PARAMETRIC EXCITATION OF SURFACE OSCILLATIONS OF A PLASMA BY AN EXTERNAL HIGH FREQUENCY FIELD

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Starting from the hydrodynamic equations and the Maxwell equations, a system of equations is derived, which describes the linear oscillations of a bounded plasma in the presence of an external HF field. The dispersion relation for potential surface oscillations of both a plane layer and a cylinder is obtained from this system. It is shown that new branches arise in the spectrum of plasma surface oscillations when an external HF field is present. The surface oscillations are unstable over a vast range of frequencies of the external field.

THE extremely interesting properties of plasma located in a strong high frequency (HF) electric field have stimulated a broad range of theoretical investigations in this promising field of physics. A broad class of instabilities was predicted to accompany the processes taking place in a plasma placed in a HF field. On the other hand, for a number of cases the stabilizing effect of an HF field on unstable plasma states was pointed out. A detailed review of the theory of the plasma—HF field interaction was given in[1]. However, in the mentioned investigations the plasma was assumed infinite and homogeneous. This restriction limits to a certain extent the possibilities of comparison of the theory with experiment, in which the plasma is always bounded. It is well known that a bounded plasma, even in the absence of an HF field, has an oscillation spectrum that differs from the oscillation spectrum of an infinite plasma. For example, in a plane layer of plasma, beside the usual plasma oscillations with the frequency near the Langmuir frequency ω_{Le}, there also exist surface oscillations whose frequency is near $\omega_{Le}/\sqrt{2}$. The appearance of new types of oscillations should necessarily be reflected in the stability analysis of the bounded plasma in an HF field.

The present paper is concerned with the investigation of the oscillations and stability of a plasma layer in a strong HF field. For this purpose we derive equations for the electromagnetic fields of plasma perturbations, together with boundary conditions on the free plasma—vacuum interface. Using the derived equations, the potential surface oscillations of a plasma layer are considered. It is shown that the HF field changes the known surface-oscillation spectrum and results in the appearance of a new low-frequency spectrum. When the frequencies of the external HF field are close to or less than $\omega_{Le}/\sqrt{2}$, the surface perturbations of the plasma grow exponentially. Their growth rate can be substantially higher than that of the volume oscillations, which are also unstable under these conditions. This fact should be taken into account in experiments on the interaction of nontransparent plasma objects with HF fields. Similar results are obtained for a plasma cylinder placed in an HF field.

1. We consider a bounded plasma in a strong HF field. Neglecting the thermal motion of the particles, we assume the plasma boundary to be sharp. The vector of the homogeneous electric field $\mathbf{E}$ ($E_0 \sin \omega_0 t$, 0, 0) is aligned tangent to the boundary. The density is assumed to be uniform, $n_e^{(0)} = n_i^{(0)} = \text{const}$. Under the influence of the HF field, the particles acquire the velocities

$$\mathbf{v}_\alpha^{(0)} = \frac{e_\alpha}{m_\alpha} \int_{-\infty} \mathbf{E}(t')\,dt'. \tag{1}$$

Linearizing the equations describing the plasma state for small perturbations, we obtain

$$\frac{\partial n_\alpha}{\partial t} + v_\alpha^{(0)} \frac{\partial n_\alpha}{\partial x} = -\operatorname{div}(n_\alpha^{(0)} \mathbf{v}_\alpha),$$

$$\frac{\partial \mathbf{v}_\alpha}{\partial t} + v_\alpha^{(0)} \frac{\partial \mathbf{v}_\alpha}{\partial x} = \frac{e_\alpha}{m_\alpha}\left(\mathbf{E} + \frac{1}{c}[\mathbf{v}_\alpha^{(0)}\mathbf{B}]\right), \tag{2*}$$

$$\operatorname{rot}\mathbf{B} = \frac{4\pi}{c}\mathbf{j} + \frac{1}{c}\frac{\partial \mathbf{E}}{\partial t}, \quad \operatorname{rot}\mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t},$$

where

$$\mathbf{j} = \sum_{\alpha=e,i} e_\alpha (n_\alpha^{(0)} \mathbf{v}_\alpha + n_\alpha \mathbf{v}_\alpha^{(0)}),$$

n_α and $\mathbf{v}_\alpha$ are the perturbed density and velocity, respectively, of the particles of type α, and $\mathbf{E}$ and $\mathbf{B}$ are the perturbed fields. The coefficients of the system (2) are periodic in time, and thus we can assume for the time dependence of the solutions of (2)

$$\mathbf{E} = \sum_{n=-\infty}^{\infty} \mathbf{E}^{(n)}(\mathbf{r})\, e^{-i(\omega+n\omega_0)t}$$

etc. The dependence of the solutions on space coordinates both inside and outside the plasma shall be represented in the following form:

$$\mathbf{E}^{(n)}(\mathbf{r}) = \sum_{\mathbf{k}} \mathbf{E}^{(n)}(\mathbf{k})\, e^{i\mathbf{kr}}.$$

We obtain then the following system of homogeneous equations, which describes the spectrum of eigenvalues of the wave vectors $\mathbf{k}_i$ (k_x, k_y, k_{xi}) inside the plasma:

$$\left(1 + \frac{\omega_L^2}{c^2 k_x^2}\right)(k_y B_z^{(s)} - k_{zi} B_y^{(s)}) - \frac{\omega_s}{c^2 k_x^2} \sum_{m,n,\alpha} \frac{J_{s-n} J_{m-n} \omega_{L\alpha}^2}{\omega_n} (k_y B_z^{(m)} - k_{zi} B_y^{(m)})$$

$$+ \frac{\omega_s}{c} E_x^{(s)} - \frac{1}{c} \sum_{m,n,\alpha} \frac{\overset{\alpha}{J}_{s-n} \overset{\alpha}{J}_{m-n}}{\omega_n} \omega_{L\alpha}^2 E_x^{(m)}$$

$$- \frac{k_i^2}{c k_x^2} \sum_{m,n,\alpha} \frac{(\omega_s - \omega_n)\omega_{L\alpha}^2}{\omega_n^2} J_{s-n}^\alpha J_{m-n}^\alpha E_x^{(m)} = 0, \tag{3}$$

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*$[\mathbf{v}_\alpha^{(0)}\mathbf{B}] \equiv \mathbf{v}_\alpha \times \mathbf{B}$.

$$\left(k_i^2 - \frac{\omega_s^2 - \omega_L^2}{c^2}\right)B_z^{(s)} = (k_y B_z^{(s)} - k_{zi} B_y^{(s)}) + \frac{\omega_s}{c} E_x^{(s)} - \frac{1}{c}\sum_{m,n,\alpha} J_{s-n}^{\alpha} J_{m-n}^{\alpha} \frac{\omega_{L\alpha}^2}{\omega_n} E_x^{(m)}, \quad (4)$$

$$\left(k_i^2 - \frac{\omega_s^2 - \omega_L^2}{c^2}\right)(k_y B_y^{(s)} + k_{zi} B_z^{(s)}) = 0, \quad (5)$$

where J_n^{α} is the Bessel function of order n of the argument $a_{\alpha} = k_x e_{\alpha} E_0 / m_{\alpha} \omega_0^2$; e_{α} and m_{α} are the charge and mass of the plasma particles of type α;

$$\omega_{L\alpha}^2 = 4\pi n_{\alpha}^{(0)} e_{\alpha}^2 / m_{\alpha}; \; \omega_L^2 = \omega_{Le}^2 + \omega_{Li}^2; \; \omega_s = \omega + s\omega_0; \; s = 0, \pm 1, \pm 2, \ldots.$$

If we set the density equal to zero in the system of equations (3), (4), and (5), then we obtain a system of homogeneous equations for the vacuum fields. Equating to zero the determinant of this system, we obtain the spectrum of eigenvalues of the wave vectors k_e (k_x, k_y, k_{ze}) outside the plasma. It is necessary to supplement the resulting system of equations with the boundary conditions:

$$\sum_{k_{ze}} E_x^{(s)}\big|_e = \sum_{k_{zi}} E_x^{(s)}\big|_i, \quad \sum_{k_{ze}} B_z^{(s)}\big|_e = \sum_{k_{zi}} B_z^{(s)}\big|_i,$$
$$\sum_{k_{ze}} (k_y B_y^{(s)} + k_{ze} B_z^{(s)})\big|_e = \sum_{k_{zi}} (k_y B_y^{(s)} + k_{zi} B_z^{(s)})\big|_i, \quad (6)$$
$$\sum_{k_{ze}} B_y^{(s)}\big|_e - \sum_{k_{zi}} B_y^{(s)}\big|_i = \frac{1}{ck_x^2}\Big\{\sum_{n,m,\alpha,k_{zi}} \frac{(\omega_s - \omega_n)\,\omega_{L\alpha}^2}{\omega_n^2} J_{s-n}^{\alpha} J_{m-n}^{\alpha} k_{zi} E_x^{(m)} + \frac{\omega_L^2}{c}\sum_{k_{zi}} B_y^{(s)} - \frac{\omega_s}{c}\sum_{n,m,\alpha,k_{zi}} \frac{\omega_{L\alpha}^2}{\omega_n} J_{s-n}^{\alpha} J_{m-n}^{\alpha} B_y^{(m)}\Big\}_i.$$

Simultaneous solution of Eqs. (4)—(6) yields the spectrum of eigenfrequencies of the bounded plasma in a strong HF field.

We now consider the case of the surface oscillations. In the short wave limit $(k_y^2 + k_x^2)c^2 \gg \omega^2, \omega_L^2, \omega_0^2$, when the oscillations can be assumed to be nearly potential, we obtain from (3) and (4)

$$k_i^2\left(E_x^{(s)} - \sum_{n,m,\alpha} \frac{\omega_{L\alpha}^2}{\omega_n^2} J_{s-n}^{\alpha} J_{m-n}^{\alpha} E_x^{(m)}\right) = 0. \quad (7)$$

By equating to zero the second factor in (7), we determine the spectrum of the volume potential oscillations of plasma in a strong HF field. This spectrum was studied in[2,3].

We consider now the second solution of (7):

$$k_i^2 = k_x^2 + k_y^2 + k_{zi}^2 = 0.$$

Hence we find the possible eigenvalues of the wave vector k_{zi}:

$$k_{zi} = \pm i(k_x^2 + k_y^2)^{1/2} = \pm i k_{\parallel}.$$

In the same approximation, we obtain for the wave vector k_{ze} outside the plasma:

$$k_{ze} = \pm i k_{\parallel}.$$

Consider a plasma layer of thickness d. For the field inside the layer, we assume

$$E_x^{(s)} = E_{xi}^{(s)}(k_{\parallel}) e^{k_{\parallel} z} + E_{xi}^{(s)}(-k_{\parallel}) e^{-k_{\parallel} z}, \qquad (0 < z < d). \quad (8)$$

In the region outside the plasma, using the fact that the fields must be finite at z = ±∞, we obtain

$$E_x^{(s)} = E_{xe}^{(s)}(k_{\parallel}) e^{k_{\parallel} z} \qquad (z < 0).$$
$$E_x^{(s)} = E_{xe}^{(s)}(-k_{\parallel}) e^{-k_{\parallel} z} \qquad (z > d) \quad (9)$$

Substituting the expressions (8) and (9) for the fields in the boundary conditions (6), we obtain the following dispersion equation for the surface oscillations of a plasma layer in a strong HF field[1]:

$$\operatorname{Det}\|A_{mn}\| = 0, \quad (10)$$

where

$$A_{mn} = \delta_{mn} - \frac{\Delta\varepsilon_i(\omega_n)}{1 + \Delta\varepsilon_i(\omega_n)} \sum_s J_{m-s}(a) J_{n-s}(a) \frac{\Delta\varepsilon_e(\omega_s)}{1 + \Delta\varepsilon_e(\omega_s)},$$

$$\Delta\varepsilon_{\alpha}^{(n)} = -\frac{\omega_{L\alpha}^2 (1 \pm e^{-k_{\parallel} d})}{2(\omega + n\omega_0)^2}, \quad \alpha = e, i, \quad a = \frac{k_x E_0}{\omega_0^2}\left(\frac{e_e}{m_e} - \frac{e_i}{m_i}\right).$$

Comparing the derived system of equations with the dispersion relation for the volume oscillations[3], we note that the presence of plasma boundaries results in a renormalization of the Langmuir frequencies:

$$\omega_{L\alpha}^2 \to \bar{\omega}_{L\alpha}^2 = \frac{\omega_{L\alpha}^2}{2}(1 \pm e^{-k_{\parallel} d}).$$

This fact enables us to use the method developed in[3] to solve the infinite system of equations (10). For the case of high-frequency surface oscillations, we obtain:

$$\omega^2 = \bar{\omega}_{Le}^2 \Big\{1 + \sum_{l=-\infty}^{\infty} J_l^2 \frac{\bar{\omega}_{Li}^2}{(l\omega_0 + \bar{\omega}_{Le})^2}\Big\}. \quad (11)$$

In the limit of an infinitely thick layer ($k_{\parallel} d \gg 1$), the oscillation interaction occurring on each of the boundaries becomes negligible, and we obtain the spectrum of the surface oscillations for a semi-infinite plasma:

$$\omega^2 = \frac{\omega_{Le}^2}{2}\Big\{1 + \sum_{l=-\infty}^{\infty} J_l^2 \frac{\omega_{Li}^2}{(l\omega_0\sqrt{2} + \omega_{Le})^2}\Big\}. \quad (12)$$

In the opposite limiting case ($k_{\parallel} d \ll 1$), we get from (11) an oscillation spectrum similar to that of the volume oscillations of an infinite plasma located in an external HF field[3] and also an oscillation spectrum that depends substantially on the thickness of the layer

$$\omega^2 = \frac{k_{\parallel} d}{2}\omega_{Le}^2 \Big\{1 + k_{\parallel} d \sum_{l=-\infty}^{\infty} J_l^2 \frac{\omega_{Li}^2}{[l\omega_0\sqrt{2} + \omega_{Le}\sqrt{k_{\parallel} d}]^2}\Big\}. \quad (13)$$

When the frequency of the external field is much higher than the ion Langmuir frequency, we obtain from the dispersion relation (10) the following simpler equation:

$$1 = \frac{\Delta\varepsilon_i(\omega)}{1 + \Delta\varepsilon_i(\omega)} \sum_{l=-\infty}^{\infty} \frac{J_l^2 \Delta\varepsilon_e(\omega_l)}{1 + \Delta\varepsilon_e(\omega_l)}. \quad (14)$$

If the frequency of the external field and its higher harmonics differ substantially from $\bar{\omega}_{Le}$, we obtain from this equation the spectrum of the low-frequency surface oscillations, which exist only in the presence of an external HF field:

$$\omega^2 = \frac{\omega_{Li}^2}{2}(1 \pm e^{-k_{\parallel} d})\Big\{1 - (1 \pm e^{-k_{\parallel} d}) \sum_{l=-\infty}^{\infty} \frac{\omega_{Le}^2 J_l^2}{\omega_{Le}^2 (1 \pm e^{-k_{\parallel} d}) - 2(l\omega_0)^2}\Big\}. \quad (15)$$

When a harmonic of the external field becomes close to $\bar{\omega}_{Le}$, it follows from this relation that the square of the eigenfrequency ω^2 becomes negative, and an aperiodic growth of the surface oscillations takes place.

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[1] Given a plasma cylinder of radius R, placed in a HF field with the electric field vector along the x axis, we seek all quantities as functions of $\exp(ik_x x + im\varphi)$. In that case

$$\Delta\varepsilon_{\alpha}^{(n)} = -\frac{\omega_{L\alpha}^2 k_x R}{(\omega + n\omega_0)^2} K_m(k_x R) I_m'(k_x R), \quad m = 0, \pm 1, \pm 2, \ldots,$$

where I_m is the Bessel function of imaginary argument, K_m is the Macdonald function, and the derivative is taken with respect to the argument.

In the immediate proximity of a resonance, formula (15) can not be applied. In this case we obtain from (14) the following frequency spectrum:

$$\omega^2 = \frac{1}{8}\bar{\omega}_{Le}^2\zeta_n^2\left\{1 \pm \sqrt{1+\frac{32}{\zeta_n^3}J_n^2(a)\frac{\omega_{Li}^2}{\bar{\omega}_{Le}^2}}\right\}, \quad (16)$$

where

$$\zeta_n = \frac{\bar{\omega}_{Le}^2}{(n\omega_0)^2} - 1.$$

When the plus sign is taken, growth of the oscillations is possible, if

$$-2\left[4J_n^2(a)\frac{\omega_{Li}^2}{\bar{\omega}_{Le}^2}\right]^{1/3} < \zeta_n < 0.$$

This means that when the n-th harmonic of the external field approaches $\omega_{Le}(1 + e^{-k_{\parallel}d})/\sqrt{2}$ from the high-frequency side, the surface oscillations grow, provided

$$n\omega_0 = \frac{\omega_{Le}}{\sqrt{2}}(1+e^{-k_{\parallel}d})^{1/2}\left(1+\left[4J_n^2(a)\frac{\omega_{Li}^2}{\bar{\omega}_{Le}^2}\right]^{1/3}\right). \quad (17)$$

After the threshold is reached, the growth rate of the surface oscillations increases, and when

$$n\omega_0 = \frac{\omega_{Le}}{\sqrt{2}}(1+e^{-k_{\parallel}d})^{1/2}\left(1+\left[\frac{1}{4}J_n^2(a)\frac{\omega_{Li}^2}{\bar{\omega}_{Le}^2}\right]^{1/3}\right) \quad (18)$$

it reaches its maximum value

$$\gamma = \frac{\omega_{Le}}{\sqrt{2}}(1+e^{-k_{\parallel}d})^{1/2}\left(\frac{\sqrt{27}}{32}J_n^2(a)\frac{\omega_{Li}^2}{\bar{\omega}_{Le}^2}\right)^{1/3}. \quad (19)$$

At positive values of the quantity ζ_n, formula (16) describes the growth of the oscillations when the minus sign is taken. We have then the following expression for the growth rate of the oscillations

$$\gamma = \frac{\omega_{Le}}{4}(1+e^{-k_{\parallel}d})^{1/2}\zeta_n\left\{\sqrt{1+\frac{32}{\zeta_n^3}J_n^2\frac{\omega_{Li}^2}{\bar{\omega}_{Le}^2}}-1\right\}^{1/2}. \quad (20)$$

This quantity reaches its maximum value

$$\gamma = \frac{\omega_{Le}}{\sqrt{2}}(1+e^{-k_{\parallel}d})^{1/2}\left[\frac{1}{2}J_n^2(a)\frac{\omega_{Li}^2}{\bar{\omega}_{Le}^2}\right]^{1/3} \quad (21)$$

when

$$n\omega_0 = \frac{\omega_{Le}}{\sqrt{2}}(1+e^{-k_{\parallel}d})^{1/2}\left\{1-\left[\frac{1}{2}J_n^2(a)\frac{\omega_{Li}^2}{\bar{\omega}_{Le}^2}\right]^{1/3}\right\}. \quad (22)$$

When the harmonic of the external frequency moves farther from the value $\omega_{Le}(1 + e^{-k_{\parallel}d})/\sqrt{2}$, formula (15) becomes applicable. Finally, when the harmonic of the external field approaches $\omega_{Le}(1 - e^{-k_{\parallel}d})/\sqrt{2}$, it is necessary to use formulas (17)—(22), in which, however, the quantity $(1 + e^{-k_{\parallel}d})$ must be replaced by $(1 - e^{-k_{\parallel}d})$.

Thus, a bounded plasma placed in a strong HF field is unstable against excitation of surface oscillations. The short-wave surface oscillations ($k_{\parallel}d \gg 1$) are unstable to external field frequencies ω_0 that are close to or lower than $\omega_{Le}/\sqrt{2}$. In the same range of frequencies ω_0, as previously noted[3], the volume oscillations are also unstable. However the growth rate of the volume oscillations is then of the order of the ion Langmuir frequency. It follows from the formulas derived above that when the external field frequency approaches $\omega_{Le}/\sqrt{2}$, only short wavelength ($k_{\parallel}d \gg 1$) surface oscillations will be excited, with a growth rate

$$\sim \frac{\omega_{Le}}{\sqrt{2}}\left(\frac{m_e}{m_i}\right)^{1/3}J_1^{2/3}.$$

When the external field frequencies are much lower than the electron Langmuir frequency, then the long-wave ($k_{\parallel}d \ll 1$) surface oscillations are also unstable. The fastest to grow, with a growth rate

$$\sim \frac{k_{\parallel}d}{\sqrt{2}}\omega_{Le}\left(\frac{m_e}{m_i}\right)^{1/3}J_1^{2/3}$$

are the oscillations with the wave vector

$$k_{\parallel} = \frac{\sqrt{2}\,\omega_0}{\omega_{Le}}d^{-1}.$$

Over a wide range of frequencies of the external field, the growth rate of such oscillations is also higher than that of the volume oscillations.

Therefore it can be concluded that in a nontransparent plasma ($\omega_0 \lesssim \omega_{Le}/\sqrt{2}$) the most dangerous type of instability caused by the presence of an external HF field will be the excitation of surface oscillations.

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