STATISTICS OF APPEARANCE OF ULTRASHORT LIGHT PULSES IN A LASER WITH A BLEACHABLE FILTER

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A probabilistic approach to mode-locking in lasers is developed. The transformation of radiation with random time characteristics into a sequence of ultrashort pulses is described. Cases of complete and partial mode-locking are considered and conditions for complete mode-locking in a laser with a bleachable filter are determined.

THE early studies of time characteristics of radiation generated under mode-locking conditions by a laser with a bleachable filter[1] led us to expect that such lasers may be used to obtain extremely short light pulses up to 10^{-2}—10^{-13} sec. Several successive experiments[2,3] established the assumption that this is always possible, i.e., that the available lasers with bleachable filters always emit light in the form of a sequence of identical pulses whose spacing is completely determined by the resonator geometry. A number of theoretical papers, such as[4,5], presented computations of the length and shape of such pulses, while the existence of a regular time pattern itself was accepted as a fact. However the literature does not include a theoretical foundation of the assumption that the time structure of the emission should represent a periodic sequence of single short pulses[1)]. It was merely known[1] that the bleachable filter exhibits a tendency to emphasize the most intense fluctuations of the initial emission. However the extent to which this tendency is manifested, and whether a single intense spike or several emerge from the initial time structure of the fluctuation, have not yet been studied. As to the experimental investigation of mode-locking in lasers, the conclusion that a regular time structure was observed was based on erroneous methods of recording the time characteristics of the emission (a critique of these methods is given in[8]). The first study of laser emission with a high time resolution was only recently reported in an experimental paper[9]. This study revealed that, against all expectations, a laser with a bleachable filter emits a very irregular time structure rather than a "regular" sequence of pulses. In reference to this result we wish to emphasize that mode-locked lasers can have various operating regimes with more or less regular time structure.

The present work attempts to describe the process of formation of emission with definite time characteristics from the initial random time structure due to spontaneous noise. Cases of complete and partial mode-locking are described and conditions for the emission of "true" ultrashort pulses by a bleachable filter laser are indicated.

1. TRANSFORMATION OF THE RADIATION FIELD WITH MODE-LOCKING

As generation develops in a laser with bleachable filter, the changes in the field profile of interest occur within a time interval with the following characteristics. During this time interval the filter is bleached so that its absorption coefficient falls to zero from some initial magnitude. The gain of the active element remains practically constant during that time and is not frequency dependent; the contribution of spontaneous radiation to the field intensity is negligibly small.

At the beginning of the bleaching process the time dependence of the radiation field $E_0(t)$ at the laser output can be represented as follows:

$$E_0(t) = \mathrm{Re}(e^{-i\omega t}\mathscr{E}_0(t)\exp\{(\beta - \alpha_{ln} - \alpha_{nl})[t/T]\}). \quad (1)$$

Here e^{β} is the field gain in the active element, $\exp(-\alpha_{ln})$ is the attenuation in reflection from mirrors and from other possible linear losses, $e^{-\alpha}$ is the initial transmission of the filter, and T is the resonator period equal to the time of traversal by light of two lengths of the resonator, $T = 2L/c$. Square brackets designate the integral part of the number, i.e., $[t/T]$ is the number of periods. The high-frequency oscillations of the field are defined by the factor $e^{-i\omega t}$ where ω is one of the natural frequencies of the resonator, $\omega = 2\pi N/T$, and N is an integer, $N \gg 1$. $\mathscr{E}_0(t)$ is a slow envelope that is a periodic function of time with a period T. The field increase during the period T is characterized by the quantity $\exp(\beta - \alpha_{ln} - \alpha_{nl})$.

At the end of the bleaching process the time dependence of the field has the form

$$E(t) = \mathrm{Re}(e^{i\omega t}\mathscr{E}(t)\exp\{(\beta - \alpha_{ln})[t/T]\})\cdot\mathrm{const}. \quad (2)$$

The field increase per period is characterized in (2) by the quantity $\exp(\beta - \alpha_{ln})$ and not by $\exp(\beta - \alpha_{nl})$ as in (1), in accordance with the fact that the filter becomes transparent at the end of the bleaching process. The function $\mathscr{E}(t)$ as before remains periodic with period T but it does not coincide with $\mathscr{E}_0(t)$.

1)We consider a simple laser resonator geometry where complete mode-locking means the presence of a single pulse with an intermode-beat period. The presence of parasitic reflections in the resonator or inexact placement of the filter near the mirror are known to render the time structure of the emission more complex in a fully defined manner [6,7]. Such distortions of the time structure are not considered here.

We call the function $\mathscr{E}_0(t)$ the initial field profile and $\mathscr{E}(t)$ the final profile. The successive stages of generation after bleaching can be accompanied by additional profile distortions; however these are not discussed here because they are not connected with the filter-induced field variation under consideration. Considering the deformation of the field profile we merely compare $\mathscr{E}(t)$ with $\mathscr{E}_0(t)$. The relation of $\mathscr{E}(t)$ to $\mathscr{E}_0(t)$ depends on the characteristics of the filter material and can be quite complex. From the mode-locking point of view the best is the inertialess (relative to low-frequency field variation) filter operating according to the two-level scheme. The law of deformation of the field profile was determined in[10] for such a filter assuming certain laser geometries. According to[10] the final profile is expressed in terms of the initial profile by the formula

$$\mathscr{E}(t) = \mathscr{E}_0(t)\,|\mathscr{E}_0(t)|^{p-1}, \tag{3}$$

$$p = 1 + \alpha_{\rm ln} / (\beta - \alpha_{\rm ln} - \alpha_{\rm ln}). \tag{4}$$

According to (3) the time structure of the field amplitude modulus is distorted: the amplitude modulus is raised to the power p and the time structure of the phase remains unchanged. Larger values of p obviously entails a larger deformation of the field. We note that the expression for p includes the product of coefficient α_{nl} characterizing absorption per period and the quantity $(\beta - \alpha_{\rm ln} = \alpha_{\rm nl})^{-1}$ approximately determining the number of periods during which the field effectively interacts with the filter (see (1)). If the gain β is significantly higher than the threshold value of $\alpha_{\rm ln} + \alpha_{\rm nl}$, the field increases rapidly and quickly reaches the intensity at which the filter becomes transparent. The profile variation is small in this case. On the other hand if the gain excess over threshold is negligible, i.e., $\beta \approx \alpha_{\rm ln} + \alpha_{\rm ln}$, the field increases slowly and interacts with the filter for a long period of time. Here p, the index of nonlinear transformation of the initial field, is a large number.

2. INITIAL FIELD PROFILE

The initial field profile generated by spontaneous noise is formed within the linear portion of generation development that precedes bleaching. Owing to the linearity of the regime and the Gaussian nature of the spontaneous emission process, the field in this region is also a Gaussian random process. The ensemble-averaged characteristics of this process were computed in[11]. In our problem we select an ensemble of initial functions and emphasize the spectral width using the results of[11] for quantitative computations; we also see to it that the probability distribution for the field amplitude is close to Gaussian.

We consider that the initial field profile has the form

$$\mathscr{E}_0(t) = \sum_{n=1}^{m} A_n e^{i\varphi_n} f(t - \tau - t_n), \tag{5}$$

where f(t) is a pulse function such that

$$f(t) = \sin(\pi m t / T) \text{ for } 0 \leqslant t \leqslant T/m,$$
$$f(t) = 0 \text{ for } T/m \leqslant t \leqslant T;$$
$$f(t+T) = f(t), \tag{6}$$

and the pulses are considered equidistant

$$t_n = \frac{T}{m}(n-1). \tag{7}$$

It is assumed that τ, A_n, and φ_n are independent random quantities; the magnitude of the shift τ is uniformly distributed in the interval $(0, T/m)$, the phases φ_n are uniformly distributed in the interval $(0, 2\pi)$, and the amplitudes A_n obey the Rayleigh distribution

$$w(A_n^2)\,d(A_n^2) = \exp(-A_n^2)\,d(A_n^2). \tag{8}$$

To abbreviate the notation the normalization is selected such as to render the ensemble-averaged values $\langle A_n^2 \rangle = 1$. We now consider the spectral representation of the function $\mathscr{E}_0(t)$. Expanding the periodic function $\mathscr{E}_0(t)$ into a Fourier series:

$$\mathscr{E}_0(t) = \sum_{k=-\infty} e^{2i\pi kt/T} B_k.$$

Taking (5)–(7) into account, the Fourier coefficients can be expressed in terms of the amplifides and phases of the pulses:

$$B_k = \frac{1}{\pi m}\frac{1 + e^{2i\pi k/m}}{1 - (2k/m)^2} e^{-2i\pi\tau/T} \sum_{n=1}^{m} A_n \exp\left(i\varphi_n - 2i\pi k\frac{n}{m}\right). \tag{9}$$

Using the distribution for φ_n and for A_n we can readily obtain from (9) the ensemble-averaged values of Fourier coefficients and their mean squares S_k:

$$\langle B_k \rangle = 0, \quad S_k = \langle B_k B_k^* \rangle = \frac{2}{\pi^2 m}\frac{1 + \cos(2\pi k/m)}{\{1 - (2k/m)^2\}^2}. \tag{10}$$

The dependence of the mean squares of Fourier coefficients on the number k expressed by (10) is shown in Fig. 1. We note that the spectral width is determined by the number m which is the number of pulses in a period. Indeed, the figure shows that the quantity S_k reaches one-half of its maximum value when $k = 0.6m$. Therefore by selecting the parameter m we can make the width of the initial spectrum correspond to any given specific laser. It should be remembered that this width is much smaller than that of the luminescence line of the active material (see[11]). This is due to the fact that the selected initial field profile is that which is established in the beginning of the bleaching process as a result of prolonged and spectrally inhomogeneous amplification of spontaneous noise.

3. FINAL FIELD PROFILE

We consider the time characteristics of the field that are established by the filter bleaching process. Using (5)–(7) for the initial profile and the law of transformation of the initial field (3) we can represent the final profile in the form

$$\mathscr{E}(t) = \sum_{n=1}^{m} A_n^p e^{i\varphi_n} \{f(t - \tau - t_n)\}^p. \tag{11}$$

The action of the filter on the radiation field causes pulse narrowing and variation in the relative intensities of the pulses.

Pulse narrowing is indicated in (11) by the fact that the pulse function f(t) is transformed into the function $\{f(t)\}^p$. We determine the spectral broadening caused by this transformation of the pulse function. The expansion of $\mathscr{E}(t)$ into a Fourier series is written as

$$\mathscr{E}(t) = \sum_{k=-\infty}^{\infty} b_k e^{2i\pi kt/T}.$$

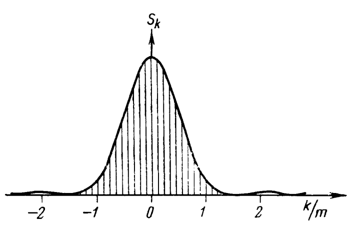

FIG. 1. Spectrum of initial field in a laser corresponding to random process (5). The magnitude S_k of the spectral component intensity (10) is laid off along the abscissa axis and the number k of the spectral component divided by the effective number of modes m is laid off along the ordinate axis.

It follows from (6), (7), and (11) that

$$b_k = \frac{1}{m2^p}\frac{\Gamma(p+1)}{\Gamma(1+p/2-k/m)\Gamma(1+p/2+k/m)} \times \exp\left(-2i\pi k\frac{\tau}{T}+i\pi\frac{k}{m}\right)\sum_{n=1} A_n{}^p \exp\left(i\varphi_n - 2i\pi k\frac{n}{m}\right). \quad (12)$$

Using probability distribution for φ_n and A_n we obtain an expression for the mean squares of Fourier coefficients from (12)

$$\langle b_k b_k^*\rangle = \frac{1}{m2^p}\{\Gamma(p+1)\}^3\left\{\Gamma\left(1+\frac{p}{2}-\frac{k}{m}\right)\Gamma\left(1+\frac{p}{2}+\frac{k}{m}\right)\right\}^{-2}. \quad (13)$$

It follows from (13) for the case of $p \gg 1$ that the spectral component intensity comprises one-half of the intensity of the maximum component ($\langle b_k b_k^*\rangle = \frac{1}{2}\langle b_0 b_0^*\rangle$) for $|k| = 0.3m\sqrt{p}$. The width of the spectrum thus increases in proportion to $\sqrt{p}$. In the general case the broadening of the spectrum corresponds to a shortening of the characteristic time scale of field variation. In our case this means narrowing of the pulse functions. The ratio of the final to the initial pulse widths is

$$\frac{\Delta t}{\Delta_0 t} = \int_0^{T/m} \sin\pi m\frac{t}{T}\,dt \Big/ \int_0^{T/m}\left\{\sin\pi m\frac{t}{T}\right\}^p dt = \frac{\sqrt{\pi}}{2}\frac{\Gamma(p/2+1/2)}{\Gamma(p/2+1)} \approx \sqrt{\frac{\pi}{2p}}. \quad (14)$$

Beside the pulse narrowing effect the filter has another and more important effect of emphasizing the most intense pulses. We see from (5) and (11) that if the ratio of amplitudes of two pulses was A_1/A_2 before bleaching, it is changed after bleaching to $(A_1/A_2)^p$. This means that the initial energy distribution among the pulses can be strongly affected. The most interesting is the situation in which one of the pulses that underwent change in one period turns out to be much more intense than all the others and captures almost all the energy of emission. It is indeed in this case that mode-locking can be considered complete. At the same time our approach also allows us to analyze the case of partial mode-locking. This is the case in which several intense pulses, instead of one, remain in the final field profile.

We emphasize that our definition of complete mode-locking coincides with the generally accepted definition when formulated on the basis of the spectral rather than the temporal approach. In fact if we assume that the maximum pulse is numbered n_1 and so $A_{n_1}^p \gg \Sigma A_n^p$ (summation over $n \neq n_1$), an expression for the spectral components is readily obtained from (12)

$$b_k = \frac{1}{m2^p}\frac{\Gamma(p+1)}{\Gamma(1+p/2-k/m)\Gamma(1+p/2+k/m)}A_{n_1}{}^p \cdot \times \exp\left(-2i\pi k\frac{\tau}{T}+i\pi\frac{k}{m}+i\varphi_{n_1}-2i\pi k\frac{n_1}{m}\right) \quad (15)$$

It follows from (15) for the phase components

$$\arg b_k = \varphi_{n_1} + k(\pi/m - 2\pi\tau/T - 2\pi n_1/m),$$

i.e., the spectral component phases depend linearly on the number k which corresponds to the generally accepted definition of mode-locking.

We note that both effects, the narrowing of pulses and the separation of intense pulses, occur simultaneously in the same field-filter interaction process. However only the second effect is equivalent to mode-locking of the initial spectrum. It is this effect that can result in the formation of a regular time structure from the initially random time structure. On the other hand, the pulse narrowing effect represents field spectrum broadening and has no direct relation to mode-locking of the initial spectrum[2)]. We can easily imagine a laser model in which mode-locking is not accompanied by spectral broadening. For example spectral broadening can be prevented by the dispersion of gain. If the initial field is artificially created in a laser with a bleachable filter and the field spectral width coincides with the gain bandwidth, the spectral broadening of the field is hampered although the mode-locking effect remains and is manifested qualitatively in the preferential growth of the most intense pulses, as in our laser model.

4. CONDITIONS FOR COMPLETE MODE-LOCKING

We derive the quantitative formulation of the above definition of complete mode-locking. We consider that complete mode-locking has occurred if the energy in the most intense pulse is at least M times larger than the energy emitted in a period T in all the remaining pulses:

$$I_{n1} \geqslant M\sum_{n\neq n_1} I_n. \quad (16)$$

Here I_n is the energy of the n-th pulse, $I_n \equiv A_n^{2p}$, n_1 is the number of the maximum pulse, and M is a large number (we set M = 9 in specific cases).

By virtue of the statistical nature of the problem we can require that inequality (16) be satisfied only within a certain accuracy limit η. Therefore we may expect that the laser regime ensures complete mode-locking if the probability W of satisfying (16) is larger than a given value of η

$$W\left\{I_{n_1} \geqslant M\sum_{n\neq n_1} I_n\right\} \geqslant \eta. \quad (17)$$

(In the computations we assumed η = 0.5). The problem is to determine the parameters m and p which satisfy inequality (17); the values of m and p, turning (17) into an equality, yield the boundary between the regions of complete and partial mode-locking.

We first obtain the sufficient condition of complete

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2)It is appropriate to note that in [4,12] spectral broadening is erroneously identified with mode-locking.

mode-locking, i.e., we find at least a part of the region (p, m) in which (17) is satisfied. It is readily seen that (16) is satisfied by definition if the intensity of the maximum pulse is M(m − 1) times larger than that of any other pulse

$$I_{n_1} \geqslant M(m-1) I_n, \quad n \neq n_1. \qquad (18)$$

We now consider the probability W_1 that (18) is satisfied. Since (16) follows from (18), $W \geq W_1$ and the requirement that

$$W_1\{I_{n_1} \geqslant M(m-1) I_n\} \geqslant \eta \qquad (19)$$

is a sufficient condition of mode-locking. As shown in the Appendix, the quantity W_1 is expressed in terms of p, m, and M in the following manner:

$$W_1\{I_{n_1} \geqslant M(m-1) I_n\} = \Gamma(a+1)\Gamma(m+1)/\Gamma(m+a), \qquad (20)$$

where

$$a = \{M(m-1)\}^{1/p}. \qquad (21)$$

Thus the sufficient condition (19) for mode-locking assumes the form

$$\Gamma(a+1)\Gamma(m+1)/\Gamma(m+a) \geqslant \eta. \qquad (22)$$

Equations (22) and (21) determine the range of values of the nonlinear transformation index p and the number of modes in the spectrum m[3], for which complete mode-locking is defined. When the spectral width is large ($m \gg 1$) the sufficient condition for mode-locking can be obtained from (21) and (22) in the form of a dependence of p on m:

$$p \geqslant \frac{(\ln m)^2 + \ln m \cdot \ln M}{\ln(1/\eta)} \qquad (23)$$

We now proceed to find the necessary condition for complete mode-locking. The definition of complete mode-locking (16) leads to the inequality

$$I_{n_1} \geqslant M I_n, \quad n \neq n_1. \qquad (24)$$

Therefore the probability W_2 of satisfying (24) is larger than the probability W of satisfying (16): $W_2 \geq W$ and if the inequality

$$W_2\{I_{n_1} \geqslant M I_n\} > \eta, \qquad (25)$$

is satisfied in some region (p, m) the condition of complete mode-locking is also satisfied with a probability W smaller than η.

In the Appendix we derive an expression for W_2 in terms of the parameters m, p, and M:

$$W_2 = \Gamma(b+1)\Gamma(m+1)/\Gamma(m+b), \qquad (26)$$

where

$$b = M^{1/p}. \qquad (27)$$

We find from (25) that at least for

$$\Gamma(b+1)\Gamma(m+1)/\Gamma(m+b) < \eta \qquad (28)$$

complete mode-locking does not occur. Equations (28) and (27) allow us to determine the range of values of parameters p and m in which a partial mode-locking occurs. When $m \gg 1$ we find an explicit dependence of p on m from (27) and (28):

$$p < \frac{\ln m \cdot \ln M}{\ln(1/\eta)}. \qquad (29)$$

Both (29) and (23) are approximate formulas although they can be used in practice even when m = 10.

The obtained results are conveniently plotted in the coordinates p and m; see Fig. 2. Here curve 1 corresponds to the sign of equality in (22) and curve 2 to the sign of equality in (28).

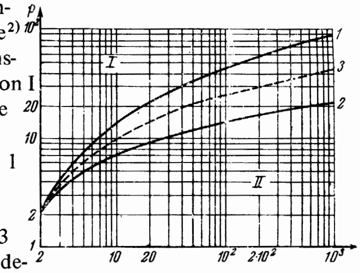

FIG. 2. Two regions of values of the parameters m and p (m is the initial number of modes, see footnote[2]) and p is the index of nonlinear transformation of the initial field). Region I corresponds to the case of complete mode-locking and Region II to the case of partial mode-locking. Curve 1 corresponds to the sign of equality in (22), Curve 2 corresponds to the sign of equality in (28), and Curve 3 is the assumed boundary of the mode-locking region.

In accordance with the above the region of values (p, m) above curve 1 corresponds to the case of complete mode-locking. Given these values of p and m, a period of T = 2L/c contains with a probability greater than 0.5 only a single intense pulse $\Delta t \approx T/m\sqrt{p}$ long and the total energy contained in this pulse exceeds at least 9 time the energy of all the remaining pulses.

The region of values (p, m) below curve 2 corresponds by definition to the case of partial mode-locking. This means that for those values of p and m there are, with a probability greater than 0.5, several pulses with commensurate intensities in the period T = 2L/c.

The true boundary between the regions of values of (p, m) corresponding to complete and partial mode-locking runs between curves 1 and 2. Additional research is necessary for an exact determination of this boundary. The curve is plotted as a geometric mean of the values p(m) determined by curves 1 and 2.

Figure 2 clearly demonstrates the fact that the larger the number m of the modes the more difficult the achievement of mode-locking. The graphs contained in the figure show that for a large m complete mode-locking requires larger values of the nonlinear transformation index p. Indeed the quantity p is given for each specific laser and is exceedingly difficult to change.

We note that the literature contains an erroneous notion in this matter. It is claimed in[12] that the broader the initial spectrum (other laser parameters being the same) the easier the mode-locking. As we have shown above the opposite relationship is true. The nature of the dependence of p on m can be explained qualitatively by the fact that a large initial number of modes increases the probability of several commensurable high-intensity spikes appearing in the initial field and therefore a higher order nonlinearity is necessary to separate the most intense single spike in a reliable manner.

[3]The initial number of modes could be defined more precisely by the quantity (m + 1) (see Fig. 1 and discussion of (10)); however since we are mainly interested in the case of $m \gg 1$, we consider it possible for the sake of brevity to designate the parameter m as the initial number of modes.

5. EXPERIMENTAL METHODS TO OBTAIN COMPLETE MODE-LOCKING

Detailed evaluation of mode-locking experiments requires accurate data on the relaxation constants of the bleachable filters used in lasers. In the absence of such data we first assume for rough evaluation that two-level bleachable filters are being used in existing lasers, i.e., that our results are directly applicable to actual systems. As noted before, the basic parameters p and m are related to those laser characteristics that are established at the beginning of the bleaching process. The quantities p and m can be obtained by considering a linear region of generation development and performing computations analogous to those presented in[11]. It follows from the computations that

$$p = \frac{\alpha_{nl}}{\alpha_{ln} + \alpha_{nl}} \left(\frac{Y}{Z} \right)^{1/2}, \quad m = T\Delta\omega_{lum} (2\pi)^{-3/2} (YZ)^{-1/4};$$

$$Y = (\alpha_{ln} + \alpha_{nl}) \frac{t_{pump}}{T} \left(\frac{1}{2} \frac{U}{U_{th}} \right)^{-1/2} \left(\frac{U}{U_{th}} - 1 \right)^{-1/2}$$

$$Z = \ln \left\{ \frac{I_{bl}}{I_{sp}} Y^{-3/4} (\alpha_{ln} + \alpha_{nl})^2 \right\} + \frac{3}{4} \ln \ln \left\{ \frac{I_{bl}}{I_{sp}} Y^{-3/4} (\alpha_{ln} + \alpha_{nl})^2 \right\}$$

Here $\Delta\omega_{lum}$ is the width of luminescence line of the active specimen, t_{pump} is the duration of pumping, U and U_{th} are the pumping energy and its threshold value, I_{bl} is the radiation flux density necessary to bleach the filter, I_{sp} is the density of flux emitted by spontaneous sources of the active specimen into a solid angle corresponding to axial modes, and α_{nl} and α_{ln} are loss coefficients determined as follows: $e^{-\alpha}_{nl}$ is the initial filter transmission (based on attenuation of wave intensity in a single traversal of the entire absorbing layer), and $e^{-\alpha}_{ln}$ is the attenuation of wave intensity in a single traversal of the resonator length due to other losses; this quantity is usually expressed in terms of mirror reflection coefficients r_1 and r_2: $e^{-\alpha} = \sqrt{r_1 r_2}$.

Considering the existing solid-state lasers operating in the mode-locking regime we set $t_{pump} = 0.5 \times 10^{-3}$ sec, $U/U_{th} = 1.03$, $I_{bl}/I_{sp} = 10^9$, $e^{-\alpha}_{nl} = 0.70$, $e^{-\alpha} = 0.90$, and the resonator period $T = 2L/c = 6 \times 10^{-9}$ sec (laser 100 cm long). The computed nonlinear transformation index here is $p = 85$. We assume the width of the neodymium glass luminescence line $\Delta\omega_{lum} = 6 \times 10^{13}$ sec^{-1}; the initial number of modes then is $m = 600$ (the initial spectral width is $\Delta\omega = 6.5 \times 10^{11}$ sec^{-1}). For ruby, assuming $\Delta\omega_{lum} = 2 \times 10^{12}$ sec^{-1}, we obtain $m = 20$ (initial spectral width $\Delta\omega = 2.2 \times 10^{10}$ sec^{-1}). According to Fig. 2 both cases $p = 85$, $m = 600$ and $p = 85$, $m = 20$ correspond to the region of complete mode-locking. The figure also shows that the mode-locking condition is met to excess in the ruby laser case. On the other hand in the case of the neodymium laser a slight variation in the initial parameters can, by reducing the p index, disrupt the conditions required by complete mode-locking. Therefore the neodymium laser requires a stricter control of the pump power and the ratio of linear and nonlinear losses than does a ruby laser.

Thus we can expect complete mode-locking in existing ruby and neodymium glass lasers provided two-level inertialess filters are used. The use of inertial filters however significantly degrades the conditions of mode-locking. If the relaxation rate of γ of the population difference of the filter is smaller than the initial field spectral width $\Delta\omega = 2\pi m/T$ the exponential law of field transformation (3) does not hold. Nevertheless the law can be used for rough estimates of a single pulse probability by reducing $\gamma/\Delta\omega$ times the value of nonlinear losses per period in (4), i.e., by introducing in this manner an effective nonlinear transformation index $p_{eff} = 1 + (p - 1)\gamma/\Delta\omega$. Such a reduction of p means that filter inertia weakens the effectiveness of mode-locking to an extraordinary degree. Deviations from the two-level scheme of filter action also degrade the mode-locking regime: relaxation to other levels, depopulating the working level pair, can significantly decrease the nonlinear effect of the filter on the field.

In connection with this one can expect that at least not all lasers operating in the mode-locking regime can achieve complete mode-locking. Apparently the nonregularity of time characteristics of emission observed in[9] is not an accidental phenomenon and the problem of experimental production of complete mode-locking requires special attention. The solution of this problem can obviously be promoted by a search for bleachable materials with optimal characteristics. However it also seems desirable, using existing bleachable filters, to increase the effectiveness of mode-locking by readjusting the operating regime of the laser.

One of the operating regimes suitable for mode-locking corresponds to an increased number of periods during which the filter-field interaction is effective. This can be achieved by reducing the rate of field increase in the laser using a feedback circuit producing additional losses in the laser and working on the principle of an inertial darkening filter.

Another variant of such a regime is a system in which a partially mode-locked emission generated by another laser, and not the spontaneous noise, is the initial laser field. A correct selection of the time delay of the driving and passively excited laser can produce a sum of indices ($p_{eff} = p_1 + p_2$) that are achievable in each laser separately.

APPENDIX

COMPUTATION OF PROBABILITIES OF COMPLETE PARTIAL MODE-LOCKING

According to Sec. 4, to find the sufficient condition for complete mode-locking we must compute the probability W_1 of satisfying (18); this means that the intensity of the maximum pulse is $M(m - 1)$ times larger than the intensity of any other pulse. Since $I_n = (I_{0n})^p$, (18) computed in terms of initial intensities is equivalent to

$$I_{0n_1} \geqslant aI_{0n}, \quad a = \{M(m-1)\}^{1/p}. \tag{A.1}$$

The probability W_1 of satisfying (18) (or (A.1), which is the same thing) consists of m identical terms. The first yields the probability of the first pulse being maximal so that inequality (A.1) is satisfied for $n_1 = 1$, the second yields the same result for $n_1 = 2$, etc.:

$$W_1 = mW_1^{(n_1=1)}. \tag{A.2}$$

Since the amplitudes of different pulses are independent, the probability $W_1^{(n_1=1)}$ is determined by the integral

$$W_1^{(n_1=1)}=\int_0^\infty dx\, w(x)\left\{P\left(\frac{x}{a}\right)\right\}^{m-1}. \tag{A.3}$$

Here $x = I_{01}$, $w(x) = e^{-x}$ is the probability distribution of the initial pulse intensity (see (8)), and $\{P(x/a)\}^{m-1}$ is the probability that each of the $m - 1$ independent random values of I_{0n} is less than x/a; here

$$P(\xi)=\int_0^\xi w(\xi')\,d\xi'=1-e^{-\xi}.$$

Finally, computing the integral in (A.3), we obtain expression (20) given in the text.

In finding the necessary condition for complete mode-locking (see Sec. 4) analogous considerations lead to an expression for probability W_2 of satisfying (24):

$$W_2=m\int_0^\infty dx\, w(x)\left\{P\left(\frac{x}{b}\right)\right\}^{m-1}. \tag{A.4}$$

Computing the integral in (A.4), we obtain (26) given in the text.

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