CONTRIBUTION TO THE THEORY OF THE SKIN EFFECT IN SEMIMETALS

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The skin effect is considered in a semimetal possessing a multi-valley carrier spectrum. The redistribution of the carriers among the valleys leads to the result that the value of the surface impedance depends on the relation between the skin depth δ and the diffusion length L. For L > δ the skin effect is determined by a conductivity which is appreciably smaller than the conductivity of the bulk sample and which depends on the orientation of the valleys. Under the conditions for normal and anomalous skin effects the electromagnetic field, whose amplitude depends on the type of scattering at the surface, penetrates to a depth ~L.

INTRODUCTION

In a plate of semimetal possessing a multi-valley electron spectrum, the current density j_x caused by an external field E parallel to the surface of the plate is the sum of the currents j_β of the electrons in separate valleys (β is a subscript labeling the valley):

$$j_x = \sum_\beta j_{\beta x}. \tag{1}$$

By virtue of the anisotropy of the Fermi surfaces the currents $\mathbf{j}_\beta$ in the general case are directed at an angle to the total current. The transverse components $j_{\beta z}$ (z denotes the coordinate along the normal to the surface z = 0) cancel in the sum

$$j_z = \sum_\beta j_{\beta z} = 0. \tag{2}$$

Condition (2) can be satisfied only in that case when, in addition to the transverse electric field E_z, changes δn_β of the electron concentrations in the valleys also appear. In this connection, no volume charge is present:

$$\sum_\beta \delta n_\beta = 0. \tag{3}$$

Equilibrium is established inside the valleys because of intervalley transitions; as is well-known,[1] an equalizing of the concentrations takes place over distances of the order of the intervalley diffusion length $L \sim l\sqrt{T/\tau}$ where l is the mean free path, and τ and T are the intravalley and intervalley relaxation times. Since a large momentum transfer is required for an intervalley transition, then one can assume that $T \gg \tau$ and $L \gg l$. It is essential that the currents $j_{\beta z}$ also vary over distances ~L according to the equations of continuity

$$\left(\frac{1}{e}\frac{\partial j_{\beta z}}{\partial z} \approx \frac{\delta n_\beta}{T}\right).$$

A nonlocal coupling of the current with the effective field exists near the surface of the sample, at distances $\sim l$. In this connection, an important characteristic feature exists in the case of several valleys. The point is that the current densities depend both on the longitudinal and on the transverse fields and on δn_β. Therefore, a certain part of the longitudinal current $j_{\beta x}$ varies over distances ~L. Formally this part of $j_{\beta x}$ can be distinguished upon the elimination of E_z and δn_β with the utilization of $j_{\beta z}$. A general expression for $j_{\beta x}$ follows from the solution of the kinetic equations for a many-valley system:[2]

$$j_{\beta x} = j_{\beta s} + a^\beta j_{\beta z}. \tag{4}$$

Here $j_{\beta s}$ only depends on the longitudinal field; the kernel of this integral operator varies over distances $\sim l$ (the expression for $j_{\beta s}$ and the anisotropy factor a^β will be specifically defined in Sec. 1). In the static electrical conductivity of thin films, the noted characteristic feature of a many-valley spectrum leads to size effects at thicknesses ~L and $\sim l$.[1,2]

The problem of the effect of a high-frequency electromagnetic field on a semimetal is considered in the present article. The characteristic features of the multi-valley spectrum may appear in different ways depending on the relation between the lengths l, L and the skin depth δ. It is obvious that if $\delta \gg L, l$, then the skin effect (normal) will be determined by the conductivity of the bulk sample because the equalizing of the concentrations takes place inside the skin-effect layer. At higher frequencies normal ($L > \delta > l$) and anomalous ($L > l > \delta$) skin effects are possible. In this situation the effective conductivity is determined by the currents $j_{\beta s}$ in (1) (because $j_{\beta z} \approx$ const), and it turns out to be smaller than the conductivity of the bulk sample, and in addition it depends on the orientation of the valleys. Therefore, the value of the surface impedance must be sensitive to the orientation of the valleys even for the normal skin effect. For matching characteristic lengths ($L \sim \delta$) the surface impedance may possess a frequency dependence which differs from the usual one. Another consequence is the unusual, anomalous penetration of the electromagnetic wave beyond the limits of the skin-effect layer by a depth ~L. Such a behavior of the field, associated with nonequilibrium of the concentrations in the valleys, formally follows from Maxwell's equation

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{4\pi i\omega}{c^2} j_x \tag{5}$$

upon taking into account the slow variation of part of the current (1). It is obvious that the amplitude E_L of this field is proportional to $\sum_\beta a^\beta j_{\beta z}$. It turns out that the quantity E_L is extremely sensitive to the kind of scattering at the surface of the sample. If intervalley Umklapp processes involving electrons are possible at a boundary then, as is shown in article[2], individual currents $j^0_{\beta z}$ appear on the surface (which are canceled out only in their total sum). If, however, a boundary scatters only into its "own" valley, then it is obvious that on it the currents $j^0_{\beta z}$ must vanish separately. Below it will be shown that in this connection the field E_L turns out to be δ/L times smaller than in the case of intervalley scattering at a surface.

1. BASIC EQUATIONS

Let us consider the problem of the normal incidence of an electromagnetic wave, polarized along the x axis, on the surface of a sample which occupies the half-space $z > 0$. In order to determine the current (1) it is necessary to solve the kinetic equations. In[2] 1) this is done in the fairly general case of ellipsoidal electron and hole Fermi surfaces for an arbitrary type of scattering at a boundary of the crystal, taking into account intervalley transitions on the surface and in the volume. Here we use a number of the results of[2], having taken the time dependence $\sim e^{-i\omega t}$ into consideration. The principal features of the problem, which were mentioned in the Introduction, are associated with the multiply-connected nature of the Fermi surface and the anisotropy in each of the valleys. In order to make the results easy to visualize, we perform the calculation for a very simple model possessing the indicated properties.

Let us confine our attention to the special case of only electron valleys, represented by identical ellipsoids of revolution lying in the xz plane. Let us assume that the boundary scatters diffusely. Also we shall not take surface bending of the band into consideration. Taking into account what was stated above, in accordance with Eq. (2.20) expression (4) for $j_{\beta s}$ has the form

$$j_{\beta s}=\frac{3}{4}(A_{xx}{}^{\beta})^2\sigma_0\int_0^\infty\frac{dz'}{l_\beta}E_x(z')\left[E_{i1}\left(\frac{|z-z'|}{l_\beta}\right)-E_{i3}\left(\frac{|z-z'|}{l_\beta}\right)\right],$$
$$E_{in}(x)=\int_1^\infty dt\,t^{-n}e^{-xt}. \tag{6}$$

Here the A^β_{ik} are the matrix elements transforming ellipsoids into spheres,[3] and in Eq. (4) $a^\beta = a^\beta_{xz} = A^\beta_{xz}/A^\beta_{zz}$; in our case

$$A_{xx}{}^\beta=\frac{\sqrt{\varepsilon_1\varepsilon_3}}{A_{zz}{}^\beta},\quad A_{xz}{}^\beta=\frac{(\varepsilon_1-\varepsilon_3)s_\beta c_\beta}{A_{zz}{}^\beta} \tag{7}$$
$$A_{zz}{}^\beta=\sqrt{\varepsilon_1 s_\beta^2+\varepsilon_3 c_\beta^2},$$

$\epsilon_i = m/m_i$, m_i are the principal values of the mass tensor, m is the free electron mass; $s_\beta = \sin\theta_\beta$, $c_\beta = \cos\theta_\beta$, and the angle θ_β determines the slope of the principal axis of the β^{th} ellipsoid to the z axis. Then

$$l_\beta=l_0A_{zz}{}^\beta,\quad l_0=\sqrt{\frac{2\zeta}{m}}\tau,\quad \sigma_0=\frac{e^2n\tau}{m}, \tag{8}$$

ζ is the Fermi level. The density of the transverse current satisfies the equation of continuity

$$\frac{1}{e}\frac{\partial j_{\beta z}}{\partial z}=\frac{N}{T}\delta n_\beta, \tag{9}$$

1) Below all references to formulas of [2] are marked with the number 2.

where N is the number of valleys. A dependence on the frequency is included in the relaxation times τ and T

$$\frac{1}{\tau}=\frac{1}{\tau_0}+\frac{N-1}{T_0}-i\omega,\quad \frac{N}{T}=\frac{N}{T_0}-i\omega. \tag{10}$$

Here we shall not cite the operator expression for $j_{\beta z}$ (see Eq. (2.21)). In what follows we shall need formulas for $j_{\beta z}$ and $j_{\beta s}$ in the diffuse approximation:

$$j_{\beta z}=\sigma_0\left[A_2{}^\beta E_x+A_1{}^\beta\left(E_z+\frac{1}{e}\frac{\partial\zeta}{\partial n}\frac{\partial\delta n_\beta}{\partial z}\right)\right], \tag{11}$$
$$j_{\beta s}=\sigma_0A_3{}^\beta E_x, \tag{12}$$
$$A_1{}^\beta=(A_{zz}{}^\beta)^2,\ A_2{}^\beta=A_{zz}{}^\beta A_{xz}{}^\beta,\ A_3{}^\beta=(A_{xx}{}^\beta)^2. \tag{13}$$

One can use expressions (11) and (12) at distances $z \gg l_\beta$ from the boundary. They are obtained from the nonlocal relations (2.21) and (6) under the hypothesis that the fields vary much more slowly than the kernel of the integral operators. If we denote the characteristic scale of variation of the field by δ_E, then for all β the inequality

$$\delta_E\gg l_\beta. \tag{14}$$

must be satisfied.

2. THE NORMAL SKIN EFFECT

The normal skin effect takes place if condition (14) is valid. In this connection one can use the diffuse approximation everywhere with the exception of the crystal surface layer $z \sim l_\beta$. The system of equations in the diffuse approximation, which follow from Eqs. (1), (4), (5), (9), (11), and (12), has the form

$$E_x''=\delta_0^{-2}\Big[E_x+C\sum_\beta A_2{}^\beta\delta n_\beta'\Big],$$
$$E_x'+CA_1{}^\beta\sum_\gamma A_1{}^\gamma(\delta n_\beta''-\delta n_\gamma'')=\frac{eN}{T\sigma_0}\delta n_\beta, \tag{15}$$
$$\delta_0^{-2}=-\frac{4\pi i\omega}{c^2}\sigma_0\sum_\beta A_1{}^\beta,\quad C=\frac{1}{e}\frac{\partial\zeta}{\partial n}\Big(\sum_\gamma A_1{}^\gamma\Big)^{-1}. \tag{16}$$

The field E_z is eliminated with the aid of the condition $\sum_\beta j_{\beta z} = 0$ (taking into consideration that $\sum_\beta A_2^\beta = 0$). Substituting $E_x(z) = Ee^{-qz/\delta_0}$, $\delta n(z) = \delta n e^{-qz/\delta_0}$, we find

$$E=-\frac{C}{\delta_0}\frac{q}{q^2-1}\sum_\beta A_2{}^\beta\delta n_\beta, \tag{17}$$
$$q\cdot\left[A_1{}^\beta\sum_\gamma A_1{}^\gamma(\delta n_\beta-\delta n_\gamma)+\frac{A_2{}^\beta}{q^2-1}\sum_\gamma A_2{}^\gamma\delta n_\gamma\right]=2u^2\delta n_\beta, \tag{18}$$
$$u^2=\frac{eN}{2T\sigma_0}\frac{\delta_0^2}{C}=\frac{3N^2}{4}(\varepsilon_1+\varepsilon_3)\frac{\tau}{T}\frac{\delta_0^2}{l_0^2}. \tag{19}$$

For clearness we limit our attention to the simplest case of two ellipsoids oriented at right angles to each other ($\theta_2 = \theta_1 + \pi/2$). Having used expressions (7) and (13) and having introduced the notation

$$\delta n_1=-\delta n_2\equiv\delta n\qquad \gamma^2=\sum_\beta A_1{}^\beta\Big/\sum_\beta A_3{}^\beta=1+\frac{(\varepsilon_1-\varepsilon_3)^2}{\varepsilon_1\varepsilon_3}s_1{}^2c_1{}^2, \tag{20}$$

we obtain the dispersion equation

$$q^4-q^2\frac{1+u_1{}^2}{\gamma^2}+\frac{u_1{}^2}{\gamma^2}=0,\quad u_1{}^2=\frac{u^2}{\varepsilon_1\varepsilon_3}=\frac{\delta_0{}^2}{L^2},\quad L^2=\frac{T}{3\tau}l_0{}^2\frac{\varepsilon_1\varepsilon_3}{\varepsilon_1+\varepsilon_3}; \tag{21}$$

its roots are given by

$$q^2_{1,2}=\frac{1}{2\gamma^2}[1+u_1{}^2\pm\sqrt{(1+u_1{}^2)^2-4u_1{}^2\gamma^2}]. \tag{22}$$

The boundary condition for the field E_x under the conditions of the normal skin effect obviously may be transferred to the real surface z = 0:

$$\sum_i E^{(i)} = E_x(0). \tag{23}$$

In the diffuse approximation the values of $j_{\beta z}$ and δn_β are usually related by a phenomenological boundary condition at z = 0:

$$\frac{1}{e} j_{\beta z}(0) = \sum_{\beta'} S_{\beta\beta'}[\delta n_\beta(0) - \delta n_{\beta'}(0)]; \tag{24}$$

here $S_{\beta\beta'}$ denotes the rate of intervalley surface scattering. By matching $j_{\beta z}$ and δn_β at the boundary of the region of applicability of the diffuse approximation with the currents and concentrations inside the surface layer, one can determine the parameters $S_{\beta\beta'}$. This is done in[2] (see Sec. 4). Let us cite the value of S_{12} which is obtained from Eq. (2.44) for the system of two ellipsoids:

$$S_{12} = S = \frac{l_1 d_{12}}{4\tau[1 - 0{,}47(d_{12} + d_{21})]}. \tag{25}$$

Here $d_{\beta\beta'}$ is the probability for intervalley scattering at the surface (we recall that $\sum_{\beta'} d_{\beta\beta'} = 1$, where $d_{\beta\beta}$ denotes the probability of diffuse scattering into "its own" valley). With the aid of the relations

$$j_{\beta z}(0) = -\frac{eN\delta_0}{T}\sum_i \frac{\delta n_\beta^i}{q_i}, \quad \delta n_\beta(0) = \sum_i \delta n_\beta^i,$$

(17), (23), and (24) we find the amplitudes of the field $E^{(i)}$:

$$\frac{E^{(1,2)}}{E_x(0)} = \pm\left(1 + \frac{\delta_0}{TSq_{2,1}}\right)\frac{q_{1,2}(q_{2,1}^2 - 1)}{(q_2 - q_1)F_1}, \tag{26}$$

$$F_1 = 1 + q_1 q_2 + \frac{\delta_0}{TS}\frac{q_1 + q_2}{q_1 q_2}.$$

Now one can write down an expression for the surface impedance

$$Z = \frac{4\pi i\omega}{c^2}\frac{E_x(0)}{E_x'(0)}.$$

Having utilized Eq. (26) we obtain

$$Z = Z_0\frac{F_1}{F_2}; \quad Z_0 = -\frac{4\pi i\omega}{c^2}\delta_0, \tag{27}$$

$$F_2 = q_1 + q_2 + \frac{\delta_0}{TS}\left(1 + \frac{q_1}{q_2} + \frac{q_2}{q_1} - q_1 q_2\right).$$

The parameter γ defined in (20) characterizes the orientation of a valley and its anisotropy. For $\gamma \approx 1$, i.e., for an orientation which does not reduce to a redistribution ($\theta_i \approx 0$) or for weak anisotropy ($(\epsilon_1 - \epsilon_3)^2/\epsilon_1\epsilon_3 \ll 1$) in (27) we have $F_1/F_2 = 1$ and Z is equal to its usual value for the normal skin effect and isotropic (in the xz plane) conductivity.

Let us consider the most favorable orientation ($\theta \approx \pi/4$) and strong anisotropy, when $\gamma^2 \gg 1$. The quantity u_1 defined in (21) may be varied over broad limits during a change of the frequency ω ($u_1^2 \sim 1/\omega$). The following cases are characteristic.

1. Low frequencies:

$$u_1 = \frac{\delta_0}{L} \gg \gamma, \quad \gamma L \gg l_1,$$

$$q_1 \approx 1, \quad q_2 \approx \frac{\delta_0}{\gamma L}, \quad \frac{F_1}{F_2} \approx \begin{cases} 1 + \dfrac{\gamma^2}{2u_1^2} \\ 1 + \dfrac{\gamma}{u_1} \end{cases}. \tag{28}$$

2. Intermediate frequencies (matching of the lengths L and δ_0):

$$u_1 = \frac{\delta_0}{L} \sim 1, \quad u_1\gamma = \frac{\delta_0\gamma}{L} \gg 1,$$

$$\frac{\gamma L\delta_0}{l_1^2} \gg 1, \quad |q_{1,2}| \approx \sqrt{\frac{\delta_0}{L\gamma}}, \quad \frac{F_1}{F_2} \approx \begin{cases} {}^1/_2\sqrt{\gamma L/\delta_0} \\ {}^2/_3\sqrt{\gamma L/\delta_0} \end{cases}. \tag{29}$$

3. Higher frequencies:

$$u_1\gamma = \frac{\delta_0}{L}\gamma \ll 1, \quad \frac{\delta_0}{l_1} \gg \frac{1}{\gamma},$$

$$q_1 \approx \frac{1}{\gamma}, \quad q_2 \approx \frac{\delta_0}{L}, \quad \frac{F_1}{F_2} \approx \gamma\begin{cases} 1 - u_1\gamma \\ 1 + u_1^2 \end{cases}. \tag{30}$$

The second inequalities in expressions (28)—(30) guarantee the fulfillment of the criterion (14). The upper and lower values for F_1/F_2 correspond to the cases of large and small rates of intervalley scattering S (i.e., to values $\delta_0/TSu_1 = L/TS \approx d_{12}^{-1}\sqrt{\tau/T} \ll 1$ or $\gg 1$). The corrections to unity in these expressions are associated with a more deeply penetrating component of the electric field. As already mentioned in the Introduction, the amplitude of this field is sensitive to the type of surface scattering. The field $E^{(2)}$ in case 3, which penetrates to a depth $\sim L \gg \delta_0$, is of interest; its amplitude $E^{(2)} \approx -\gamma\delta_0 E_x(0)/L$ for an appreciable part of the intervalley scattering and $\approx -\gamma^2\delta_0^2 \cdot E_x(0)/L^2$ as $d_{12} \to 0$.

3. THE ANOMALOUS SKIN EFFECT

If inequality (14) is violated, then in order to solve Maxwell's equation (5) it is necessary to use nonlocal expressions for the current given by expressions (1), (4), and (6). In contrast to the usual problem for the anomalous skin effect,[4] in our case a comparatively slowly varying term with $j_{\beta z}$ is contained in expression (1). Under the conditions of the anomalous skin effect, when $\delta \ll l \ll L$, one cannot take into consideration the changes of $j_{\beta z}$ within the limits of the skin layer. The circumstance enables one, in connection with the solution of Eq. (5), to represent the field in the form of a sum of rapidly and slowly-varying parts:

$$E_x = E_s + E_L. \tag{31}$$

The function E_s satisfies the usual integro-differential equation

$$\frac{\partial^2 E_s}{\partial z^2} = -\frac{4\pi i\omega}{c^2}\sum_\beta j_{xs}^\beta(E_s). \tag{32}$$

In order to determine the field E_L one can omit its second derivative and, in expression (6), take E_L out from under the integral sign. As a result we obtain

$$E_L = -\left(\sigma_0\sum_\beta A_3^\beta\right)^{-1}\sum_\beta a_{xz}^\beta j_{\beta z}. \tag{33}$$

In order to determine $j_{\beta z}$ it is necessary to solve Eqs. (9) with (31) and (33) taken into consideration. For this purpose let us distinguish in the sample a surface region of dimension $\ll$ L but $\gg l_0$. In this region the intervalley transitions are not important and, according to Eq. (9), $j_{\beta z} = j_{\beta z}^0$; here $\delta n_\beta(z)$ can be evaluated exactly (see[2], Sec. 4). The diffuse approximation is applicable to the remaining part of the crystal, where in expression (11) one should use E_L given by (33) as E_x because the field E_s can be regarded as attenuated beyond the limits of the surface region. After eliminating the field

E_z from Eqs. (9) and (11) we obtain the following system of equations:

$$\delta n_\beta(z) = \delta n_\beta e^{-z/L},$$
$$\left(L^2 - \frac{Tl_\beta^2}{3N\tau}\right)\delta n_\beta + \frac{Tl_\beta^2}{3N\tau}\left(\sum_\gamma A_1^\gamma\right)^{-1}(A_1^\gamma + A_2^\gamma a_{xz}^\beta)\delta n_\gamma = 0. \quad (34)$$

A dispersion equation that determines the characteristic lengths L_i^2 ($i = 1, 2, \ldots N-1$) follows from (34). It is obvious that all $L_i^2 \sim Tl^2/3N\tau$. For the current density we have:

$$\frac{1}{e} j_{\beta z}(z) = -\frac{N}{T}\sum_{i=1} L_i\,\delta n_{\beta i}\,e^{-z/L_i}. \quad (35)$$

The boundary conditions are obtained by matching $j_{\beta z}$ and δn_β with the solutions in the surface region. In the present case, owing to the presence of the rapidly changing field E_s, the boundary condition differs from (24). Thus, just as is done in[2] (Sec. 4), when (31) is taken into account we obtain the result that the difference consists in the replacement in (24) of $\delta n_\beta(0)$ by

$$\delta n_\beta(0) + \frac{\partial n}{\partial \zeta}\left(a_{xz}^\beta - \frac{1}{N}\sum_\gamma a_{xz}^\gamma\right)\int_0 eE_s(z)\,dz.$$

For an appreciable fraction of the intervalley scattering, when $S_{\beta\beta'} \sim l/\tau$, in the change of the boundary condition (24) one can drop the left-hand part which, with (35) taken into consideration, is roughly $\sqrt{\tau/T}$ times smaller than the corresponding terms on the right-hand side. In this case

$$\delta n_\beta(0) = -\frac{\partial n}{\partial \zeta}\left(a_{xz}^\beta - \frac{1}{N}\sum_\gamma a_{xz}^\gamma\right)\delta_1 eE_s(0). \quad (36)$$

Here the following equation, which one can obtain with the aid of (32), has been taken into consideration:

$$\int_0^\infty E_s(z)\,dz = \delta_1 E_s(0), \qquad \delta_1^2 = \delta_0^2\gamma^2. \quad (37)$$

The amplitude of the field E_L given by Eq. (33) is, as one can easily estimate, of the order of $\delta_1 E(0)/L$; in the case of two ellipsoids

$$E_L = \frac{\gamma^2 - 1}{\gamma^2}\frac{\delta_1}{L}E(0).$$

If the intervalley scattering at the surface is negligible ($S_{\beta\beta'} \to 0$) then, with the accuracy used above in order to obtain E_L given by (33) $j_{\beta z}(0) \approx 0$, and the penetrating field is negligibly small. Taking into account the terms omitted here of the next order of smallness in δ/L, one can show that, just like in Sec. 2, here $E_L \sim \delta_1^2 E(0)/L^2$.

By using (31) and (32) one can write the surface impedance of the plate in the form

$$Z = Z_\infty\left[1 - \frac{E_L(0)}{E(0)}\right], \quad Z_\infty = \frac{4\pi i\omega}{c^2}\frac{E_s(0)}{E_s'(0)}. \quad (38)$$

The value of Z_∞ should be obtained in the usual way from a solution of (32) under the conditions for the anomalous skin effect. The problem of the anomalous skin effect for a system of two ellipsoids was solved by Sondheimer,[5] who used a non-self-consistent method of solution (which did not take into account the redistribution of the electrons, and the solutions of the kinetic equation in[5] did not satisfy the conditions $\delta n_\beta = 0$) and obtained a system of Maxwell's equations from which it is impossible to eliminate the transverse field. Then he successfully discarded a number of the terms which did not permit one to utilize the usual methods of the theory of the anomalous skin effect, and obtained an equation which agrees with Eq. (32). Of course, effects associated with the penetration of a wave into the depths of the sample dropped out of the investigation. In the limiting case of the anomalous skin effect, i.e., for

$$\frac{3\pi i\omega}{c^2}\sigma_0 A_3^\beta l_\beta^2 = \frac{3}{4}\frac{l_0^2}{\delta_0^2}\varepsilon_1\varepsilon_3 \gg 1, \quad (39)$$

from (38), just like in[5], it follows that

$$Z_\infty = (1 + i\sqrt{3})\left(\frac{\sqrt{3}\,\pi\omega^2}{\sigma_0 c^4}\right)^{1/3}\left(\sum_\beta \frac{A_3^\beta}{l_\beta}\right)^{-1/3}. \quad (40)$$

According to estimates for the field, the corrections to Z_∞ in (38) are of the order of δ_1/L.

4. DISCUSSION

The most important qualitative effect for the surface impedance consists in its pronounced dependence on orientation and the substantial increase associated with an increase of the frequency under the conditions for the normal skin effect. According to Eqs. (28)–(30), upon a change of the relation of the lengths δ_0 and L the ratio Z/Z_0 given by Eq. (27) increases γ times. If the orientation of the crystallographic axes relative to the surface is such that normal currents $j_{\beta z}$ exist, then $\gamma \sim \sqrt{\epsilon_1/\epsilon_3}$ for $\epsilon_1 \gg \epsilon_3$ (in Bi, for example, $\epsilon_1/\epsilon_3 \sim 10^2$). If, however, from symmetry considerations it is known that appreciable currents $j_{\beta z}$ cannot be produced by an external field, then $\gamma \approx 1$. In the intermediate case (29), upon a comparison of the characteristic lengths and fulfillment of the condition $\omega T_0 \ll 1$, the dependence of Z on the frequency ($Z \sim \omega^{3/4}$) may change qualitatively whereas $Z_0 \sim \omega^{1/2}$.

The corrections to the values of the impedance in (30) and (38), associated with an electromagnetic wave which penetrates a distance $\sim L$, are small in magnitude. In the case when the thickness of the layer b is comparable with L (but $b > \delta$), these corrections upon symmetric illumination of the surface are, as is not difficult to show, $\sim(\gamma\delta_0/L)\coth(b/L)$. It is possible that one can observe their dependence on size and frequency ($\sim\omega^{-1/2}$). The amplitude of the deeply-penetrating wave (in the presence of intervalley scattering at the surface) has the same magnitude with respect to the field at the surface. One can attempt to observe the penetrating field in experiments on the transmission of a high-frequency wave through thin films. Its observation would give important information about intervalley transitions in the volume and on the surface.

Let us briefly discuss a more general situation than the one considered in Secs. 2 and 3. We did not take into account the role of specular scattering at the surface; it is obvious that in this connection the results qualitatively agree with the case of weak intervalley scattering (in addition, in Eq. (40) Z_∞ is replaced by $(8/9)Z_\infty$). Only the electron valleys were taken into account whereas in real semimetals the presence of the holes is essential. Taking account of the hole surfaces leads to the appearance in the theory of new characteristic dimensions (associated with recombination and hole intervalley transitions); qualitatively the physical picture of this does not change. As shown in[2], surface bending of the band leads to an increase of the fraction of specular scattering for those carriers whose approach to the surface is impeded. The influence of this effect in the case considered by us of plates which are thick in comparison with l has already been noted. In thin films ($b \ll l$) where the bending leads to an effective change of the conductivity,[2] in the first scheme effects appear which are not related to the many-valley nature of the carrier spectrum. Consideration of this interesting problem goes outside the framework of the present article. We did not investigate the question of the behavior of the field E_z and of the gradients of the concentrations at the boundaries of the plate. The situation here is analogous to the one investigated in[2]: for $z \ll l$ these quantities must have logarithmic singularities. For example, for E_z it is not difficult to obtain

$$E_z \approx E_x(0)\left(1+\frac{\delta}{L}\ln\frac{l}{z}\right)$$

in the presence of intervalley scattering at the surface.

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