EDDY CURRENTS IN THE THERMOMAGNETIC AND ACOUSTOMAGNETIC EFFECTS

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Eddy currents arise in a conductor in which a temperature gradient or sound beam exists and which is located in an external magnetic field perpendicular to these vectors. The eddy currents flow in this case in a plane perpendicular to the magnetic field. The currents produce an additional magnetic field whose strength may be of the same order of magnitude or in some cases even larger than that of the external field. In a plane perpendicular to the magnetic field, the conductor is divided into domains arranged in the ∇T direction, the eddy-current magnetic field being oppositely directed in neighboring domains.

It is shown in the present paper that electric eddy current with magnetic fields comparable with or larger than the external magnetic field are produced in a number of metals and semimetals in the presence of an external magnetic field $\mathbf{H}_0$ and a temperature gradient ∇T or an acoustic flux of density $\mathbf{S}$. The Nernst coefficient α_1 measured in such conditions, or the analogous transverse acousto-electric coefficient β_1, depends strongly on ∇T or $\mathbf{S}$.

The reason for the eddy currents lies in the following. If $\mathbf{H}_0$ (the z axis) and ∇T (the x axis) are mutually perpendicular, then the thermomagnetic part $\alpha_1 \nabla T \times \mathbf{H}_0$ of the electric field*

$$\mathbf{E} = \alpha\nabla T + \alpha_1[\nabla T\mathbf{H}_0]$$

is not potential if

$$\frac{\partial}{\partial x}\left(\alpha_1\frac{\partial T}{\partial x}\right) = \varkappa\left(\frac{\partial T}{\partial x}\right)^2\frac{\partial(\alpha_1/\varkappa)}{\partial T} \neq 0$$

(κ is the thermal conductivity). The condition that the electric field must be potential leads to the necessity for the occurrence of eddy currents, such that

$$\operatorname{rot}(\eta\mathbf{j} + \alpha_1[\nabla T\mathbf{H}]) = 0 \qquad (1)$$

(η is the resistivity). On the boundary of the conductor, the current component normal to the boundary vanishes and therefore the current paths are closed inside the conductor. To obtain the eddy-current part of the acoustoelectric field it is sufficient to take into account the dependence of S on the coordinates, due to the absorption of the sound $S = S_0 e^{-x/l}$ (l—absorption length of the sound).

We carry out the calculation assuming a weak magnetic field H_j of the eddy currents, $c^{-1}\mu_\mp H_j \ll 1$ ($\mu_\mp$—mobilities of the electrons and holes), and a small relative temperature drop or a relative change of the acoustic flux. In this case we can disregard the dependence of the kinetic coefficients on H_j. The calculation is the same for the cases of the thermoelectric and acoustoelectric fields; $\alpha_1\partial T/\partial x$ and $\beta_1 S$ will be denoted by J. Let the crystal be a rectangular parallelepiped, and let its dimension in the z direction be much larger than the other dimensions $2L_x$ and $2L_y$. Then the eddy currents flow in the xy plane, and their magnetic field H_j is parallel to the z axis. From (1) we obtain an equation determining the distribution of the field H_j inside the conductor:

$$\Delta H_j + a\partial H_j/\partial x + b(H_j + H_0) = 0, \qquad (2)$$

where

$$a = -\frac{1}{\eta}\left(\frac{\partial\eta}{\partial x} + \frac{4\pi J}{c}\right) \qquad b = -\frac{4\pi}{c\eta}\frac{\partial J}{\partial x}$$

On the boundaries of the conductor, the normal component of the current is equal to zero, meaning that the magnetic field is constant along the boundary; from this it follows in turn that outside the conductor $H_j = 0$; consequently, $H_j = 0$ at $x = \pm L_x$ and arbitrary y and at $y = \pm L_y$ and arbitrary x. The x axis is directed in such a way as to make $a > 0$.

Let us expand the field H_j and the inhomogeneous term bH_0 in (2) in terms of the eigenfunctions H_{mn} of the operator $\Delta + a\partial/\partial x$ satisfying the zero boundary conditions. It can be readily shown that

$$H_{mn} = e^{-ax/2}(e^{i(m\pi/2L_x)x} - (-1)^m e^{-i(m\pi/2L_x)x})\cos[(n + 1/2)\pi y/L_y],$$

and the eigenvalues of the operator $\Delta + a\partial/\partial x$ are equal to

$$\lambda_{mn} = -\frac{a^2}{4} - \frac{\pi^2}{4}\left[\frac{m^2}{L_x^2} + \frac{(2n+1)^2}{L_y^2}\right], \qquad m, n = 0, 1, 2, \ldots \qquad (3)$$

Since the eigenfunctions are not orthogonal, the expansion of the inhomogeneity bH_0 can be obtained by starting from the equality

$$1 = \frac{8}{\pi^2}e^{-ax/2}\sum_{n=0}^{\infty}(-1)^n\frac{\cos[(n+1/2)\pi y/L_y]}{n+1/2}$$
$$\times\sum_{m=0}^{\infty}(-1)^m\{A_{2m+1}\operatorname{ch}(aL_x/2)\cos[(m+1/2)\pi x/L_x] - A_{2m}\operatorname{sh}(aL_x/2)\sin(m\pi x/L_x)\};$$

here

$$A_k = \frac{1}{k[1 + (aL_x/k\pi)^2]}.$$

If

$$bH_0 = \sum_{m,n} c_{mn}H_{mn}, \qquad H_j = \sum_{m,n}\gamma_{mn}H_{mn},$$

then

$$\gamma_{mn} = -\frac{c_{mn}}{\lambda_{mn} + b},$$
$$c_{mn} = 8\pi^{-2}(-1)^{m+n}(n + 1/2)^{-1}A_m[e^{aL_x/2} + (-1)^{m+1}e^{-aL_x/2}]bH_0.$$

*$[\nabla T\mathbf{H}_0] \equiv \nabla T \times \mathbf{H}_0$.

The weak dependence of a and b on the coordinates is insignificant if the functions H_{mn} depend on the coordinates more strongly than a and b, i.e., if $aL \gg 1$ (L is the characteristic scale of the inhomogeneity; $L = T/|\nabla T|$ in the case of the temperature gradient, and $L = l$ in the case of the acoustic flux). The inequality $aL \gg 1$ will be assumed satisfied. In order of magnitude, $b \approx a/L \ll a^2$.

The average magnetic field of the currents is

$$\overline{H}_j = \frac{1}{4L_xL_y}\int\int H_j\,dx\,dy = -16\frac{bH_0}{\pi^4}\sum_{m,\,n=0}^{\infty}\frac{1}{(n+{}^1/_2)^2} \times\left\{\frac{A_{2m+1}^2\operatorname{ch}^2(aL_x/2)}{b+\lambda_{2m+1,\,n}} - \frac{A_{2m}^2\operatorname{sh}^2(aL_x/2)}{b+\lambda_{mn}}\right\} \quad (4)$$

The experimentally measured coefficients $\widetilde{\alpha}_1$ and $\widetilde{\beta}_1$ will differ from their true values α_1 and β_1 because of the eddy currents:

$$\widetilde{\alpha}_1 = \alpha_1(1+\overline{H}_j/H_0), \quad \widetilde{\beta}_1 = \beta_1(1+\overline{H}_j/H_0).$$

If $aL_x/\pi \ll 1 \ll aL/\pi$, then

$$\overline{H}_j = \frac{2^8}{\pi^6}\frac{bH_0}{L_x^{-2}+L_y^{-2}}[1+O(aL_x/\pi)].$$

When $L_x \approx L_y \approx L_0$ we have

$$\overline{H}_j \approx 0{,}1H_0bL_0^2 \approx 0{,}1H_0|aL_0|L_0/L \ll H_0.$$

This result coincides with the order-of-magnitude estimate that can be obtained from (1) by replacing there H by H_0 in the term with the vector product.

Let us consider now the case when $aL_x \gg \pi$; in this case it is impossible to iterate in (2). Replacing in (4) summation over m by integration over $m\pi/aL_x$, which introduces in the result an error of the order of $\pi/aL_x \ll 1$, we obtain (when $L_y \geq L_x$)

$$\frac{\overline{H}_j}{H_0} = \frac{16}{\pi^2}\frac{bL_x^2}{(aL_x)^4}e^{aL_x}. \quad (5)$$

Within the limits of applicability of this formula, $\overline{H}_j$ increases monotonically with increasing ∇T and S. The sign of $\overline{H}_j$ coincides with the sign of $\partial(\alpha_1/\kappa)/\partial T$ in the case of ∇T; in the case of the acoustic flux, H_j is opposite to H_0.

Let us investigate the geometry of the eddy currents when $aL_x/\pi \gg 1$, $L_y \geq L_x$. In this case γ_{mn} has a rather sharp maximum at n = 0 and $m = aL_x/\pi$. Therefore the expression for the magnetic field can be reduced to the form

$$\frac{H_j}{H_0} = \frac{16}{\pi^4}e^{aL_x/2}e^{-ax/2}\sum_{m=0}^{\infty}(-1)^m \times\left\{\frac{A_{2m+1}\cos[(m+{}^1/_2)\pi x/L_x]}{m+{}^1/_2} - \frac{A_{2m}\sin(m\pi x/L_x)}{m}\right\} \times\sum_{n=0}^{aL_y/\pi}\frac{(-1)^n\cos[(n+{}^1/_2)\pi y/L_y]}{n+{}^1/_2} = f(x)\varphi(y).$$

The number of essential terms in the first factor is of the order of aL_x/π. The function $\varphi(y)$ is approximately constant inside the crystal and vanishes on its boundary. The function f(x) oscillates, reversing sign approximately aL_x/π times. The magnetic field is constant along the current line $\mathbf{j} = (c/4\pi)\operatorname{curl}\mathbf{H}_j$, since the scalar product $\mathbf{H}_j\cdot\operatorname{curl}\mathbf{H}_j$ vanishes when the direction of $\mathbf{H}_j$ is constant. Consequently, the current lines H_j = const have a nearly rectangular form (since $H_j \sim f(x)\varphi(y)$); the crystal is subdivided into aL_x/π domains in the x direction. In neighboring domains, the field H_j has opposite signs and its absolute value increases from zero on the boundary of the domain to a maximum at its center. These maximum values decrease in the positive direction (i.e., in the direction in which $a > 0$) like $\exp(-ax_n/2)$ (x_n is the coordinate at the center of the n-th domain).

In the presence of ∇T, the foregoing effects can be realized in a number of metals and semimetals at temperatures on the order of 10°K and below. If $L_y \geq L_x$, then the effect is determined by the parameter $aL_x/\pi = 4c^{-1}\sigma\alpha_l\Delta T$ (T—temperature drop). In Bi with $\sigma_{4.2}/\sigma_{300} = 200$ at T = 4°K, the coefficient is $a_1 = 10^{-5}$ V/Oe-deg[1]; therefore when $\Delta T = 1°$ we have $aL_x/\pi \approx 10$ and according to (5) we have $H_j \approx 3H_0$. In Bi at $\mu_\mp H/c \gg 1$, the resistivity η increases with increasing magnetic field; therefore the effect is realizable only if $\mu_+H_0/c \lesssim 1$. This also imposes a limitation on the possible values of H_j, the order of magnitude of which cannot exceed c/μ_+.

In Cu with small admixture of Fe at T = 4°K, the thermal emf is anomalously large and equals $\alpha = 2\mu$V/deg (the Kondo effect), and $\sigma = 10^{21}$ sec^{-1}[2]. Estimating $\alpha_l \approx \alpha\mu/c$ and taking $\Delta T = 1°$, we obtain $aL_x/\pi \approx 1$ and $H_j \approx 0.1H_0$. The field H_0 should be smaller than $c/\mu \approx 500$ Oe.

The effect under consideration is realizable in semimetals in the presence of acoustic flux. We shall estimate the acousto-electric coefficients by starting from the Weinreich relation[3]; then

$$\frac{aL_x}{\pi} \approx 4\frac{\mu^2 S}{sc^2}\frac{L_x}{l}$$

(s is the speed of sound). If the acoustic flux in Bi at helium temperature is S = 1 W/cm², then at a sound frequency $\omega_s \approx 3\times 10^7$ Hz, with l = 5 cm[4], we get $aL_x/\pi \approx 4$ and $H_j \approx 0.3H_0$.

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