INFLUENCE OF DISSIPATIVE EFFECTS ON ELECTROACOUSTIC WAVES IN A PLASMA

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Asymptotic formulas are obtained which describe the evolution of solitary electroacoustic waves in a plasma due to electron-ion collisions.

It was shown in[1] that in a plasma with negative dielectric permittivity, there may propagate "electroacoustic solitons" - stationary, solitary waves of density rarefactions with high frequency electromagnetic fields trapped in them. In[2], a general nonstationary solution of the equations for electroacoustic waves of sufficiently small amplitude was obtained. That solution described the dynamics of the formation of electroacoustic solitons as a result of the incidence of a modulated electromagnetic wave on the plasma boundary.

It was assumed in the cited papers that the dissipative processes could be neglected. The conditions for these processes to be small were considered briefly in[1]. However, weak dissipative effects, which do not substantially influence the process of formation of the electroacoustic solitons, do have a considerable effect on their further evolution.

The present note is concerned with an investigation of the role of electron-ion collisions (EIC) in the dynamics of electroacoustic solitons of small amplitude (in a rarefied plasma with $T_e \gg T_i$, this dissipative effect is dominant over a vast range of parameters[1]).

In order to derive the equations describing the evolution of electroacoustic waves in the case of small (but finite) frequency of the EIC, we write the electric field in the following form

$$\mathscr{E}(x,t) = \mathrm{Re}\,[E(x,t)e^{-i\omega t}],$$

where $E(x, t)$ is a slowly varying complex amplitude.

The equation for E is of the form (see[1], Eq. (1.19))

$$\left[\omega^2\varepsilon_0 + \omega^2\frac{\partial\varepsilon_0}{\partial\rho_0}(\rho-\rho_0) + c^2\frac{\partial^2}{\partial x^2}\right]E + i\frac{\partial(\omega^2\varepsilon_0)}{\partial\omega}\frac{\partial E}{\partial t} = 0. \tag{1}$$

Here $\epsilon_0(\omega, \rho_0)$ is the dielectric permittivity of a plasma with unperturbed density ρ_0; ϵ_0 is determined by

$$\varepsilon_0 = 1 - \frac{\omega_0^2}{\omega^2} + \frac{i}{\tau\omega}. \tag{2}$$

The quantity τ is the characteristic time of the EIC[3]

$$\tau^{-1} \approx \frac{\pi e^4 n_0}{T_e^{3/2} m_e^{1/2}} \ln\left(0.37\frac{T_e}{e^2 n_0^{1/3}}\right). \tag{3}$$

It is assumed here that

$$\tau\omega \gg 1. \tag{4}$$

Expressing the complex amplitude of the field in the form

$$E = a(x,t)e^{i\varphi(x,t)},$$

where $a(x, t)$ and $\varphi(x, t)$ are real functions, we obtain from (1) and (2), by omitting the term in $\partial E/\partial t$ which is small in comparison with $c^2\partial^2E/\partial x^2$ (see[1], Sec. 3), the following equations:

$$a_{xx} - \mu^2\gamma^{-2}(\gamma^2+\nu)a - \varphi_x{}^2 a = 0,$$

$$(a^2\varphi_x)_x = -\omega a^2/c^2\tau.$$

For waves of sufficiently small (but finite) amplitude, propagating in the positive direction of the x axis, the quantity $\nu = (\rho - \rho_0)/\rho_0$ satisfies the equation[4]

$$\nu_t + c_s\nu_x = -c_s(a^2)_x/2E_c{}^2. \tag{6}$$

The following notation is used in (5) and (6):

$$\mu^2 = (\omega_0{}^2-\omega^2)/c^2,$$

$$\gamma^2 = (\omega_0{}^2-\omega^2)/\omega_0{}^2, \tag{7}$$

$$E_c{}^2 = 16\pi\rho_0 c_s{}^2, \quad c_s{}^2 = T_e/m_i.$$

For $\tau = \infty$, the system (5) and (6) coincides with the full set of equations for the electroacoustic waves of sufficiently small amplitude, which was considered in[2,4]. Accordingly, the range of applicability of (5) and (6) is limited by the same conditions as in[2,4].

In the static case, when the plasma density is time-independent, Eqs. (5) and (6) coincide with those for the stationary skin layer, which have been derived and discussed by Silin[5].

The system (5) and (6) has a solution of the form

$$a(x,t) = e^{-t/2\tau}A(\xi), \quad \varphi = \varphi(\xi),$$

$$\nu(x,t) = \frac{A_0{}^2 - A^2(\xi)}{2E_c{}^2(1-w/c_s)} - \gamma^2, \tag{8}$$

where

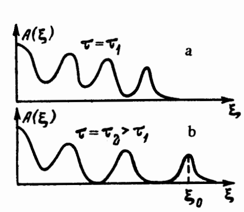
$$\xi = x - c_s t + \tau(c_s - w)(1 - e^{-t/\tau}), \tag{9}$$

$$A_{\xi\xi} - \mu^2\frac{(A_0{}^2 - A^2)A}{2E_c{}^2\gamma^2(1-w/c_s)} - \frac{P^2(\xi)}{A^3} = 0, \tag{10}$$

$$dP/d\xi = -A^2\omega/\tau c^2, \quad P(\xi) = \varphi_\xi A^2. \tag{11}$$

Here w and A_0 are arbitrary constants.

The system of ordinary differential equations (10) and (11) for the functions $A(\xi)$ and $P(\xi)$ has the same form as Eqs. (5.2) and (5.3) of the paper by Silin[5]. Using results of that paper, we find that for sufficiently small frequences of the EIC, τ^{-1}, the general form of the function $A(\xi)$ is as displayed in the figure. For



clarity, the form of the solution for two different values of τ is presented there.

As $\omega\tau \to \infty$, the separation of the individual oscillations in the front part of the wave increases. The leading oscillations in the front, which are well separated from each other, can then be described by the following asymptotic formula[1]:

$$A(\xi) = 2E_c\gamma(1 - w/c_s)^{1/2}\operatorname{sech}\mu(\xi - \xi_0), \tag{12}$$

where ξ_0 is the coordinate of the peak of the oscillation under consideration (see the figure). The constant A_0 appearing in (10) then has the following limit:

$$\lim_{(\omega\tau)\to\infty} A_0 = \sqrt{2}E_c\gamma(1 - w/c_s)^{1/2}. \tag{13}$$

Inserting (12) and (13) into formulas (8), we obtain the following asymptotic expression describing the amplitude of the field and the relative change of the plasma density in the leading oscillation, for large $\omega\tau$:

$$a(x, t) = e^{-t/2\tau}A_m \operatorname{sech}\mu(\xi - \xi_0), \tag{14}$$

$$A_m = 2E_c\gamma(1 - w/c_s)^{1/2}, \tag{15}$$

$$\nu = -2\gamma^2 \operatorname{sech}^2\mu(\xi - \xi_0). \tag{16}$$

The formulas (14)—(16), together with the expression for the quantity ξ, describe an electroacoustic soliton having at $t = 0$ an amplitude A_m and velocity w (cf. the corresponding expressions in[2,4]). At the subsequent instants of time, the wave described by formulas (14)—(16) and (9) conserves its soliton-like profile. Its "Mach number" is determined by the expressions

$$M(t) = \frac{1}{c_s}\left(\frac{dx}{dt}\right)_{\xi=\xi_0} = 1 - [1 - M(0)]e^{-t/\tau}, \tag{17}$$

$$M(0) = w/c_s, \tag{18}$$

and the maximum amplitude of the electric field is equal to

$$a_m(t) = A_m \exp(-t/2\tau). \tag{19}$$

In accordance with (15), (17) and (19), the relation between the amplitude $a_m(t)$ and the Mach number at any instant is of the form

$$a_m(t) = 2E_c\gamma[1 - M(t)]^{1/2}, \tag{20}$$

which coincides with the corresponding expression for the electroacoustic solitons of small amplitude without the damping taken into account.

Allowance for the EIC thus leads to an exponential damping of amplitude in the soliton[2]; accordingly the Mach number $M(t)$, in accordance with formula (17), tends to unity exponentially when $t \to \infty$.

As far as the profile of the relative density, $\nu(\xi)$, is concerned, it follows from (16) that its form does not depend on time. This is connected, first, with the fact that we do not take into account the slower dissipative processes, viz., the ion-acoustic Landau damping and the plasma viscosity caused by ion-ion collisions. Second, we neglect the nonlinear steepening of the density profile of the ion-acoustic wave (Eq. (6) is obtained from the linearized equations of the ion-acoustic fluid dynamics).

The condition of applicability of expressions (14)—(16) and (9) is of the form[3] (see the Appendix)

$$(\tau\omega) \gg (10/\gamma)^2. \tag{21}$$

We now briefly consider the role of other dissipative effects. If the viscosity and heat transfer are the most important processes next to the EIC, then the evolution of an electroacoustic soliton will proceed as follows: first the electromagnetic field trapped in the soliton will attenuate and the speed of that field will become close to the speed of sound; then the joint influence of the viscosity, heat transfer, and the nonlinear steepening of the wave profile will result in a triangular form of the density profile.

We take the opportunity to express our gratitude to V. P. Sokolov for useful discussion of the results.

APPENDIX

We now determine the domain of validity of solution (12). Let us substitute (12) into (11). With the boundary condition $P(\infty) = 0$, we obtain

$$P(\xi) = \frac{\omega A_m^2}{c^2\tau\mu}[1 - \operatorname{th}\mu(\xi - \xi_0)]. \tag{A.1}$$

Using (A.1) and (12) to calculate the ratio of the last term to the first in (10), we get

$$\frac{P^2(\xi)}{A^3(\xi)A_{\xi\xi}} = \frac{\omega^2}{c^4\tau^2\mu^4}\frac{e^{-2\mu(\xi-\xi_0)}}{[\operatorname{sh}^2\mu(\xi - \xi_0) - 1]}\operatorname{ch}^4\mu(\xi - \xi_0). \tag{A.2}$$

It follows from this expression that

$$\frac{P^2(\xi)}{A^3 A_{\xi\xi}} \sim \begin{cases} (\gamma^2\tau\omega)^{-2}, & \xi > \xi_0 \\ (\gamma^2\tau\omega)^{-2}e^{4\mu(\xi_0-\xi)}, & \xi < \xi_0 \end{cases} \tag{A.3}$$

In order that the term in $P^2(\xi)$ in (10) be negligible in the range of values of ξ that correspond to a solution with its peak at the point $\xi = \xi_0$, it is necessary that the ratio appearing on the left side of (A.3) be sufficiently small at the distance of a few soliton lengths from the peak[4]. Putting $\xi_0 - \xi = k\mu^{-1}$, where $k \sim 1$, we obtain the corresponding condition in the form

$$(\tau\omega) \gg \gamma^{-2}e^{2k}. \tag{A.4}$$

Assuming for concreteness that $k = 2$, we obtain (21).

[1]For values of $(\omega\tau)^{-1}$ sufficiently small, it follows from (11) that $dP/d\xi \approx 0$. Taking account of this fact and of the boundary condition $A(\infty) = 0$, we obtain from (10) the approximate expression (12). The domain of applicability of that relation is considered in the Appendix.

[2]The particular solution considered here describes, as can be seen in the figure, electroacoustic waves which can be excited on the boundary of the plasma under certain special boundary conditions, which can be determined with the values of the functions $a(x,t)$, $P(x,t)$ and $\nu(x,t)$ at $x = 0$. However, it follows from the formulas describing this solution that the leading wave at sufficiently large values of t does not depend on the initial condition any more, and assumes the form of a soliton, so that the formulas (14) – (16) and (9) describe the propagation of a soliton, with the EIC taken into account, in the case $(\omega\tau) \gg 1$.

[3]For the frequency of the EIC to be much larger than the Landau damping decrement for the ion-acoustic wave, τ_L^{-1}, the inequality $\tau^{-1} \gg \tau_L^{-1} \approx 10^{-2}\omega\gamma/c$ should hold. It follows from the condition of consistency of the last relation with (21) that $c_s/c\gamma \ll 1$. This inequality is, at the same time, the condition of validity of the initial equations in (5) (see [1]), and thus is assumed to be true from the very beginning.

[4]The quantity (A.2) also becomes large in the neighborhood of the point of inflexion of the soliton profile (where $A_{\xi\xi} = 0$). However the size of that domain is of the order of $\delta\xi \approx \mu^{-1}(\gamma^2\tau\omega)^{-2} \ll \mu^{-1}$, i.e., much smaller than the size of the soliton.

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170