneous magnetic field at low frequencies $\omega < \omega_{Hi}$, in the long-wave part of the spectrum, was first considered in [16], and also in [17]. Our Eq. (7) makes it possible to advance further in the region of short-wave and high-frequency oscillations. In a weakly-inhomogeneous magnetic field, $|\nabla H|/H \ll |\nabla n|/n$, Eq. (4) yields, in particular, the drift-cyclotron instability investigated in [14,18]. To establish the limiting transition from the previously investigated oscillations, it would be necessary to add in (7) the residue connected with the longitudinal motion of the ions. We shall consider, however, almost transverse oscillations, $k_\parallel \ll k(m/M)^{1/2}\kappa\rho_{Hi}$, for which the longitudinal motion of the ions is of no importance.

For non-magnetized oscillations $\omega_{Hi} \ll \omega_1^*$, the frequency ω and the increment γ are determined by the expressions

$$\omega=\left(\omega^*-2\frac{cT_i}{eH^2}k_\perp\frac{dH}{dr}\right)\alpha,\quad \alpha=(1+k_\perp^2\rho_e^2)^{-1}, \tag{8}$$
$$\gamma=-k_\perp^2\rho_e^2\nu\alpha-\sqrt{\pi}\,\omega\alpha\frac{\omega-\omega^*(1+\eta)}{kv_{Ti}}, \tag{9}$$

where

$$\rho_e=(T_i/m)^{1/2}\omega_{He}^{-1},\quad \nu=4\pi ne^2/m\sigma.$$

The condition for the stability of such oscillations is

$$\left(2\frac{cT_i}{eH^2}k_\perp\frac{dH}{dr}+\omega^*\eta\right)\Big/kv_{Ti}>\frac{k_\perp^2\rho_e^2}{\sqrt{\pi}}\frac{\nu}{\omega}. \tag{10}$$

Let us explain the meaning of the foregoing inequality. In its left side are terms favoring the instability, and their sum is proportional to the transverse drift current $j_\perp$; consequently, the reason for the instability is the transverse current. The term in the right side of the inequality (10) causes damping of the oscillations as a result of collisions. Formally we can take ν to mean the effective frequency of any type of collisions, particularly those due to collective effects, and not only Coulomb collisions. This is connected with the fact that the Coulomb dissipation was taken into account in the derivation of (4) only via the friction force. Besides the collision damping, it would be necessary to take into account in the dispersion equation (4) also the Cerenkov absorption of the oscillations by the electrons. It is extremely small if the oscillations are almost transverse, $k_\parallel \ll k(m/M)^{1/2}\kappa\rho_{Hi}$.

The instability condition (10) contains the free parameter $k_\perp$, which can be eliminated by using the non-magnetization condition (6). Putting $k_\perp^{-1} \sim \kappa\rho_{Hi}^2$, $\kappa \approx \kappa_H$, we obtain the sufficient condition for the instability:

$$\varkappa\rho_{Hi}>(\nu/\omega_{He})^{1/3}. \tag{11}$$

Substituting in this inequality the characteristic parameters of the experiment[6,8-11,13] $\nu = \nu_{e,i} \sim (1-3)\times10^6$ sec^{-1}, $H \sim 10^3$ Oe, $n \sim 10^{13}$ cm^{-3}, $\kappa^{-1} \sim 20$ cm, $T_e \approx (10-20)$ eV, and $T_i \sim (50-100)$ eV, we obtain $\kappa\rho_{Hi} > 5\times10^{-2}$, which holds true even for protons. Strictly speaking, it is necessary to substitute the initial temperatures of the ions and the electrons in the condition (11) for the instability limit, and then condition (11) becomes more stringent, but at the same time the non-magnetization condition can be satisfied only for short waves, for which $k_\perp\rho_e > 1$. Taking into account in inequality (10) the fact that the quantity $k_\perp\rho_e$ is finite, and maximizing with respect to $k^{-1} \sim \kappa\rho_{Hi}^2$, we would obtain a less stringent criterion of instability, which can be satisfied for lower temperatures than in (11), namely

$$\varkappa\rho_{Hi}>\left(\frac{\nu}{\omega_{He}}\frac{m}{M}\right)^{1/5}. \tag{11'}$$

In the region of longer-wavelength oscillations, when the non-magnetization criterion is not satisfied, the influence of the cyclotron harmonics becomes significant. An investigation of this limiting case with $k_\perp\rho_{Hi} < (\rho_{Hi}\kappa)^{-1}$ shows that a large term proportional to $(k_\perp\rho_{Hi}^2\kappa)^{-1}$ appears in the imaginary part for the ions. This is connected formally with the residue at the zeroth cyclotron harmonics. The frequencies of the unstable oscillations do not exceed in this case the cyclotron frequency, $\omega < \omega_{Hi}$, and the instability condition

$$\left(\frac{2cT_i}{eH^2}k_\perp\frac{dH}{dr}+\omega^*\eta\right)\exp\left\{-\left|\frac{\omega}{\omega_1^*}\right|\right\}\Big/ kv_{Ti}k_\perp\varkappa\rho_{Hi}^2 > \frac{\nu}{\omega^*}\frac{k_\perp^2\rho_e^2}{\sqrt{\pi}} \quad (12)$$

is formally analogous to the condition (10). Eliminating the free parameter $k_\perp$ with the aid of the inequality $\rho_{Hi}^2 k_\perp\kappa < 1$, we obtain a sufficient instability condition that coincides with (11).

2. HEATING OF HEAVY IMPURITIES AND COOLING OF PROTONS

The non-magnetization criterion (6) is particularly easy to satisfy for impurity ions. It is satisfied not only for the analyzed oscillations, but also for oscillations with longer wavelengths, which have been investigated experimentally.[6,9] In such oscillations, the turbulent heating considered earlier in [1-4] becomes possible. We present an equation describing the heating of the ions as the result of Cerenkov-absorption processes—radiation of oscillations of the type under consideration:[4]

$$\frac{\partial f_0}{dt}+c\frac{[\mathbf{EH}]}{H^2}\nabla f_0=\sum_{\mathbf{k}}\left(\frac{\omega}{k_\perp v_\perp}\frac{\partial}{\partial v_\perp}-\frac{\nabla}{\omega_{Hi}}\right)D_{\mathbf{k}}\left(\frac{\omega}{k_\perp v_\perp}\frac{\partial f_0}{\partial v_\perp}-\frac{\nabla f_0}{\omega_{Hi}}\right), \quad (13)$$

$$D_{\mathbf{k}}=\frac{\omega_{pi}^2|\mathbf{E_k}|^2}{4nMk_\perp v_\perp}.$$

The index designating the type of impurity is omitted from now on; E_k is the Fourier component of the amplitude of the electric-field oscillations, in terms of which the energy density of the excited noise is expressed in the following manner:

$$W_{\mathbf{k}}=\frac{|\mathbf{E_k}|^2}{8\pi}\omega_{\mathbf{k}}\frac{\partial\varepsilon}{\partial\omega_{\mathbf{k}}}=N_{\mathbf{k}}\omega_{\mathbf{k}}, \quad (14)$$

where N_k denotes the number of waves in $(\mathbf{k}, \mathbf{r})$ space. The rate of growth of the noise before it becomes limited by nonlinear effects is determined entirely by the linear increment of the buildup of the oscillations (9), the ion part of which has the following form:

$$\gamma_i=\pi\omega\int\left(\frac{\omega}{k_\perp v_\perp}\frac{\partial}{\partial v_\perp}-\frac{\nabla}{\omega_{Hi}}\right)f_0\,dv_\perp. \quad (15)$$

Equation (13) has been derived under the assumption that $\omega \ll kv_{Ti}$, $k_\parallel \ll k$, the motion of the ions in the oscillations is not magnetized, and the characteristic time of heating is much longer than $2\pi/\omega_{Hi}$. With the aid of this equation it is possible to show that, owing to the quasielastic scattering by oscillations of the type under consideration, the criterion of which is the inequality $\omega \ll kv_{Ti}$, most ions (protons) become cooled and the heavy components of the impurities are heated. To this end, and to clarify the picture of the process, we calculate with the aid of (13) the following moments of the distribution function f_0:

$$n=\int f_0 v_\perp\,dv_\perp, \quad nT_i=\frac{1}{2}\int Mv_\perp^2 f_0 v_\perp\,dv_\perp.$$

Carrying out the corresponding integration with respect to the velocity in the initial equation, we obtain ultimately

$$\frac{\partial n}{\partial t}+\operatorname{div}\frac{nc}{H}\left[\left(\mathbf{E}-\frac{1}{ne}\sum_{\mathbf{k}}\gamma_i\mathbf{k}N_{\mathbf{k}}\right),\mathbf{h}\right]=0, \quad (16)$$

$$\frac{\partial}{\partial t}(nT_i)+c\frac{[\mathbf{EH}]}{H^2}\nabla(nT_i)=\sum_{\mathbf{k}}\gamma_i\omega_{\mathbf{k}}N_{\mathbf{k}}-\operatorname{div}\mathbf{q}_\perp, \quad (17)$$

$\mathbf{q}_\perp$ is the heat flux due to the quasielastic scattering of the ions by the oscillations excited as the result of the instability. It can be obtained by multiplying the ion drift velocity

$$-\frac{c}{enH}\left[\sum_{\mathbf{k}}\gamma_i\mathbf{k}N_{\mathbf{k}},\mathbf{h}\right],$$

produced under the influence of the noise, by the density of the internal energy nT_i:

$$\mathbf{q}_\perp=\sum_{\mathbf{k}}\frac{\pi e^2|\mathbf{E_k}|^2[\mathbf{k},\mathbf{h}]}{2M\omega_{Hi}k_\perp^2}\int v_\perp^2\,dv_\perp\left(\frac{\omega}{k_\perp v_\perp}\frac{\partial}{\partial v_\perp}-\frac{\nabla}{\omega_{Hi}}\right)f_0. \quad (18)$$

The heat flux $\mathbf{q}_\perp$ is directed along the radius of the plasma column towards the decreasing temperature gradient, and leads to an effective cooling. The restraining mechanism, which will be shown below to determine the thermal equilibrium, is the convective heat transfer $cH^{-2}[\mathbf{E}\times\mathbf{H}]\nabla(nT_i)$ towards the center of the plasma column.

Equation (16) expresses the continuity law in a turbulent plasma. It is seen from it that the ions experience additional drift under the influence of the turbulent friction force

$$\mathbf{R}_{ti}=-\sum_{\mathbf{k}}\gamma_i\mathbf{k}N_{\mathbf{k}}, \quad (19)$$

which results from scattering of the ions by the noise.

Equation (17) expresses the law of energy transport for the ions. It is seen from it that the internal energy of the ions changes as a result of a number of factors. The increase of the energy depends on the "inelastic collisions" of the ions with the oscillations, and the rate of heating due to this cause is

$$Q_i=\sum_{\mathbf{k}}\gamma_i\omega_{\mathbf{k}}N_{\mathbf{k}}.$$

The cooling of the ions, as already noted above, is due to the heat flux directed across the force lines of the magnetic field. In addition, it is necessary to take into account in the particle heat balance the fact that the transverse component of the electric field $\mathbf{E}_\perp$, which exists under the conditions of the experiment (6), causes an electric drift that draws the particles towards the center of the plasma column. This effect is governed by the second term in (17).

If we substitute in (17) the values of the frequency ω from (8) and of the increment γ from (15), and integrate with respect to the velocities, then the right side of the heat-balance equation (17) turns out to be negative

$$\frac{\partial}{\partial t}(nT_i)+c\frac{[\mathbf{EH}]}{H^2}\nabla(nT_i)<0. \quad (20)$$

This means that the protons become cooled. The cooling process continues until the second term balances the heat flux. It is therefore impossible to explain the heating of the protons on the basis of (13); this equation can serve only as a basis for their cooling. However, if it is assumed that the protons become heated, then their limiting temperature can be established from other conditions.

A condition for the limiting proton temperature may be the instability criterion (11). Indeed, the time ($\tau \approx \gamma^{-1} \sim 10^{-5}$ sec) of the development of the drift instability, which determines the rate of plasma transport to the walls, turns out to be much smaller than the skin time $\sigma(r_0/c)^2 \sim 10^{-4}$ sec, which determines the rate of plasma heating. Under these conditions, the excess of pressure over the stable value will be immediately "dropped" and the proton temperature will be determined by the equation

$$T_i = Mr_0^2\omega_{Hi}^2(\nu / \omega_{He})^{2/3}. \tag{21}$$

A similar situation was indicated in [19] for the case of development of large-scale turbulence under conditions of strong-current discharge in the Zeta apparatus.

Substituting in (21) the experimental data $r_0 \sim 20$ cm, $H \sim 10^3$ Oe, and $\nu \sim (10^6–10^7)$ sec^{-1}, we obtain $T_i \sim (80–150)$ eV. This estimate is in satisfactory agreement with the experimental value.

For heavy impurities, the rate of heating due to the Cerenkov absorption of the oscillations in the considered frequency interval may greatly exceed their diffusion losses, because we have for these impurities $\omega > k_\perp v_{Ti}^2 \kappa \omega_{Hi}^{-1} = kv_{dr}^{\alpha}$; for protons, to the contrary, we have $\omega < kv_{dr}$. We have in mind here the frequency ω from the spectrum of the turbulent pulsations registered in the experiment.[6] From the condition for the competition between the heating and cooling process, resulting from the radial diffusion, the particle energy does not exceed the value

$$ZeHr_0\omega / kc,$$

where ω/k is the characteristic phase velocity of the oscillations, which determine the value of the diffusion coefficient in equation (13). If we substitute in this formula the values of the frequency $\omega \sim 10^6$ sec^{-1} and wavelength $\lambda = 2\pi/k \approx 5$ cm which are characteristic of the oscillations in the discussed experiments (see, for example, [6,8,13]), and also $H \sim 2 \times 10^3$ Oe and $r_0 = 20$ cm, then we get

$$T_i \lesssim Z\cdot 10^2 \text{ eV}. \tag{22}$$

This estimate agrees with the experimental data. It is easy to establish here that the non-magnetization criterion

$$\frac{k_\perp c T_i}{ZeH^2}|\nabla H| \gg \frac{ZeH}{Mc}$$

is satisfied with a large margin for heavy impurities in oscillations with $\lambda = 2\pi/k \approx 5$ cm.

3. TURBULENT RESISTANCE

Owing to the development of the considered microinstabilities, a certain quasistationary level of the turbulent pulsations is established. The noise will first increase exponentially, in accordance with (15), after which nonlinear processes come into play and cause a limitation on the noise level. The scattering of the electrons and ions by the noise leads to a number of macroscopic effects, to a change in the average values of the plasma parameters. To clarify the picture of the phenomenon, let us consider the hydrodynamic equations averaged over the oscillations. For ions, these equations can be derived with the aid of the moments of the distribution function f_0 (see (16) and (17)). Thus, it can be established that the ions are acted upon by a turbulent friction force $\mathbf{R}_{fi} = -\sum_k \gamma_k k N_k$. The heat released in the ion gas as the result of scattering by the noise is $Q_i = \sum_k \gamma_i \omega_k N_k$. From the condition for the conservation of the total momentum, it can be shown that in the steady state the electrons are acted upon by a friction force $\mathbf{R}_{fe} = -\mathbf{R}_{fi}$. The momentum transfer to the electrons can be the result of attenuation of the oscillation by Coulomb collisions and of nonlinear effect of scattering and absorption of oscillations by the electrons. In analogy we can establish that the release of heat in the electrons is the result of nonlinear processes is equal to $Q_e = \sum_k \gamma_e(N) N_k \omega_k$, where $\gamma_e(N)$ is the nonlinear damping decrement. Taking the foregoing into account, we write down the equations expressing the momentum and energy balance for a gas of electrons and ions:

$$-\nabla(nT_i + nT_e) + \frac{1}{4\pi}[\operatorname{rot}\mathbf{H}, \mathbf{H}] = 0, \tag{23}$$

$$ne\left(\mathbf{E} - \frac{\mathbf{R}_f}{ne}\right) = \frac{1}{c}[\mathbf{jH}] + \frac{ne}{\sigma}\mathbf{j}, \tag{24}$$

$$n\dot{T}_e = \mathbf{j}\left(\mathbf{E} - \frac{\mathbf{R}_f}{ne}\right) + \sum_k \gamma_e N_k \omega_k, \tag{25}$$

$$n\dot{T}_i = -\operatorname{div}\mathbf{q}_\perp + \sum_k \gamma_i N_k \omega_k. \tag{26}$$

It should be noted that the first two equations were already discussed in the first section as the conditions for equilibrium when $\mathbf{R}_f \equiv 0$. In the presence of noise, the action of the external electric field $\mathbf{E}_\perp$ decreases by an amount $\mathbf{R}_f/ne$. Equation (24) expresses Ohm's law in a turbulent plasma. With the aid of this equation we can estimate the turbulent conductivity σ_f. The last two equations express the energy balances for the electrons and ions. When these are added, we obtain the energy balance for the plasma as a whole:

$$\frac{\partial}{\partial t}\left(nT_e + nT_i + \sum_k \omega_k N_k\right) = -\operatorname{div}\mathbf{q}_\perp + \mathbf{j}\left(\mathbf{E} - \frac{\mathbf{R}_f}{ne}\right). \tag{27}$$

On the left side is the rate of change of the density of the internal and noise energies. The energy decreases as the result of the heat flux $\mathbf{q}_\perp$, the energy increment is ensured by the work performed on the total current $\mathbf{j}_\perp$ by the "acting" electric field $(\mathbf{E} - \mathbf{R}_f/ne)_\perp$.

In the steady state, the turbulent friction force $\mathbf{R}_f$ compensates for the action of the electric force $ne\mathbf{E}_\perp$. Physically this phenomenon is understandable. Indeed, the turbulent friction force leads to a drift of the particles towards the wall chambers. The constraining mechanism is the drift of the particles towards the axis of the plasma column under the influence of the components of the electric field $\mathbf{E}_\perp$. When these drifts are equal, the particles are not displaced on the average and a stationary state is established, in which the noise level has exactly the value necessary to satisfy the condition

$$\mathbf{R}_f = en\mathbf{E}_\perp.$$

If it turns out that $\mathbf{R}_f > ne\mathbf{E}_\perp$. Then as the result of the fact that the rate of heating of the protons is much smaller than the rate of heat escape due to the radial diffusion, the excess of pressure over equilibrium will be "dropped" and the system will arrive at the stability boundary, determined by the condition (21):

$$T_i = Mr_0^2\omega_{Hi}^2(\nu / \omega_{He})^{2/3}.$$

The noise level decreases to the value determined by (28). If it turns out that $R_f < neE_\perp$, then the electric drift exceeds the drift under the influence of R_f. This will cause the pressure towards the column axis to exceed the equilibrium values (21). In this case the rate of growth of the noise increases and the equality (28) is restored. Thus, the system will adjust itself to an equilibrium state that is determined completely by the conditions (21) and (28).

A simultaneous solution of Eqs. (21) and (28) makes it possible to establish the limiting temperature of the protons and the noise level. However, it may turn out that the noise level will not suffice to satisfy Eq. (28), in other words, the instability in question will not ensure the required level. Then the equilibrium state (21), (28) becomes impossible. We shall show, however, that under the experimental conditions the noise will suffice in the case of development of the instability in question. To this end, let us estimate the turbulent friction force

$$\mathbf{R}_f = \sum_{\mathbf{k}} \gamma_e \mathbf{k} N_{\mathbf{k}} = \sum_{\mathbf{k}} \frac{\mathbf{k}}{\omega} \gamma_i W_{\mathbf{k}}. \tag{29}$$

To calculate this force we must know the noise energy density, which, according to conditions (7) and (14), equals

$$W_{\mathbf{k}} = \frac{|\mathbf{E}_{\mathbf{k}}|^2}{8\pi} \omega_{\mathbf{k}} \frac{\partial \varepsilon}{\partial \omega_{\mathbf{k}}} = \frac{mnv_\sim^2}{2} \frac{1}{(k\rho_e)^2}, \tag{30}$$

where $v_\sim = cE_k/H$ as the amplitude of the electron-velocity oscillations. This quantity does not exceed the phase velocity of the unstable oscillations within the framework of the developed theory, which has the same order of magnitude as the drift velocity (v_{dr})

$$v_\sim \leqslant \omega / k \sim v_{\text{дp}} \approx \varkappa \rho_{Hi} v_{Ti}.$$

Therefore the noise energy density does not exceed

$$W_{\mathbf{k},\, max} < \frac{nMv_{Ti}^2}{2} \left(\frac{\varkappa}{k}\right)^2. \tag{31}$$

The instability increment γ_i, in accordance with condition (9), is of the order of

$$\gamma_k \sim kv_{\text{дp}}^2 / v_{Ti} = k(\varkappa\rho_{Hi})^2 v_{Ti}. \tag{32}$$

Substituting (31) and (32) in (29) and maximizing with respect to the wave vector $\mathbf{k}_\perp$, using the non-magnetization condition (6), we obtain the maximum friction force attainable for the given instability

$$R_{f,\, max} = \sum_k \frac{nMv_{Ti}^2}{2} \varkappa\rho_{Hi} \frac{\varkappa^2}{k} \sim \frac{nMv_{Ti}^2}{2} \varkappa(\varkappa\rho_{Hi})^3. \tag{33}$$

Let us compare its numerical value with the electric force for the following characteristic experimental parameters: $\kappa^{-1} \sim 20$ cm, $\rho_{Hi} \sim 1$ cm, $v_{Ti} \sim 10^7$ cm/sec, and $E_\perp \sim 0.01$ V.

As a net result we find that in a wide range of variation of the presented parameters we get the inequality

$$\mathbf{R}_{f,\, max} > \mathbf{E}_\perp ne.$$

This means that there will be enough noise to establish an equilibrium state determined by Eqs. (21) and (28).

It is now easy to estimate the turbulent conductivity. It is determined from the following relation:

$$\frac{nej_\perp}{\sigma_j} = R_{f,\, max} = \frac{nMv_{Ti}^2}{2} \varkappa(\varkappa\rho_{Hi})^3$$

and turns out to be

$$\sigma_f = \frac{ne^2}{m\omega_{He}(\varkappa\rho_{Hi})^3}. \tag{34}$$

Its calculated value, corresponding to formula (34), is close to the experimental value $\sigma_f \sim 10^{15}$.

Thus, the general picture of the phenomenon is as follows: In the case of a strong-current discharge, at small values of β, a "force-free" magnetic-field configuration is established. The inhomogeneity of the magnetic field is just as large in this case as the inhomogeneity of the density. Under these conditions, high-frequency drift instabilities are excited, in which the ions are not magnetized. The development of the instability leads to establishment of a quasistationary turbulence level. The ions experience quasielastic scattering by the turbulent pulsations, and this leads to a number of macroscopic effects. First, the heavy impurities, for which the heating rate exceeds the cooling rate due to radial diffusion, become heated to limiting temperatures that depend on the charge number of the impurity. The protons, which constitute the bulk of the ions, are cooled as a result of scattering by the noise. However, the limiting proton temperature is established at a level corresponding to the instability of the oscillations. The heating of the protons cannot be explained on the basis of the considered instabilities. One cannot exclude the possibility that this heating is due to other types of instability.

The energy consumed in heating the impurities is equal to the work performed on the transverse current by the "acting" electric field in the plasma ($\mathbf{E} - \mathbf{R}_f/ne$). In the steady state, the turbulent friction force balances the electric force and the heating stops. This determines a certain stable equilibrium state, in which an increased electric resistance of the plasma is established in a direction transverse to the magnetic field. The estimated turbulent conductivity is close to the experimental value.

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