INTERACTION OF INTERSECTING LIGHT BEAMS

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The nonlinear interaction of two intersecting light waves of one and the same frequency, leading by means of the Kerr effect and electrostriction to the appearance of a part of the dielectric permittivity tensor which changes rapidly in space, causes a change of the phase velocities of these waves. The contribution of this effect is of the same order of magnitude as the contribution from the ordinarily observed spatially constant change of the dielectric permittivity tensor. As a consequence of this, in connection with the observation of double refraction induced by a powerful light beam, different results are obtained, depending on whether the frequencies of the strong pumping light beam and the weak probing light beam are the same or different.

IN articles[1-3] the question of the interaction of two oppositely-directed light waves of one and the same frequency is theoretically investigated, taking into account the appearance of a spatially-varying part of the dielectric permittivity tensor (an "optical grating"). The scattering of a light beam by an optical grating produced by an intense standing light wave of half the frequency was observed experimentally in[4]. Waves propagating at an angle to one another were considered in[5]. The change of the phase velocities of two intersecting waves of the same frequency will be studied below for different relative polarizations as a consequence of the formation of an optical grating via the mechanisms of electrostriction and the Kerr effect. In order of magnitude, the effect should be the same as that stipulated by the spatially constant change of the optical density due to the presence of the light field. In this connection, different results are obtained for a study of double refraction induced by an intense light wave depending on whether the weak probing light beam has the same or a different frequency than the intense pumping light beam. Perhaps this property explains the apparent contradiction between the work of Veduta and Kirsanov[6] and that of Paillette.[7] Later the following questions are discussed: the possible role of the formation of an optical grating in the self-focusing of light,[8,10] the interaction of two light waves of somewhat different frequencies (see[11,12]), and the violation of Fresnel's law for the reflection of light waves from a boundary.

First let us consider the propagation in an isotropic medium of two intersecting light waves of one and the same frequency (steady-state regime). The equations of electrodynamics give

$$\partial^2 \mathbf{D} / \partial t^2 + c^2 \operatorname{rot} \operatorname{rot} \mathbf{E} = 0. \tag{1}$$

For the electric displacement vector we have

$$D_i = \varepsilon_0 E_i + \delta\varepsilon_{ij} E_j, \tag{2}$$

where $\delta\epsilon_{ij}$ denotes the change of the index of refraction caused by the presence of light waves in the medium. From symmetry considerations we assume that

$$\delta\varepsilon_{ij} = A\overline{E_i E_j} + B\delta_{ij}\overline{E_k^2}, \tag{3}$$

where the upper bar denotes averaging over the rapid optical vibrations.

First let us consider the interaction of two beams of light which have one and the same direction for the electric field (for example, along the z axis). Then $\delta\epsilon_{zz} = (A + B)\overline{E}^2$. If the frequencies of both interacting waves coincide and are equal to ω, then the solutions of Eqs. (1)–(3) can be written in the form of a series

$$E = E_1 \cos(\omega t - \mathbf{k}_1\mathbf{r}) + E_2 \cos(\omega t - \mathbf{k}_2\mathbf{r}) + E_3 \cos(\omega t - 2\mathbf{k}_1\mathbf{r} + \mathbf{k}_2\mathbf{r}) + E_4 \cos(\omega t - 2\mathbf{k}_2\mathbf{r} + \mathbf{k}_1\mathbf{r}) + \ldots \tag{4}$$

A calculation gives (in the case of the Kerr effect for the present polarization, see also[12])

$$\mathbf{k}_1^2 = \varepsilon_0 \frac{\omega^2}{c^2}\left\{1 + \frac{A+B}{2\varepsilon_0}(E_1^2 + E_2^2) + \frac{A+B}{2\varepsilon_0}E_2^2\right\},$$

$$\mathbf{k}_2^2 = \varepsilon_0 \frac{\omega^2}{c^2}\left\{1 + \frac{A+B}{2\varepsilon_0}(E_1^2 + E_2^2) + \frac{A+B}{2\varepsilon_0}E_1^2\right\}. \tag{5}$$

In the derivation essentially two conditions are used:

$$|\mathbf{k}_2 - \mathbf{k}_1| L \gg 1, \quad (\mathbf{k}_2 - \mathbf{k}_1)^2 \gg (\omega^2/c^2)\,|(A+B)E_i E_j|, \tag{6}$$

where L is the characteristic size of the interaction region, and i and j take the values 1 and 2. We emphasize that the last term in each of the equations (5) represents the contribution to the phase velocity because of the presence of the optical grating (i.e., the part of the dielectric permittivity tensor which changes in space; to the first approximation the law of variation has the form $\cos(\mathbf{k}_1 \cdot \mathbf{r} - \mathbf{k}_2 \cdot \mathbf{r})$).

From Eqs. (1)–(3) it is not difficult to trace how terms appear in (5) which are associated with the formation of the optical grating. The grating obtained from Eq. (3) leads to a mutual scattering of two interacting waves at exactly the Bragg angles (for a more detailed discussion, see[1,5]). However each of the waves gives a contribution to the other wave with a phase shift of exactly $\pi/2$ with respect to the phase of the resulting radiation of the wave. Then the intensities of both beams remain unchanged, and mutual scattering only leads to an increasing phase shift which is equivalent to a change of the phase velocity. In this connection, of course, the phase velocity of the weaker wave is changed more.

In this connection it is necessary to note that the assertion contained in[1,5] about the possibility of equalizing the energies of two intersecting light waves of the same frequency without taking account of active nonlinearities of the type of nonlinear absorption[13] is erroneous. A case when an increase of the energy of one of the waves nevertheless occurs will be considered below.

In analogy with the calculation cited above, it is also easy to obtain the solution for the case of intersecting light waves of one and the same frequency when $\mathbf{E}_1 \perp \mathbf{E}_2$. We have

$$k_1^2 = \varepsilon_0 \frac{\omega^2}{c^2}\left\{1 + \frac{A+B}{2\varepsilon_0}E_1^2 + \frac{B}{2\varepsilon_0}E_2^2 + \frac{A}{4\varepsilon_0}E_2^2\right\},$$
$$k_2^2 = \varepsilon_0 \frac{\omega^2}{c^2}\left\{1 + \frac{A+B}{2\varepsilon_0}E_2^2 + \frac{B}{2\varepsilon_0}E_1^2 + \frac{A}{4\varepsilon_0}E_1^2\right\}. \quad (7)$$

But again the last terms in each of the equations (7) describe the contribution from the optical grating.

Let us specifically consider the Kerr effect. Then[14,15]

$$A = -3B > 0. \quad (8)$$

From here we obtain the following expressions for the phase velocities of propagation of a weak wave through the field of an intense wave $\mathbf{E}_1$:

$$k_2^2 = \varepsilon_0 \frac{\omega^2}{c^2}\left\{1 + \frac{2A}{3\varepsilon_0}E_1^2\right\} \text{ for } \mathbf{E}_1 \parallel \mathbf{E}_2,$$
$$k_2^2 = \varepsilon_0 \frac{\omega^2}{c^2}\left\{1 + \frac{A}{12\varepsilon_0}E_1^2\right\} \text{ for } \mathbf{E}_1 \perp \mathbf{E}_2. \quad (9)$$

In the case of the propagation of a weak wave of extremely different frequency through an intense light field (i.e., when an optical grating is not formed) we have

$$k_2^2 = \varepsilon_0 \frac{\omega^2}{c^2}\left\{1 + \frac{A}{3\varepsilon_0}E_1^2\right\} \text{ for } \mathbf{E}_1 \parallel \mathbf{E}_2,$$
$$k_2^2 = \varepsilon_0 \frac{\omega^2}{c^2}\left\{1 - \frac{A}{6\varepsilon_0}E_1^2\right\} \text{ for } \mathbf{E}_1 \perp \mathbf{E}_2. \quad (10)$$

From a comparison of Eqs. (9) and (10) we see that the change of the phase velocity associated with the propagation of a weak light wave through the field of an intense wave depends on whether their frequencies are identical or different. Possibly this is precisely the reason for the difference between the results of articles[6] and[7]. In[7] the change of the phase velocity was investigated for a light beam from a gas laser which passed through a liquid which had been exposed to intense light from a rubidium laser. The results as a whole are described well by the formulas of the orientational theory. In[6], the phase velocity of a light beam passing at an angle of 90 deg to an intense light beam from the same laser was measured. Here, of course, it would be impossible to explain the results by formulas (10), which led the authors to a conclusion about the decisive role of a mechanism differing from the Kerr effect. Formulas (9) qualitatively describe the results of[6] better (the sign of the change of the index of refraction for both polarizations now agree); however quantitative agreement is not obtained. This problem apparently requires special theoretical and especially experimental investigation. At any rate it is quite clear that in[6] it is necessary to take the formation of an optical grating into account.

Now let us turn our attention to the following interesting property. The formation of an optical grating in a standing light wave that is inhomogeneous over its cross section may lead to a change in the distribution of the light energy over the cross section. Let us consider the following simple problem. Let a standing wave of the magnetic type, polarized along the z axis, be established between perfectly conducting plates $y = \pm d$ in a liquid medium possessing strong electro-strictional properties along the z axis. Without taking the nonlinear terms into account, the solution of Eqs. (1) and (2) for such a wave would have the form

$$E = E_0 \cos \omega t \cos k_z z \cos(\pi y / 2d). \quad (11)$$

Now it is necessary to take the presence of the nonlinear terms (3) into account. From Eqs. (1)—(3) one can easily see that the steady-state solution for this case will have the form

$$E = \sum_{m,\, n=0} B_{mn} \cos \omega t \cos[(2m+1)k_z z] \cos\left[(2n+1)\frac{\pi y}{2d}\right], \quad (12)$$

where the coefficients B_{mn} with $n \neq 0$ cannot vanish simultaneously. The solution (12) describes the change of the electric field over the cross section, taking into account both the effect of the optical grating and the usual mechanism for the self-focusing of light.[8-10] However one can easily eliminate the latter if we confine our attention to times τ after the appearance of the light vibrations such that $\lambda/s \ll \tau \ll d/s$ (s is the velocity of sound, λ is the wavelength of the light). In this connection in (3) it is necessary to discard all terms not containing a factor of the form $\cos(2nk_z z)$ ($n = 1, 2, \ldots$), since deformations with a characteristic time of establishment $\sim d/s$ are not yet observed. It is easy to see that here the general form of the solution (12) is not changed, but only the specific values of the B_{mn} are changed; moreover, as before all terms with $n \neq 0$ cannot vanish simultaneously. For a sufficiently weak wave one can easily write down the terms of the type B_{01} in terms of B_{00}. However, here there is simply no need to cite the explicit form of these expressions. The use of a waveguide in this calculation is not at all fundamental since the corresponding solutions (11) and (12), which are periodic in y, also exist in an infinite medium.

The investigation which has been carried out suggests the possibility of self-focusing an intense light beam in the presence of only a reflected wave by means of the mechanism for the formation of an optical grating. A constriction of the beam due to electrostriction will then occur after a time of the order of only λ/s, and possibly there is no need (in the presence of the reflected wave) to use some additional mechanism[15] to explain the initial stage of self-focusing in solids. However, in that case the investigation carried out here is not a proof of the feasibility of self-focusing of a standing wave by means of the mechanism for the formation of an optical grating, and this question remains open for the present.

The absence of energy exchange between intersecting beams in the case of equal frequencies (without taking into consideration an active nonlinearity of the type of nonlinear absorption[13]) was due to the appropriate phase relationships between the light waves and the optical grating created by them. In the presence of a frequency difference $\Delta\omega$ the situation changes abruptly. For small values of $\Delta\omega$ the phase of the optical grating, which changes slowly with the time, will be specified not only by the phases of the light waves but also by the relaxation time for the Kerr effect or for the phonon viscosity for electrostriction. Here an exchange of energy between the interacting beams may occur. Of course, the total power of both beams will decrease since the relaxation is associated with an increase of the entropy of the system. The transfer of part of the energy (even though small) to the medium is here an inevitable element for the initiation of energy exchange between the light beams. For the case of the Kerr effect, the conditions for an increase of the weak light beam upon interaction with an intense light beam of a different frequency are obtained in articles[11,12] in connection with an investigation of stimulated Rayleigh-wing scattering.

In certain cases the last mechanism may apparently give a contribution that violates Fresnel's law for the reflection of light. These are, in the first place, the reflection of laser radiation containing many modes, and in the second place, the case when the frequency of the reflected light is changed with respect to the incident light because of a time variation of the medium's properties (for example, a nonstationary regime induced by temperature scattering of the second type[16]).

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