MACROSCOPIC QUANTIZATION AND THE PROXIMITY EFFECT IN S-N-S JUNCTIONS

I. O. KULIK

Physico-technical Institute of Low Temperatures, Ukrainian Academy of Sciences

Submitted June 10, 1969

Zh. Eksp. Teor. Fiz. 57, 1745–1759 (November, 1969)

A microscopic theory of the stationary Josephson current in plane junctions of the superconductor - normal metal - superconductor type is developed. It is shown that, owing to spatial quantization of single-particle excitation energies in the normal layer, the superconducting current contains a component that does not decrease exponentially at distances on the order of $\xi_0 \sim v_0/T_c$. The component oscillates with variation of the thickness of the normal metal layer. This is due to discontinuous variation of the number of levels within the gap at certain values of the thickness. The temperature dependence of the amplitude of the oscillating current component can be described by the expression $e^{-\pi dT/v_0}$ (d is the normal layer thickness and v_0 the Fermi velocity). The possibility of experimental observation of the effect is discussed.

1. INTRODUCTION

As shown by Andreev[1], the spectrum of the elementary excitations of a layer of normal metal in contact with super-conductors on both sides (S-N-S contact) is quantized at excitation energies not exceeding the energy gap of the superconductor Δ. Actually, this quantization implies the presence of a coherent connection between the phases of the ordering parameters of the superconductors adjacent to the normal metal, i.e., the existence of superconducting ordering for the entire S-N-S system as a whole. In the absence of phase coherence of Δ on both sides of the junction, the quantization no longer takes place. As will be shown below, the positions of the Andreev levels are determined by the relative phase of the gaps of two superconductors, $\chi = \chi_1 - \chi_2$, and a change of χ by 2π denotes a shift of the entire level system as a whole by an amount equal to their relative distance.

Thus, the existence of a discrete spectrum of single-particle excitations in the S-N-S junction is closely connected with the phase coherence for such a junction. On the other hand, phase coherence denotes the possible flow of superconducting currents (the Josephson effect) through the junction[2]. As is well known[3], the Josephson current is proportional to the derivative, with respect to χ of the junction-energy term that depends on the relative phase χ:

$$I = \frac{2e}{\hbar}\frac{\partial E}{\partial \chi}. \qquad (1.1)$$

It is clear that by virtue of the aforementioned sensitivity of the position of the quantized level to the phase, the energy of the junction will also depend essentially on χ, and this dependence remains in force also when the width of the normal layer greatly exceeds the dimension of the Cooper pair $\xi_0 \sim v_0/\Delta$. Thus, currents that do not have the usual exponential dependence on the thickness of the normal layer, of the type e^{-d/ξ_0}, can flow through the junction.[1)] Instead, the effect attenuates exponentially at distances on the order of $\xi_T \sim v_0/T$ (T—temperature, $T \ll T_c$) or l, where l is the mean free path in the normal metal, i.e., under conditions when "smearing" of the discrete levels takes place in the N layer as the result of collisions or finite temperatures. When $d \lesssim \xi_T$, l, the phases of the gaps in the two superconductors are effectively coupled, owing to the dependence of the energy of the system on $\chi_1 - \chi_2$, i.e., "phase coherence" due to the spatial quantization in the S-N-S junction, takes place.

In this paper we construct a microscopic theory of the proximity effect due to the spatial quantization in an impurity-free system ($l = \infty$). In Sec. 2 we find the energy levels and the wave functions of the single-particle excitations and analyze their dependence on the phase. In Sec. 3 we investigate the current states in a spatially inhomogeneous system. An expression is obtained for the superconducting current in the form of a quadratic functional of the ordering parameter $\Delta(\mathbf{r})$. Finally, in Sec. 4 we calculate the Josephson current with the aid of the eigenfunction expansion of the single-particle problem. It is shown that the maximum value of this current decreases exponentially with temperature like $\exp(-\pi dT/v_0)$, and oscillates as a function of the thickness of the normal layer with a period $\delta d = \pi v_0/\Delta$ (v_0—Fermi velocity). The latter effect is connected with the motion of the size-quantization E_n levels past the edge of the energy gap of the superconductor Δ.

1)A similar possibility was noted earlier by Aslamazov, Larkin, and Ovchinnikov [4] independently of the connection with the spatial quantization. According to [4], the Josephson current through an S-N-S junction can contain under certain conditions a component that does not vanish exponentially like $\exp(-d/\xi_0)$, this being connected with the conservation of the correlation of the Copper pairs in the normal metal at distances that are large compared with ξ_0. The case considered in this paper differs from that investigated in [4]. The model of [4] corresponds to a strong reflection of the electrons from the boundary of the N-layer ($1-R \ll 1$), i.e., what was actually considered in [4] was not an S-N-S junction, but an S-I-N-I-S junction. The systematics of the quantum state in the N layer is different in this case, and particularly, the position of the discrete levels is practically independent of χ. Therefore the Josephson current changes in [4] monotonically with thickness, whereas in our case it oscillates with variation of d.

2. WAVE FUNCTIONS AND ENERGY LEVELS OF SINGLE-PARTICLE EXCITATIONS

The quantum states of the elementary excitations of a superconductor are determined with the aid of the solution of the Bogolyubov-de Gennes equations[5,6] for two-component wave functions Ψ:

$$H\Psi = E\Psi, \quad \Psi = \begin{pmatrix} \psi \\ \varphi \end{pmatrix}, \quad H = \begin{pmatrix} T & \Delta \\ \Delta^* & -T \end{pmatrix}, \tag{2.1}$$

$T = -\nabla^2/2m - \zeta$ is the kinetic energy operator and ζ is the chemical potential. For the one-dimensional problem $\Delta = \Delta(z)$, the dependence of Ψ on the coordinates can be chosen in the form

$$\Psi = \exp(iq_x x)\exp(iq_y y)\begin{pmatrix} \psi(z) \\ \varphi(z) \end{pmatrix},$$

and then $\psi(z)$ and $\varphi(z)$ satisfy the equations

$$\begin{aligned} (T_z - \xi_q)\psi + \Delta(z)\varphi &= E\psi, \\ -(T_z - \xi_q)\varphi + \Delta^*(z)\psi &= E\varphi, \end{aligned} \tag{2.2}$$

and the quantities T_z and ξ_q are defined by

$$T_z = -\frac{1}{2m}\frac{d^2}{dz^2}, \quad \xi_q = \zeta - \frac{q^2}{2m}, \quad \mathbf{q} = (q_x, q_y, 0).$$

The quantity ξ_q can be assumed to be positive. Indeed, if the momentum component parallel to the surface exceeds the Fermi momentum, $q > p_0$, then the wave functions of the excitations (at energies $E \sim \Delta \ll \zeta$) will attenuate exponentially in the z direction and are of no significance in our problem.

Assume that the region $|z| < d/2$ is filled with normal metal, and in the regions $|z| > d/2$ we have superconductors, which for simplicity are assumed to be identical ($\Delta_1 = \Delta_2 = \Delta$, $T_{c1} = T_{c2} = T_c$). We assume that the transition temperature of the normal metal is equal to zero: $T_{cN} = 0$. If the thickness of the normal layer d is large compared with the coherence length ξ_0, then $\Delta(z)$ is exponentially small everywhere inside the normal region, with the exception of a layer of width $\sim\xi_0$ near the interface with the superconductor, whereas in the superconductor $\Delta(z)$ varies over distances on the order of $\xi(T) \sim v_0/\sqrt{T_c(T_c - T)}$. If the temperature is low compared with the superconducting transition temperature T_c, then the change of $\Delta(z)$ in the superconductor also occurs over distances $\sim\xi_0$ near the interface. We considering this case ($d \gg \xi_0$, $T \ll T_c$) using the simplest model, in which $\Delta(z)$ is assumed to change jumpwise on the interface between the normal metal and superconductor (a similar model was considered in[7]): $|\Delta(z)| = \Delta_0$ when $|z| > d/2$ and $|\Delta(z)| = 0$ when $|z| < d/2$. Here, however, it is important to take into account the fact that the phases of the ordering parameter in the left and right superconductors may differ: $\arg\Delta(z) = \chi_1$ when $z < -d/2$ and $\arg\Delta(z) = \chi_2$ when $z > d/2$. Thus, we assume for $\Delta(z)$ the dependence

$$\Delta(z) = \begin{cases} \Delta_0 e^{i\chi_1}, & z < -d/2 \\ 0, & -d/2 < z < d/2 \\ \Delta_0 e^{i\chi_2}, & z > d/2. \end{cases} \tag{2.3}$$

Substituting (2.3) in (2.2), we can easily find the wave functions of the excitations and the energy levels E. The energy spectrum is continuous when $E > \Delta$ and discrete when $E < \Delta_0$. The latter can be seen from the fact that states with energy inside the gap cannot exist in the superconductor. The excitations traveling from the normal metal towards the NS interface are therefore reflected if their energy (reckoned from the chemical potential ζ) is smaller than Δ_0. As analyzed by Andreev[1], the reflection is connected with the small change of the quasimomentum $\delta p \sim p_0\Delta_0/\zeta$, and therefore proceeds via a transition for a state of the "particle" type ($p > p_0$) into a state of the "hole" type ($p < p_0$) and vice-versa. This can be readily traced in a model with a jumplike termination of the gap.

Let $\Delta = \Delta_0$ when $Z > 0$ and $\Delta = 0$ when $z < 0$. Writing the incident wave for $z < 0$ in the form

$$\Psi_0 = e^{ik_0 z}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \frac{k_0^2}{2m} = \xi_q + E, \tag{2.4}$$

the reflected wave in the form of a superposition of excitations of the particle and hole type:

$$\Psi_1 = Ae^{ik_1 z}\begin{pmatrix} 0 \\ 1 \end{pmatrix} + Be^{-ik_0 z}\begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \frac{k_1^2}{2m} = \xi_q - E, \tag{2.5}$$

and making the wave functions and their derivatives continuous at $z = 0$ with the solutions of the system (2.2) at $z > 0$,

$$\Psi_2 = Ce^{i\lambda_+ z}\begin{pmatrix} 1 \\ \gamma \end{pmatrix} + De^{-i\lambda_- z}\begin{pmatrix} 1 \\ \gamma^* \end{pmatrix}, \tag{2.6}$$

where

$$\lambda_\pm^2 = 2m(\xi_q \pm i\sqrt{\Delta_0^2 - E^2}), \quad \operatorname{Im}\lambda_\pm \gtrless 0, \tag{2.7}$$

$$\gamma = \Delta_0(E + i\sqrt{\Delta_0^2 - E^2})^{-1}, \quad E < \Delta_0, \tag{2.8}$$

we obtain the values of the amplitudes A and B. A simple calculation then shows that $|A| \approx 1$ and $|B|$ is a small quantity of the order of $m\Delta_0/k_0^2 \sim \Delta_0/\zeta$. Consequently, ignoring small corrections of the order of $(\Delta_0/\zeta)^2$, we can assume $B = 0$ (and also $D = 0$). It is further necessary to take into account the fact that k_0 and k_1 are close in magnitude, so that $E \sim \Delta_0 \ll \xi_q \sim \zeta$ (k_0 and k_1 are quantities of the order of the Fermi momentum). Therefore, with the indicated accuracy, the continuity of the wave functions implies continuity of their derivatives at the point of contact.

Returning to the case of the S-N-S junction (2.3), and taking into account the statements made above concerning the character of the reflection of the excitations from the interface with the superconductor, we represent the expressions for the two-component wave functions Ψ in the form ($E < \Delta_0$)

$$\Psi_+ = \begin{cases} A_+ e^{ik_0 z}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + B_+ e^{ik_1 z}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, & |z| < d/2, \\ C_+ e^{i\lambda_+(z-d/2)}\begin{pmatrix} e^{i\chi_2} \\ \gamma \end{pmatrix}, & z > d/2, \\ D_+ e^{i\lambda_-(z+d/2)}\begin{pmatrix} \gamma \\ e^{-i\chi_1} \end{pmatrix}, & z < -d/2; \end{cases} \tag{2.9}$$

$$\Psi_- = \begin{cases} A_- e^{-ik_0 z}\begin{pmatrix} 1 \\ 0 \end{pmatrix} + B\, e^{-ik_1 z}\begin{pmatrix} 0 \\ 1 \end{pmatrix}, & |z| < d/2, \\ C_- e^{-i\lambda_-(z-d/2)}\begin{pmatrix} e^{i\chi_2} \\ \gamma^* \end{pmatrix}, & z > d/2, \\ D_- e^{-i\lambda_+(z+d/2)}\begin{pmatrix} \gamma^* \\ e^{-i\chi_1} \end{pmatrix}, & z < -d/2, \end{cases} \tag{2.10}$$

where the plus sign corresponds to excitations moving from left to right, and the minus sign to excitations from right to left ($k_z > 0$ and $k_z < 0$, respectively).

The wave functions (2.9) and (2.10) attenuate exponentially inside the superconducting regions at distances $\sim\xi_0$ that are large compared with atomic distances. Indeed, on the basis of (2.7) we can write for $\lambda_\pm$:

$$\lambda_\pm \approx \sqrt{2m\xi_q} \pm iv^{-1}\sqrt{\Delta_0^2 - E^2}, \tag{2.11}$$

where $v = \sqrt{2\xi_q/m}$ is the velocity of the excitation inside the normal layer in the direction normal to the surface of the junction: $v = m^{-1}\sqrt{2m\xi - q^2}$. The characteristic values of the imaginary part of the wave vector (2.11) are $\Delta_0/v_0 \sim \xi_0^{-1}$. In analogy with (2.11), we present the expansions for k_0 and k_1, which we shall need in the future

$$k_0 \approx \sqrt{2m\xi_q} + E/v, \quad k_1 \approx \sqrt{2m\xi_q} - E/v. \tag{2.12}$$

Making the expressions for the wave functions (2.9) and (2.10) continuous at $z = d/2$ and $z = -d/2$, we obtain the relations between the coefficients $A_\pm$, $B_\pm$, $C_\pm$, and $D_\pm$:

$$\begin{aligned} C_\pm &= A_\pm e^{\pm ik_0 d/2} e^{-i\chi_2} = B_\pm e^{\pm ik_1 d/2}\gamma^{\mp 1}, \\ D_\pm &= A_\pm e^{\mp ik_0 d/2}\gamma^{\mp 1} = B_\pm e^{\mp ik_1 d/2} e^{i\chi_1}. \end{aligned} \tag{2.13}$$

From this we readily obtain the dispersion equations that determine the allowed values of the energy E. For the states Ψ_+ and Ψ_- they take respectively the form

$$\gamma^2 e^{i(k_0 - k_1)d} e^{\pm i\chi} = 1, \quad \chi = \chi_1 - \chi_2. \tag{2.14}$$

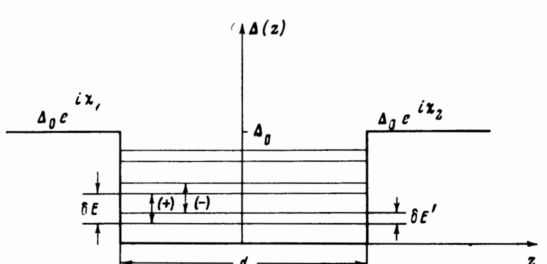
Equating the phase of the expressions in the left side of (2.15) to $2\pi n$ where $n = 0, 1, 2, \ldots$ and taking (2.8) into account, we obtain for the energy levels $E_n^\pm$ the formula

$$E_n^\pm = (v/2d)\,[2(\pi n + \varphi) \mp \chi], \quad \varphi(E) = \arccos(E/\Delta_0). \tag{2.15}$$

$\varphi(E)$ is a slow function of the energy. In particular, for states with energies $E \ll \Delta_0$, we have $\varphi \approx \pi/2$, and therefore (2.15) yields directly

$$E_n^\pm = (v/2d)\,[2\pi(n + 1/2) \mp \chi], \quad n \ll \Delta_0 d/v. \tag{2.16}$$

Thus, we obtain two systems of equidistant levels (see the figure), which are degenerate in the case $\chi = 0$. It is important that the positions of the levels depend on the relative phase χ of the gaps. A change of χ by 2π corresponds to a return to the initial state. As seen from formulas (2.15) and (2.16), the distance between the levels does not depend on the phase and can be obtained from the condition of quasiclassical



Energy levels of excitations in N-layer at a fixed value of the electron velocity on the Fermi surface $v = |v_z|$. The distance between levels is

$$\delta E = E_{n+1}^- - E_n^+ = E_{n+1}^- - E_n^- \cong \pi v/d;$$

$\delta E'$ is the splitting and is proportional to the phase difference $\chi = \chi_1 - \chi_2$:

$$\delta E' = E_n^- - E_n^+ = \delta E(\chi/\pi)$$

quantization (see[8]), $\Delta E = 2\pi/T_0$ ($\hbar = 1$), where T_0 is the period of the classical motion, equal in this case to double the time of flight of the quasiparticle between the boundaries of the N layer: $T_0 = 2d/v$.[2)]

We can obtain analogously expressions for the wave functions of the continuous spectrum $(E > \Delta_0)$. We present here the corresponding formulas, since they will be useful later in the calculation of the Josephson current. The states of the continuous spectrum are classified as $\Psi_\pm^0$ and Ψ_+^1, where Ψ^0 corresponds to a particle ($|k_z| = k_0$) and Ψ^1 corresponds to a hole ($|k_z| = k_1$), with the sign $+(-)$ corresponding to the case $k_z > 0$ (or $k_z < 0$). The functions $\Psi_\pm^0$ and $\Psi_\pm^1$ are analogous in form, and we therefore present an expression for only one of them

$$\Psi_+^0 = \text{const}\begin{cases} e^{ik_0 z}\begin{pmatrix}1\\0\end{pmatrix}, & |z| < d/2, \\ Ae^{i\varkappa_0(z-d/2)}\begin{pmatrix}e^{i\chi_2}\\ \delta\end{pmatrix} + Be^{i\varkappa_1(z-d/2)}\begin{pmatrix}\delta\\ e^{-i\chi_2}\end{pmatrix}, & z > d/2, \\ Ce^{i\varkappa_0(z+d/2)}\begin{pmatrix}e^{i\chi_1}\\ \delta\end{pmatrix} + De^{i\varkappa_1(z+d/2)}\begin{pmatrix}\delta\\ e^{-i\chi_1}\end{pmatrix}, & z < -d/2, \end{cases} \tag{2.17}$$

where the constant is determined from the normalization condition (see below), and the quantities κ_0, κ_1 and δ are defined as follows:

$$\varkappa_0^2 = 2m(\xi_q + \sqrt{E^2 - \Delta_0^2}), \quad \varkappa_1^2 = 2m(\xi_q - \sqrt{E^2 - \Delta_0^2}), \tag{2.18}$$

$$\delta = \Delta_0(E + \sqrt{E^2 - \Delta_0^2})^{-1}, \quad E > \Delta_0. \tag{2.19}$$

With quasiclassical accuracy, we can write for κ_0 and κ_1 expansions similar to (2.11) and (2.12):

$$\varkappa_0 \approx \sqrt{2m\xi_q} + v^{-1}\sqrt{E^2 - \Delta_0^2}, \quad \varkappa_1 \approx \sqrt{2m\xi_q} - v^{-1}\sqrt{E^2 - \Delta_0^2}. \tag{2.20}$$

The coefficients A, B, C, and D are determined by the formulas

$$A = \frac{e^{ik_0 d/2}}{1-\delta^2}e^{-i\chi_2}, \quad B = -\frac{\delta}{1-\delta^2}e^{ik_0 d/2}, \tag{2.21}$$

$$C = \frac{e^{-ik_0 d/2}}{1-\delta^2}e^{-i\chi_1}, \quad D = -\frac{\delta}{1-\delta^2}e^{-ik_0 d/2}.$$

As will be shown below, the continuous-spectrum states make their own contribution to the Josephson current, and in definite cases this contribution may even be the principal one (see Sec. 4). Nonetheless, this does not contradict the initial treatment of the effect, reported in the introduction and connected with the role of the spatial quantization. The character of the quantum states when $E > \Delta_0$ is determined essentially by the presence of discrete levels when $E < \Delta_0$. In the one-dimensional problem, the presence of a "potential well" $\Delta(z)$ automatically implies both the occurrence of localized states with energy $E < \Delta_0$, and a change of the scattering phase shifts for the states of the continuous spectrum $(E > \Delta_0)$, both effects being sensitive to the value of χ.

3. SUPERCONDUCTING CURRENTS IN A SPATIALLY INHOMOGENEOUS SYSTEM

We start from the well known expression for the current[10]

$$\mathbf{j} = \frac{ie}{m}T\sum_\omega[(\nabla_{\mathbf{r}'} - \nabla_{\mathbf{r}})G_\omega(\mathbf{r}, \mathbf{r}')]_{\mathbf{r}'\to\mathbf{r}}, \tag{3.1}$$

---

2)This circumstance was noted in [9] as applied to the S-N-S junction.

where $G_\omega(\mathbf{r}, \mathbf{r}')$ are the thermodynamic Green's functions of the superconductor and satisfy, together with the functions $F^+_\omega(\mathbf{r}, \mathbf{r}')$, the system of Gor'kov equations[10]. Taking into account the one-dimensional character of the problem, we write these equations in the form

$$\begin{aligned}(i\omega - T_z + \xi_q)G_\omega(z, z') + \lambda(z)F(z)F_\omega^+(z, z') &= \delta(z - z'),\\ (-i\omega - T_z + \xi_q)F_\omega^+(z, z') - \lambda(z)F^*(z)G_\omega(z, z') &= 0.\end{aligned} \tag{3.2}$$

Here $\omega = (2n + 1)\pi T$ are the odd frequencies, $\lambda(z)$ is the electron interaction constant, which will henceforth be assumed to be equal to $-|\lambda|$ when $|z| > d/2$ and to zero when $|z| < d/2$. $F(z)$ is the value of the Gor'kov function $F(z, z'; \tau - \tau')$ in the coinciding points $(z = z', \tau = \tau')$. The ordering parameter of the superconductor $\Delta^*(z)$ is defined by

$$\Delta^*(z) = |\lambda| F^*(z). \tag{3.3}$$

Introducing in (3.2) explicitly the function $U(z)$, which is equal to unity when $|z| > d/2$ and to zero when $|z| < d/2$, we rewrite these equations in the form

$$\begin{aligned}(i\omega - T_z + \xi_q)G_\omega(z, z') - \Delta(z)U(z)F_\omega^+(z, z') &= \delta(z - z'),\\ (-i\omega - T_z + \xi_q)F_\omega^+(z, z') + \Delta^*(z)U(z)G_\omega(z, z') &= 0,\end{aligned} \tag{3.4}$$

and by virtue of (3.3) $\Delta^*(z)$ satisfies the equation

$$\Delta^*(z) = \frac{m}{2\pi}|\lambda| T \sum_\omega \int_0^\zeta d\xi_q F_\omega^+(z, z) \tag{3.5}$$

(we shall henceforth not write out explicitly the dependence of the functions G_ω and F^+_ω on ξ_q).

We denote by $G^0_\omega(z - z')$ the Green's function of the normal metal, satisfying the first equation of the system (3.4) with $\Delta = 0$. Eliminating F^+_ω from (3.4), we can obtain the following closed integral equation for the function G_ω:

$$\begin{aligned}G_\omega(z, z') = G_\omega^0(z - z') - \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2 G_\omega^0(z - z_1)\Delta(z_1)U(z_1)\\ \times G_{-\omega}^0(z_1 - z_2)\Delta^*(z_2)U(z_2)G_\omega(z_2, z').\end{aligned} \tag{3.6}$$

Similarly, the equation for the ordering parameter (3.5) is rewritten in the form

$$\Delta^*(z) = \frac{m}{2\pi}|\lambda| T \sum_\omega \int_0^\zeta d\xi_q \int_{-\infty}^{\infty} dz' G_{-\omega}^0(z - z')\Delta^*(z')U(z')G_\omega(z', z). \tag{3.7}$$

We substitute (3.6) in (3.1) and calculate the superconducting current j. Naturally, for the problem of interest to us we should consider only the z-th component of the current: $j = j_z$ ($j_x = j_y = 0$), and by virtue of the continuity equation j_z does not depend on z. Therefore the current can be calculated at any point of the superconductor. From symmetry considerations it is clear that it is most convenient to carry out the calculation of the current at $z = 0$. Recognizing that in the normal state the current is equal to zero, we obtain

$$\begin{aligned}j = -\frac{ie}{2\pi} T \sum_\omega \int_0^\zeta d\xi_q \int_{-\infty} dz_1 \int_{-\infty} dz_2 \left[\left(\frac{\partial}{\partial z'} - \frac{\partial}{\partial z}\right) G_\omega^0(z - z_1)\Delta(z_1)U(z_1)\right.\\ \left.\times G_{-\omega}^0(z_1 - z_2)\Delta^*(z_2)U(z_2)G_\omega(z_2, z')\right]_{z=z'=0}.\end{aligned} \tag{3.8}$$

We now assume for the time being that the function G_ω has been expanded in a series in powers of Δ. Such a series can be readily obtained by iterating Eq. (3.6):

$$\begin{aligned}G_\omega(z, z') = G_\omega^0(z - z') - \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2 G_\omega^0(z - z_1)\Delta(z_1)U(z_1)\\ \times G_{-\omega}^0(z_1 - z_2)\Delta^*(z_2)U(z_2)G_\omega^0(z_2 - z') + \ldots\end{aligned} \tag{3.9}$$

Substituting this expansion in (3.8) and again summing the terms flanked by the expressions $G^0_\omega(z - z_1)\Delta(z_1)U(z_1)$ and $\Delta^*(z_n)U(z_n)G^0_\omega(z_n - z')$, we can verify that (3.8) can be represented identically in the form

$$\begin{aligned}j = -\frac{ie}{2\pi} T \sum_\omega \int_0^\zeta d\xi_q \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2 U(z_1)\Delta(z_1)U(z_2)\Delta^*(z_2)\\ \times\left[\left(\frac{\partial}{\partial z'} - \frac{\partial}{\partial z}\right)G_\omega^0(z - z_1)G_\omega^0(z_2 - z')\right]_{z=z'=0} G_{-\omega}(z_2, z_1).\end{aligned} \tag{3.10}$$

The expression in the square brackets, containing the Green's function of the normal metal $G^0_\omega(z - z')$, can be readily calculated on the basis of the explicit form of these functions:

$$G_\omega^0(z - z') = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{e^{ip(z-z')}}{i\omega + \xi_q - p^2/2m} dp = -\frac{m}{i\lambda_\omega}\exp(-i\lambda_\omega|z - z'|), \tag{3.11}$$

where $\lambda_\omega = [2m(\xi_q + i\omega)]^{1/2}$, and the square root sign is chosen from the condition $\operatorname{Im}\lambda_\omega < 0$. Since $\omega \sim T_c \ll \zeta$, we obtain

$$\lambda_\omega \approx -(2m\xi_q)^{1/2}\operatorname{sign}\omega - i|\omega|/v. \tag{3.12}$$

Using formula (3.11), we arrive finally at the following expression for the current:

$$\begin{aligned}j = -\frac{iem}{2\pi} T \sum_\omega \int_0^\zeta d\xi_q \int_{-\infty}^{\infty} dz_1 \int_{-\infty}^{\infty} dz_2 (\operatorname{sign} z_1 - \operatorname{sign} z_2)U(z_1)U(z_2)\\ \times\Delta(z_1)\Delta^*(z_2)G_\omega^0(z_1 - z_2)G_{-\omega}(z_2, z_1).\end{aligned} \tag{3.13}$$

Formula (3.13) was obtained earlier in connection with a microscopic calculation of the Josephson current for S-I-S junctions[11] (see also the review[12]).

The expression (3.13) is general. Thus, for example, it can be verified that for a homogeneous current state, $\Delta(z) = \Delta_0 e^{ikz}$ (in the absence of the normal-metal layer), it leads to the known dependence of the superconducting current j on the parameter of "superfluid velocity" $v_s = \hbar k/2m$. The general scheme of calculating the superconducting current in an inhomogeneous system consists of specifying a current state $\Delta = \Delta(z)$ compatible with Eq. (3.7). The Green's function $G_\omega(z, z')$ is then determined from (3.6) and must then be substituted in (3.13).

In the case of an S-N-S junction, the current state is determined by specifying the phases χ_1 and χ_2 in accordance with (2.3)[3)]. Substituting (2.3) in (3.13), we obtain

$$\begin{aligned}j = \frac{iem}{2\pi}\Delta_0^2 T \sum_\omega \int_0^\zeta d\xi_q \left[e^{i\chi}\int_{-\infty}^{-d/2} dz_1 \int_{d/2}^{\infty} dz_2 - e^{-i\chi}\int_{d/2}^{\infty} dz_1 \int_{-\infty}^{-d/2} dz_2\right]\\ \times G_\omega^0(z_1 - z_2)G_{-\omega}(z_2, z_1).\end{aligned} \tag{3.14}$$

Even from this expression we see the general

[3)]Such an approximation is valid if the critical current of the junction is small compared with the "volume" critical current $j_{cm} \sim Nev_c$, $v_c \sim \Delta_0/p_0$. In this case we can neglect the change of the phase $\chi(z)$ inside the superconductor, and consider only the jump of χ in the contact region.

character of the dependence of the current on the junction thickness d. The distance between the points z_1 and z_2 is at least equal to d. Therefore the potential term in the current is due to the product

$$G_\omega^0(d)G_{-\omega}(-d/2,\ d/2) \sim G_{-\omega}^0(d)G_{-\omega}^0(d)$$

and has, in accordance with (3.11) and (3.12), an asymptotic form

$$\exp(-2|\omega|d/v_0) \sim \exp(-2\pi dT/v_0)$$

(v_0—Fermi velocity). Indeed, inside the normal metal G_ω has the same variation as G_ω^0. Thus, at low temperatures the current does not contain the small parameter

$$\exp(-d/\xi_0) \sim \exp(-dT_c/v_0)$$

which is customarily encountered for proximity effects. As already noted, this is connected with the fact that in a pure metal at low temperatures the pairs conserve their correlation at distances that are large compared with ξ_0.[4] When $T \to T_c$, the current is proportional to

$$\exp(-2\pi dT_c/v_0) \sim \exp(-d/\xi_0).$$

Since in this case $\Delta(z)$ changes over distances $\sim\xi(T) \gg \xi_0$, and the remaining terms in the integrand of (3.14) change over distances of the order of ξ_0, we can put $\Delta(z) \approx \Delta(\pm d/2)$ and take Δ outside the integral sign[4)]. On the other hand, the value of the ordering parameter on the boundary with the normal metal is much smaller than its value far from the boundary, in a ratio $\Delta(d/2)/\Delta(\infty) \sim \xi_0/\xi(T)$ (see[6]). As the result, it turns out, as first noted by de Gennes[6] (see also[13]), that near T_c the Josephson current of an S-N-S junction is proportional to $[\Delta(\infty)\xi_0/\xi(T)]^2$ $\sim (T_c - T)^2$, i.e., to the second power of the difference $(T_c - T)$, and not to the first as in the case of the Josephson effect through a dielectric gap (S-I-S junction).

4. CALCULATION OF THE JOSEPHSON CURRENT

The connection between the Gor'kov equations (3.4) and the Bogolyubov-de Gennes equations (2.2) is given with the aid of the following relations:

$$G_\omega(z,z') = \sum_\alpha\left[\frac{\psi_\alpha(z)\psi_\alpha^*(z')}{i\omega - E_\alpha} + \frac{\varphi_\alpha^*(z)\varphi_\alpha(z')}{i\omega + E_\alpha}\right], \tag{4.1}$$

$$F_\omega^+(z,z') = \sum_\alpha\left[\frac{\varphi_\alpha(z)\psi_\alpha^*(z')}{i\omega - E_\alpha} - \frac{\psi_\alpha^*(z)\varphi_\alpha(z')}{i\omega + E_\alpha}\right]. \tag{4.2}$$

The summation here is over the eigenstates of the single-particle problem Ψ_α, corresponding to positive eigenvalues E_α. The functions ψ_α and φ_α are assumed to be normalized in accordance with the conditions

$$\int_{-\infty}^{\infty}(|\psi_\alpha(z)|^2 + |\varphi_\alpha(z)|^2)\,dz = 1. \tag{4.3}$$

The formulas (4.1) and (4.2) are obtained on the basis of the completeness and orthogonality conditions in the linear space of the functions Ψ_α. It must be recognized here that if

$$\Psi_\alpha = \begin{pmatrix}\psi_\alpha\\ \varphi_\alpha\end{pmatrix}$$

corresponds to an energy $E_\alpha > 0$, then a negative energy $-E_\alpha$ corresponds to the wave function

$$\tilde{\Psi}_\alpha = \begin{pmatrix}\varphi_\alpha^*\\ -\psi_\alpha^*\end{pmatrix}.$$

This is seen directly from the equations satisfied by ψ_α and φ_α:

$$\begin{aligned}(T_z - \xi_q)\psi_\alpha + \Delta(z)U(z)\varphi_\alpha &= E_\alpha\psi_\alpha,\\ -(T_z - \xi_q)\varphi_\alpha + \Delta^*(z)U(z)\psi_\alpha &= E_\alpha\varphi_\alpha.\end{aligned} \tag{4.4}$$

Substituting the eigenfunction expansion (4.1) in the formula (3.13) for the Josephson current and using the explicit expression for the Green's function of the normal metal G_ω^0 (3.11), we reduce j to the form

$$j = \frac{2em^2}{\pi}\Delta_0^2\,\mathrm{Re}\int_0^\zeta d\xi_q\,T\sum_\omega\frac{1}{\lambda_\omega}\sum_\alpha\left(\frac{\psi_{\alpha\omega}{}^+\psi_{\alpha\omega}{}^-}{i\omega + E_\alpha} + \frac{\varphi_{\alpha\omega}{}^+\varphi_{\alpha\omega}{}^-}{i\omega - E_\alpha}\right)e^{i\chi}, \tag{4.5}$$

where

$$\begin{aligned}\psi_{\alpha\omega}{}^+ &= \int_{d/2}\psi_\alpha(z)e^{-i\lambda_\omega z}\,dz, & \psi_{\alpha\omega}{}^- &= \int_{-\infty}^{-d/2}\psi_\alpha^*(z)e^{i\lambda_\omega z}\,dz,\\ \varphi_{\alpha\omega}{}^+ &= \int_{d/2}^{\infty}\varphi_\alpha^*(z)e^{-i\lambda_\omega z}\,dz, & \varphi_{\alpha\omega}{}^- &= \int_{-\infty}^{-d/2}\varphi_\alpha(z)e^{i\lambda_\omega z}\,dz.\end{aligned} \tag{4.6}$$

Thus, the calculation of the Josephson current is reduced to a sum over the states of the single-particle Hamiltonian (4.4).

The remainder of the calculation, using the explicit form of the wave functions ψ_α and φ_α obtained in Sec. 2, is trivial. Leaving out this calculation, we present directly the final result. The total current is divided into two terms, $j = j_1 + j_2$, where j_1 corresponds to summation over the states of the discrete spectrum $(E < \Delta_0)$, and j_2 corresponds to summation over the continuous spectrum $(E > \Delta_0)$. When calculating the last term (j_2), we must replace the sum over α $(E > \Delta_0)$, with allowance for the known formula for the state density in the BCS model, by the integral

$$\sum_\alpha(\ldots) \to \int(\ldots)\frac{L\,dk_z}{2\pi} \to \frac{mL}{2\pi\sqrt{2m\xi_q}}\int_{\Delta_0}^{\infty}(\ldots)\frac{E}{\sqrt{E^2-\Delta_0^2}}dE$$

(the wave functions of the continuous spectrum are assumed to be normalized in the segment $(-L/2, L/2)$, where $L \gg d$; naturally, the normalization length L drops out in the answer as $L \to \infty$).

The expression for j_1 takes the form

$$j_1 = \frac{2em^2}{\pi d}\Delta_0^2 T\sum_{\omega>0}\int_0^{v_0}v^2\,dv\,e^{-\omega d/v} \tag{4.7}$$

$$\times\mathrm{Re}\sum_n\left\{\frac{\exp\{i(dE_n{}^+/v + \chi)\}}{(\omega + \sqrt{\Delta_0^2 - E_n{}^{+2}})^2(E_n{}^+ - i\omega)} - \frac{\exp\{i(dE_n{}^-/v - \chi)\}}{(\omega + \sqrt{\Delta_0^2 - E_n{}^{-2}})^2(E_n{}^- - i\omega)}\right\}$$

and for j_2 we get accordingly

$$j_2 = -\frac{4em^2}{\pi^2}\Delta_0^2 T\sum_{\omega>0}\int_0^{v_0}v\,dv\,e^{-\omega d/v}\,\mathrm{Re}\int_{\Delta_0}^{\infty}dE\,\frac{\sqrt{E^2-\Delta_0^2}}{E}\frac{(\omega + iE)e^{idE/v}}{(\omega^2 + E^2 - \Delta_0^2)^2}\sin\chi. \tag{4.8}$$

These formulas are valid if $d \gg \xi_0$ and $T \ll T_c$, when the "square well" approximation (2.3) can be used. It is interesting to note that the phase depend-

4)We do not take into account the changes of $\Delta(z)$ near the surface of the NS junction at a distance $\sim\xi_0$, since these changes lead only a change of the numerical coefficients in the expression for j. This is analogous to specifying the gap in the form of a step function (2.3) in the case of low temperatures.

ence of the part of the current corresponding to summation over the continuous spectrum has the usual form $j_2 = j_S \sin\chi$, which is typical of the Josephson effect in S-I-S junctions[2,3,12], whereas the current component corresponding to summation over the discrete spectrum depends on χ in a much more complicated form (it must be recognized that $E_n^\pm$ in (4.7) are functions of χ). It is clear therefore that the fact that in the Josephson effect we always have a phase dependence of the type $\sin\chi$ is connected with the absence of localized states in the problem of the passage of the particle through a potential barrier (such states can arise, of course, if the barrier has a complicated form, but it is necessary that the distance between the corresponding discrete levels be of the order of Δ, and that the levels themselves depend on the phase χ).

We proceed to investigate the obtained expressions for the current (4.7) and (4.8). The sums over the discrete levels, which enter in (4.7), can be transformed by using the known Poisson summation formula

$$\sum_{n=n_1}^{n_2} f(n) = \sum_{p=-\infty}^{\infty} \int_{n_1-\alpha_1}^{n_2+\alpha_2} f(x) e^{2\pi i p x}\, dx, \quad 0<\alpha_{1,2}<1. \tag{4.9}$$

As a result, j_1 is represented in the form of the series

$$j_1 = \sum_{p=1}^{\infty} I_p \sin p\chi, \tag{4.10}$$

where

$$I_p = \frac{8em^2}{\pi^2}\Delta_0^2 T \sum_{\omega>0} \int_0^{v_0} v\, dv\, e^{-\omega d/v}$$
$$\times \mathrm{Re} \int_0^{\Delta_0} dE \frac{e^{-iEd/v} e^{2i\varphi(E)}}{(\omega+\sqrt{\Delta_0^2-E^2})^2 (E-i\omega)} \sin\left[2p\left(\varphi(E)-\frac{Ed}{v}\right)\right], \tag{4.11}$$

and the function $\varphi(E)$ is obtained from (2.15).

We shall assume that the thickness of the normal layer d is large not only compared with ξ_0, but also compared with the parameter ξ_T, which is determined by the relation

$$\xi_T = v_0/\pi T. \tag{4.12}$$

If $d \gg \xi_T$, then we can retain in the sum over the frequencies $\omega = (2\nu+1)\pi T$ only the lowest term $\omega = \omega_0 = \pi T$, corresponding to $\nu = 0$. Further, under the same condition, the integration with respect to v in (4.8) and (4.11) is carried out effectively over the region $v_0 - v \sim v_0 d/\xi_T \ll v_0$, leading to expressions of the form $(\omega_0 - imE)^{-1}\exp[-d(\omega_0 - imE)/v_0]$. As the result, only the single integral with respect to the energy remains in the formulas for j_1 and j_2, and it can be calculated asymptotically with allowance for the condition $\omega_0 \ll \Delta_0$ ($T \ll T_c$).

Turning to formula (4.11), we see that the integrand has sharp maxima at energy values $E = 0$ ($E \sim \omega_0$) and $E = \Delta_0$ ($\Delta_0 - E \sim \omega_0^2/\Delta_0$). Accordingly, the coefficients I_p and part of the current j_1 break up into two terms: $I_p = I_p^{(0)} + I_p^{(\Delta)}$ and $j_1 = j_1^{(0)} + j_1^{(\Delta)}$, corresponding to integration near the points $E = 0$ and $E = \Delta_0$, respectively. In calculating $j_1^{(0)}$ it is more convenient to use (4.7). After simple manipulations we obtain

$$j_1^{(0)} = \frac{6}{\pi}\frac{Nev_0}{\zeta} T e^{-d/\xi_T} F(\chi) \cos\frac{\chi}{2}, \tag{4.13}$$

where the function $F(\chi)$ has the following meaning:

$$F(\chi) = \sum_{n=-\infty}^{\infty} \frac{(-1)^n}{\nu^2 + (n + 1/2 - \chi/2\pi)^2}, \quad \nu = \frac{d}{\pi\xi_T}. \tag{4.14}$$

The last expression can be rewritten identically in the form

$$F(\chi) = -\frac{\pi i}{\nu}\sum_{s=-\infty}^{\infty} (-1)^s e^{i\chi(s+1/2)} e^{-2\pi\nu|s+1/2|}, \tag{4.15}$$

from which we see that when $\nu \gg 1$ we have asymptotically

$$F(\chi) \approx \frac{2\pi}{\nu} e^{-\pi\nu} \sin\frac{\chi}{2}. \tag{4.16}$$

We finally obtain for the part of the current $j_1^{(0)}$ the expression

$$j_1^{(0)} \approx 6(Nev_0/p_0 d) e^{-2d/\xi_T} \sin\chi, \quad v_0/d \ll T \ll \Delta_0. \tag{4.17}$$

The exponential factor in this formula $\exp(-2d/\xi_T) = \exp(-2\pi^2 T/\delta E)$ ($\delta E = \pi v_0/d$ is the distance between levels) is exactly the same as in the theory of quantum oscillation effects[14,8], i.e., it represents the usual temperature quenching of quantum oscillations. This confirms once more our treatment assumption that the investigated effect is connected with quantization of the excitation spectrum in the N layer.

The term $j_1^{(\Delta)}$ is a sum of the type (4.10), where $I_p^{(\Delta)}$ is estimated by

$$I_p^{(\Delta)} \approx -12\frac{Nev_0}{p_0 d} e^{-d/\xi_T}\left[\frac{Q_{2p-1}}{2p-1}\exp\left\{i(2p-1)\frac{\Delta_0 d}{v_0}\right\} + \frac{Q_{-2p-1}}{2p+1}\exp\left\{-i(2p+1)\frac{\Delta_0 d}{v_0}\right\}\right], \tag{4.18}$$

where

$$Q_\nu = T\int_0^\infty \frac{e^{-i\nu\varepsilon d/v_0}}{(\omega_0+\sqrt{2\Delta_0\varepsilon})^2} d\varepsilon. \tag{4.19}$$

For the coefficients Q_ν we obtain the following asymptotic expressions ($T_1 \ll T \ll T_c$)

$$Q_\nu \approx v_0/\pi i\nu\omega_0 d, \quad T \gg T_2,$$
$$Q_\nu \approx -\frac{\omega_0}{2\pi\Delta_0}\left[C + 2 + \ln\frac{\omega_0^2 d|\nu|}{2v_0\Delta_0} + \frac{\pi i}{2}\operatorname{sign}\nu\right], \quad T \ll T_2, \tag{4.20}$$

where we have introduced the characteristic temperatures $T_1 \sim v_0/d$ and $T_2 \sim (v_0\Delta_0/d)^{1/2}$, with $T_1 \ll T_2 \ll T_t$ and $C = 0.577$ the Euler constant. The term j_2 in the current is calculated in similar fashion. Since the integration begins here with $E = \Delta_0$ and is carried out effectively over the region $E - \Delta_0 \sim \omega_0^2/\Delta_0 \ll \omega_0$, this term will also contain an oscillating factor $\exp(id\Delta_0/v_0)$. In the limiting cases we obtain the following formulas

$$j_2 \approx A\cos\frac{\Delta_0 d}{v_0}, \quad T_1 \ll T \ll T_2 \ll T_c,$$
$$j_2 \approx A\pi^{-7/2}\left(\frac{2v_0\Delta_0}{dT^2}\right)^{3/2}\cos\left(\frac{\Delta_0 d}{v_0}+\frac{3\pi}{4}\right), \quad T_1 \ll T_2 \ll T \ll T_c, \tag{4.21}$$

where

$$A = (3Nev_0/p_0 d)\, e^{-d/\xi_T} \sin\chi.$$

Gathering the results together, we obtain the total junction current j. In the limit $d \gg \xi_T$, we can omit the term $j_1^{(0)}$ (formula (4.17)), since it is proportional to $\exp(-2d/\xi_T)$ and not to $\exp(-d/\xi_T)$ (compare with (4.21)). The remaining terms oscillate with the thickness of the normal metal d. Considering for simplicity the first limiting case in (4.21) ($T_1 \ll T \ll T_2$), we obtain for the critical current of the contact the estimate

$$j_c \sim (3Nev_0 / p_0 d) e^{-\pi d T/v_0} |\cos(\Delta_0 d / v_0)|. \quad (4.22)$$

The period of the oscillations with respect to thickness is $\delta d = \pi v_0/\Delta_0$. The physical reason for the oscillating $j_C(d)$ dependence is the change of the distance between levels with increasing thickness of the normal layer d. The number of levels "accommodated" in a well of height Δ_0 then increases. At definite values of d, a jumplike change takes place in the number of levels, and this leads to oscillations of the energy, and consequently also of the Josephson current, with changing thickness. Thus, the described effect is due to the motion of the size-quantization levels past the edge of the energy gap of the superconductor Δ_0. We note that in principle the current should oscillate also with temperature (owing to the temperature dependence of Δ_0), but in the region of temperatures $T \ll T_C$ the $\Delta(T)$ dependence is very weak.

As seen from (4.22), the ratio of the critical current of the junction j_C to the "volume" critical current $j_{cm} \sim Nev_0\Delta_0/\zeta$ (see footnote[3)]) is a quantity of the order of $(\zeta_0/d)\exp(-d/\xi_T) \lesssim \xi_0/d \ll 1$, thereby ensuring self-consistency of the calculation (the possibility of neglecting the gradient of the phase inside the superconductors compared with its jump on the surface)[5)]. Just as in the case of the ordinary Josephson effect[12], the magnetic field produced by the flowing currents can be neglected if $L \ll \lambda_J$, where L is the width of the junction and λ_J is the Josephson depth of penetration, $\lambda_J = (\hbar c^2/8\pi e \Delta j_C)^{1/2}$; the length $\Lambda = 2\lambda_L + d$ is determined in this case mainly by the thickness of the layer of the normal metal d. When this condition is satisfied, the problem becomes one-dimensional.

Let us make a few concluding remarks. Although in this paper we have used the model of a rectangular gap $\Delta(z)$ (2.3), it is clear in principle that allowance for a real (continuous) variation of Δ near the boundary with the normal metal does not change the nature of the considered effect, if the width of the "well" d is large compared with the region ξ_0 where its edge is diffuse. Moreover, all the formulas of Sec. 3 actually contain not the gap $\Delta(z)$ itself, but the quantity $\Delta(z)U(z)$, which vanishes identically when $|z| < d/2$, so that in reality the presence of the jump is not connected with the chosen model (allowance for the real situation changes only the magnitude of this jump). The physics of the effect under consideration does not depend on the exact form of the boundary conditions on the surface. In particular, the fact of quantization of the energy of the excitations in the normal layer and of the jumplike change of the number of levels with thickness is not connected with the concrete form of $\Delta(z)$.

Experimental observation of the discussed effect calls for the construction of S-N-S junctions containing a flat defect-free normal layer, smoothly going over into the superconducting metal (without voids or extraneous inclusions). No strong reflection of the electrons must take place on the boundary of such a layer, for otherwise the position of the Andreev levels becomes practically insensitive to the phase. In the foregoing calculations we assumed specular boundary conditions in the surface, but actually by virtue of the specific nature of the reflection of the excitations from the NS interface the "specularity" (at low energies) is retained also if the surface is not even[9]. It is known that quantization appears experimentally in the so-called Tomasch effect[15,16], namely oscillations of the tunnel current as a function of the voltage ($\delta V = \delta E/e$). Apparently, the greatest difficulty is the determination of the transition into the superconducting phase under conditions when the normal resistance is itself extremely small (since the normal layer should be pure). In addition, sufficiently low temperatures are necessary (at large thicknesses). Thus to observe superconducting currents through a normal layer of thickness $d \sim 10^2\xi_0$ it is necessary to have a temperature that is lower than T_C by at least two orders of magnitude.

In conclusion, I am grateful to A. A. Abrikosov and L. P. Gor'kov for a useful discussion of the results.

5)It should be noted that both the expression for j_{cm} and the formula (4.22) for j_C are valid in the case when the total current is sufficiently small, so that the destruction of the superconductivity is the result of unpairing processes, and not as a result of the Meissner effect. This means that we are considering sufficiently thin films or narrow contacts having a small cross section.

[1] A. F. Andreev, Zh. Eksp. Teor. Fiz. 46, 1823 (1964) and 49, 655 (1965) [Sov. Phys.-JETP 19, 1228 (1964) and 22, 455 (1966)].

[2] B. D. Josephson, Adv. Phys. 14, 419 (1965).

[3] P. W. Anderson, Ravello Lectures on the Many Body Problem v. 2, Ed. E. R. Gaianello, Acad. Press, 1963, p. 115.

[4] L. G. Aslamazov, A. I. Larkin, and Yu. N. Ovchinnikov, Zh. Eksp. Teor. Fiz. 55, 323 (1968) [Sov. Phys.-JETP 28, 171 (1969)].

[5] N. N. Bogolyubov, Usp. Fiz. Nauk 67, 549 (1959) [Sov. Phys.-Usp. 2, 236 (1959)].

[6] P. de Gennes, Superconductivity of Metals and Alloys, Benjamin, 1967.

[7] V. P. Galaĭko, Zh. Eksp. Teor. Fiz. 57, 941 (1969) [Sov. Phys.-JETP 30, 514 (1970)].

[8] I. O. Kulik, ZhETF Pis. Red. 6, 652 (1967) [JETP Lett. 6, 143 (1967)].

[9] G. A. Gogadze and I. O. Kulik, Fiz. Tverd. Tela 11, 2182 (1969) [Sov. Phys.-Solid State 11, 1762 (1970)].

[10] A. A. Abrikosov, L. P. Gor'kov, and I. E. Dzyaloshinskiĭ, Metody kvantovoi teorii polya v statisticheskoi fizike (Quantum Field Theoretical Methods in Statistical Physics), Fizmatgiz, 1962 [Pergamon, 1965].

[11] I. O. Kulik and A. E. Gorbonosov, VINITI Preprint No. 239 (1968).

[12] I. O. Kulik and I. K. Yanson, "Effekt Dzhozefsona," Fizika kondensirovannogo sostoyaniya (The Josephson Effect. Physics of the Condensed State). FTINT AN UkrSSR, No. 3, 1968, p. 3.

[13] V. P. Galaĭko, A. V. Svidzinskiĭ, and V. A. Slyusarev, Zh. Eksp. Teor. Fiz. 56, 835 (1969) [Sov. Phys.-JETP 29, 454 (1969)].

[14] I. M. Lifshitz and A. M. Kosevich, ibid. 29, 730 (1955) [2, 636 (1956)].

[15] W. J. Tomasch, Phys. Rev. Lett. 15, 672 (1965); 16, 16 (1966).

[16] W. L. McMillan and P. W. Anderson, Phys. Rev. Lett. 16, 85 (1966).

Translated by J. G. Adashko
207