SPECTRUM OF FAST ELECTRONS PRODUCED IN COLLISIONS BETWEEN PROTONS AND HEAVY ATOMS

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The spectrum of electrons emitted from heavy atoms as a result of proton impact is calculated. The number of electrons expelled with an energy ϵ varies as $d\epsilon/\epsilon^5$.

WE calculate in this paper the cross section for inelastic scattering of a proton by a heavy atom, resulting in the emission of fast electrons with energy $\epsilon \gg mv^2/2$, where m is the electron mass and v is the proton velocity. These electrons appear when a proton moves in the internal part of the atom, where the electron density is high enough, so that the classical Thomas-Fermi model can be used to describe the electrons. As the bombarding particle we choose a light nucleus (proton or helium ion), which does not produce a bound state with the electrons and therefore does not disturb the internal distribution of the electrons in this region.

In the case of the Thomas-Fermi model, the connection of the electron density n at a given point of space with the Fermi momentum p_0 and the potential φ of the self-consistent field is given by the relations[1]

$$n=\frac{p_0^3}{3\pi^2\hbar^3},\quad \varphi(r)=\frac{ze}{r}\chi\left(\frac{rz^{1/3}}{0.885a_0}\right)\quad \frac{p_0^2}{2m}=e\varphi,$$

where r is the distance to the nucleus with charge z, e is the electron charge, and a_0 is the Bohr radius. The function $\chi(x)$ is well approximated in the region $x \sim 1$ by the relation

$$\chi(x)\approx 0.45/x,$$

so that here $(r \sim a_0 z^{-1/3})$ $p_0 = 0.89\,\hbar z^{1/3}/r$.

If it is assumed that the quantity p_0 does not change when r shifts by an amount equal to the characteristic distance, then the problem reduces to the scattering of a proton in a degenerate electron gas. This problem was considered in a number of calculations of the decelerating ability of charged particles.[2-5]. The proton energy loss was determined by the elastic scattering of the electrons from the proton, and at small proton velocities v (compared with p_0/m) the only electrons that could take part in the scattering were those near the Fermi surface.

Let us employ the approach used in the cited papers to find the electron spectrum. The potential of interaction between the proton and the electron $V = e^2 e^{-\kappa\rho}/\rho$, where ρ is the distance from the electron to the proton, and the Debye screening parameter κ at large values of the Fermi momentum $p_0 a_0/\hbar \gg 1$ is given by[3] $\kappa = 2\sqrt{p_0/\pi\hbar a_0}$. The amplitude of elastic scattering of the electron by the atom is $f = -2/a_0(q^2 + \kappa^2)$, where q is the change of the wave vector of the electron upon scattering; pair scattering takes place here, since the number of electrons taking part in the transitions is small. The probability of emission of an electron with energy ϵ per unit time when a proton moves in a degenerate electron gas with Fermi momentum p_0, when $v \ll p_0/m$, and $\epsilon \le 2p_0 v$, is equal to

$$\frac{dw}{d\varepsilon}=\int\frac{2dp\,v_e\,d\sigma}{(2\pi\hbar)^3\,d\varepsilon}=\int\frac{2p^3\,dp\,do_p\,d\sigma}{(2\pi\hbar)^3\,md\varepsilon},$$

where p is the electron momentum and $d\sigma$ is the cross section for the elastic scattering of the electron by the proton, leading to a final electron energy $p_0^2/2m + \epsilon$.

Using this result, we obtain for the cross section for the scattering of the proton by the atom with emission of an electron having an energy ϵ:

$$\frac{d\sigma_{\text{ion}}}{d\varepsilon}=\int\limits_0^{R_0}2\pi\rho\,d\rho\int\limits_\rho^{R_0}\frac{dw}{d\varepsilon}\frac{2r\,dr}{v\sqrt{r^2-\rho^2}}=\frac{4\pi}{v}\int\limits_0^{R_0}r^2\,dr\frac{dw}{d\varepsilon}$$

The maximum energy of the released electron is equal to $2p_0 v$, so that the maximum distance to the nucleus, at which emission of an electron with a given energy it is possible, as determined by the relation $2p_0(R_0) = \epsilon/v$. Integrating the obtained expression, we obtain finally

$$d\sigma=6\,m^2e^4\frac{zv^4}{\varepsilon^5}d\varepsilon,\quad \varepsilon\gg mv^2.$$

This expression is valid when $mv \ll p_0(R_0)$ and $a_0/z \ll R_0 \ll a_0$, leading to the following conditions for the applicability of the result:

$$\varepsilon\gg mv^2,\quad z^{4/3}\frac{me^2}{\hbar}\gg\frac{\varepsilon}{v}\gg z^{1/3}\frac{me^2}{\hbar},\ \text{i.e.}\ v\ll\frac{z^{4/3}e^2}{\hbar}.$$

These results are of practical interest when $v \sim e^2/\hbar$. They are valid if the distance over which the electron-proton scattering takes place ($\sim me^2/p_0^2$) is small compared with the characteristic dimensions ($r \sim z^{1/3}\hbar/p_0$), as is indeed the case.

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[5] B. A. Trubnikov and Yu. N. Yavlinskii, ibid. 48, 253 (1965) [21, 167 (1965)].

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204