A CALCULATION OF SPACE CHARGE LIMITED FIELD EMISSION CURRENT OF RELATIVISTIC ELECTRONS

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Submitted April 10, 1969

Zh. Eksp. Teor. Fiz. 57, 1795–1798 (November, 1969)

The field emission current from metallic surfaces is calculated for relativistic electrons, taking into account the space charge effect. Maximal values of the field emission currents for tungsten for energies between 0.5 and 50 MeV are obtained by computer calculations based on the Fowler-Nordheim law and the Poisson equation in its relativistic form. It is shown that the maximal current flowing from a field emitter is significantly less than that calculated recently by a number of authors without taking into account the relativism of the electrons.

WITH recent experiments studying the interaction of bunches of charged particles with plasmas[1,2], superpowerful accelerators with energies of 1–10 MeV and currents 10^4–10^6 A have acquired much more significance. The only known mechanism allowing one to get such currents with the necessary transverse dimension for the bunches, $d_0 \leq 1$ cm, is field emission. The calculational scheme discussed by Dyke and Dolan[3] for field emission from cathodes does not allow for the fact that the electrons may be relativistic, which is important when working with accelerating potentials greater than 0.5–1.0 MV. In this paper, the field-emission current of a planar diode is calculated taking into account the space charge and relativistic effects. The results obtained for the plane model are generalized to the case of pointed field emitters by the method of[3].

We start from the following: the Poisson equation

$$\frac{d^2\varphi}{dx^2} = \frac{\rho}{\varepsilon_0} = \frac{J}{\varepsilon_0 v} \qquad (1)$$

where ϵ_0 is the dielectric constant of the vacuum, J the current density, and v the speed of the electrons; the Fowler-Nordsheim equation, connecting the field-emission current density J with the field E at the cathode

$$J = aE^2 \exp(-b/E), \qquad (2)$$

where a and b are constants depending on the material of the emitter; and the law of conservation of energy

$$mc^2 + e\varphi = \frac{mc^2}{\sqrt{1 - v^2/c^2}}. \qquad (3)$$

The boundary conditions are

$$\varphi|_{x=0} = 0, \quad \varphi|_{x=d} = V,$$

where d is the separation of the electrodes, and V is the potential difference applied between anode and cathode. The cathode is taken in the plane x = 0, with $E = d\varphi/dx|_{x=0}$.

Integrating the system of equations (1)–(3), we get expressions connecting the field E at the cathode, with the quantity V:

$$\int_0^V \left[\frac{4}{\varepsilon_0}\sqrt{\frac{m}{2e}}\left(\varphi + \frac{e\varphi^2}{2mc^2}\right)^{1/2} a \exp\left(-\frac{b}{E}\right) + 1\right]^{-1/2} d\varphi = Ed. \qquad (4)$$

In the absence of relativistic effects ($eV \ll mc^2$), we get from Eq. (4) the relation found previously in[3]:

$$V - \frac{4a}{3\varepsilon_0} V^{3/2} \sqrt{\frac{m}{2e}} \exp\left(-\frac{b}{E}\right) + \frac{3ma^2d^2E^2}{2\varepsilon_0^2 e} \exp\left(-\frac{2b}{E}\right) = Ed. \qquad (5)$$

The magnitude of the field-emission current density can be obtained from Eq. (2) by substituting the value of E found from Eq. (4).

Consider the asymptotic behavior of the volt-ampere characteristic of the field-emission diode in the case when $V \gg Ed$, i.e., when the field E at the cathode is much less than the vacuum field $E_0 = V/d$. In this case there is a strong screening of the external field by the space charge, and the volt-ampere characteristic of

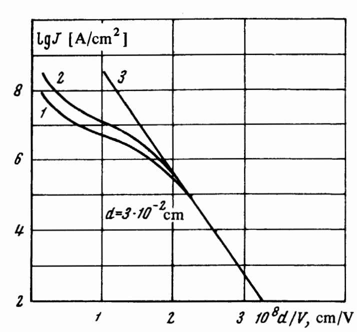

FIG. 1

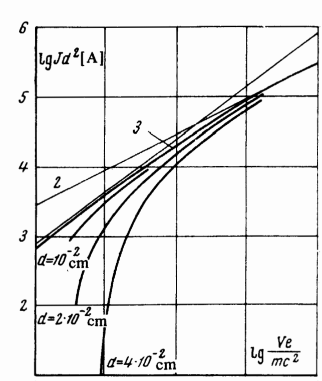

FIG. 2

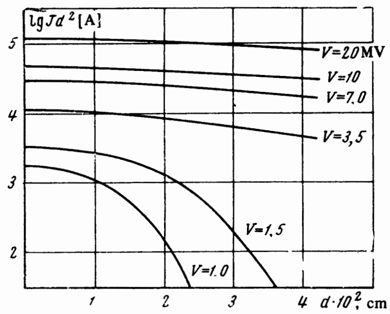

FIG. 3

the field-emission diode, when no account is taken of relativistic effects, degenerates to the Child-Langmuir behavior (the "$^3/_2$ power law"). In the general, relativistic case, we get an upper limit for the field-emission current density for $V \gg Ed$ by setting $E \to 0$ (for given current J) in Eq. (4):

$$J_0 = \frac{\varepsilon_0 \sqrt{2e/m}}{4d^2} \left[\int_0^V \left(\varphi + \frac{e\varphi^2}{2mc^2} \right)^{-1/4} d\varphi \right]^2. \quad (6)$$

Equation (6) is the generalization of the Child-Langmuir equation for relativistic electrons. When the relativistic effects are weak ($eV \ll mc^2$) or strong ($eV \gg mc^2$), we obtain from Eq. (6)

$$J_0 = \frac{4}{9} \varepsilon_0 \sqrt{\frac{2e}{m}} \frac{V^{3/2}}{d^2} \left[1 - \frac{3}{28} \frac{eV}{mc^2} \right], \quad (7)$$

$$J_0 = 2\varepsilon_0 cV / d^2. \quad (8)$$

The limits of applicability of the expressions (7) and (8) can be assessed from the results of numerical calculation with Eq. (6), shown in Fig. 2.

The calculation was carried out with an electronic computer for a tungsten emitter (in the c.m. system, $a = 2.64 \times 10^{-5}$; $b = 6.02 \times 10^{10}$) in the range $0.5 \leq V \leq 50$ MV. The results of the numerical calculation of the field-emission current density, depending on the applied tension, are presented in Fig. 1. The curve 1 is obtained on solving the system of equations (2) and (4) with allowance for relativistic effects, curve 2 without allowing for them (Eqs. (2) and (5)), and curve 3 neglecting the influence of the space charge (Eq. (2)). It can be seen from Fig. 1 that relativistic effects lead to a significant reduction in the magnitude of J. The observed difference is greater the higher the applied potential. Thus, for V = 20 MV, the value of J obtained from Eqs. (2) and (4) is less by a half than the J determined by Eqs. (2) and (5). In Fig. 2, the volt-ampere characteristics of the field-emission diode for various values of d are presented. The solution of Eq. (6) (curve 3) and two of its asymptotes, without relativistic effects - "$^3/_2$ law" (line 1), and for strong relativistic effects - Eq. (8) (line 2), are also given there.

The fields necessary to provide really effective field-emission are usually achieved by using sharp cathodes. In this connection, the question arises as to what is the absolute magnitude of the current that can be obtained from a single pointed field-emitter. Dyke and Dolan have shown[3] that for electrons from a source whose tip has radius of curvature r, auto-emission occurs in a solid angle ~1 sr, which corresponds to an emitting area equal to r^2. However, in going from the plane problem to field-emission from a point, the factor d equals the radius of the tip, r, in the first approximation. Thus, the values J_0d^2 given in Fig. 2 determine the maximum value of the current of a pointed field emitter. It can also be seen from Fig. 2 that for a given voltage V, the maximal current that can be obtained from a single field emitter is bounded above by the limit J_0d^2, determined by Eq. (6) (curve 3). How close real field-emission diodes are to saturation by the space-charge can be seen from Fig. 3, where the dependence of the field-emission current on the factor d is presented. In the limiting case of small d, Jd^2 has a slower growth rate and tends to an upper limit for $d = 0$, corresponding to a current J_0d^2.

It remains to note that the minimal values of r are limited by Joule heating of the tip. The choice of radius of the tip is determined by a compromise between the admissible heating in the current flow, and the reduction in the field-emission current if r is consequently chosen large.

In conclusion, we point out that operation of the diode in the regime where the current is limited by the space charge is distinguished by the stability of the field-emission current. In this case, owing to the compensating effect of the space charge, the field-emission current instability due to changes in the applied potential, temperature and work function of the emitter, and also to the erosion of the emitter, is considerably less than if the space charge were absent.

The authors thank V. I. Bogdanov for carrying out the numerical calculation, and A. A. Rukhadze for supplying results.

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Translated by C. Hindle
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