TEMPERATURE DEPENDENCE OF THE MAGNETIC RESISTANCE OF ALUMINUM

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The temperature dependence of the transverse components of the resistance tensor of high purity aluminum is investigated for various values of the intensity and direction of the magnetic field and for temperatures between 4 and 20°K. It is found that the mechanisms of conduction-electron scattering are not isotropic. The anomalous phenomenon of increase of the relative magneto-resistance $\Delta\rho_H/\rho_0$ with increase of temperature is investigated in detail. The results are interpreted on the basis of the concepts of magnetic breakdown and small-angle scattering, and by taking into account the contribution of U-processes.

INTRODUCTION

THE anomalous behavior of the magnetoresistance of aluminum in large effective fields[1-3] was the basis for the hypothesis that open trajectories exist in aluminum. One of the causes of their appearance may be magnetic breakdown. Giant oscillations of the transverse resistance, observed by Parker and Balcombe[4], point to the presence of breakdown at the corners (W points) of the Brillouin zone in fields exceeding 30 kOe. However, these data (breakdown at the W points only; the values of the width of the layer of open trajectories and of the breakdown field estimated by Parker and Balcombe) do not suffice to explain the nature of the anomalies, observed in weaker fields, of the monotonic component of the field dependence of the transverse resistance ρ_H. In particular, the causes for the absence of a sharp anisotropy of ρ_H, which should take place in the model of narrow layers of open trajectories at the W points, remain unexplained. Also unexplained are the appreciable magnitude of ρ_H (sometimes up to $10\rho_0$ (ρ_0 is the resistance and the absence of the field) at those directions of H for which the realization of the open trajectories is impossible within the framework of this model, the deviations from the Kohler rule, etc.

It is shown in[5,6] that the formation of narrow layers of open trajectories should be accompanied by the appearance of a non-isotropic character of the scattering of electrons in a magnetic field. In this connection, measurements of the temperature dependence of the magnetoresistance, together with usual measurements, may contribute to the clarification of the nature of the indicated anomalies. No such investigations have been carried out so far. We report in this article such investigations on single crystals of high purity aluminum. A comparison of the results on the anisotropy and the field and temperature dependences of the resistance of aluminum in a magnetic field makes it possible to deduce the presence of magnetic breakdown not only at the corners of the Brillouin zone, which leads to the formation of not too narrow layers of open trajectories, and to understand the nature of certain singularities in the behavior of the magnetoresistance of aluminum, observed in magnetic fields $\omega\tau < 100$.

EXPERIMENT

Measurement technique. The investigations were performed in the magnetic field intensity range 100–20,000 Oe and in the temperature range 4.2–20°K. In view of the use of samples of very high purity, and consequently of the need for measuring very small potential differences, a high-sensitivity procedure was used, described in detail earlier in[7] and modified for measurements in a magnetic field. The setup was used as a null indicator. An attempt has been made to use a system with a superconducting modulator for such measurements,[8] without additional technical complications, but it is very difficult to attain in this method a sensitivity better than 10^{-9} V, owing to the presence of electromagnetic induced noise. It was established that the most significant source of this noise are the mechanical vibrations of the cryostat (mainly as a result of the boiling of the liquids), leading to the appearance of parasitic induction emf. By eliminating them it is possible to attain a sensitivity $\sim 10^{-11}$ V without using any additional screening measures.

Figure 1 shows schematically the cryostat in which the measurements were performed. Part of the measuring system with the modulator is taken out to the region of the stray field, whose intensity is smaller than H_{cr} of the material used for the shields of the solenoid 8 and the transformer 9. To maintain the superconducting state over as great a length of the measuring circuit in the entire working field interval and temperature interval, part of the circuit 6 was made of niobium-zirconium wire, for which the critical field necessary to destroy superconductivity exceeds 50 kOe.

The operating frequency of modulator switching was 540 Hz and all the electric characteristics of the circuit and of its elements were close to those described in analogous devices[8,9]. It must be recalled once more that in all such devices the relaxation time of the measuring circuit is smaller by not less than one order of magnitude than the switching period of the modulator, so that there is practically no alternating current flowing through its sample. As a check, the modulator switching frequency was varied over a wide range (from 10 to 1000 Hz), and this did not affect in any way the measurement of the sample resistance measurement.

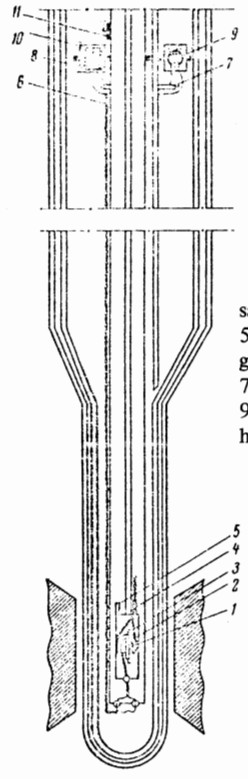

FIG. 1. Cryostat: 1 – thermometer, 2 – sample, 3 – heater, 4 – evacuated volume, 5 – lead for admission of the heat-exchange gas, 6 – section of circuit of Nb + Zr wire, 7 – calibrated resistance, 8 – solenoid, 9 – transformer, 10 – modulator, 11 – helium-level pickup.

The calibrated resistance 7 (Fig. 1) was a brass rod 25 mm long and 3 mm in diameter, and was selected with the aid of the R308 instrument; at T = 4.2°K its resistance was 8.5×10^{-7} ohms and remained the same during the course of all the measurements.

Samples. We investigated single crystals of aluminum with a room to helium temperature resistance ratio $(1.1-2.1) \times 10^4$. The sample had a square cross section exceeding 2 × 2 mm (4 × 4 mm in the case of the sample Al-23); the distance between the potential contacts was 10 mm. The sample axes were close to the principal crystallographic directions [100], [110], and [111], and were oriented with the aid of the Laue patterns. The measuring current did not exceed 1 A and was directed along the sample axis. Data on the samples are listed in the table.

The sample preparation included the operations of cutting the samples from the ingot by the electric-erosion method, etching, and annealing in air for several days at 500–600°C.

The measurements were performed in stationary fields of an electron magnet with a pole-piece diameter 65 mm and a distance between pole pieces 35 mm. To exclude the components of the hole emf and the thermal emf, the field directions and the current directions were alternated. To determine the role of the extraneous effects (bending of the current lines, geometrical shape effect, position of the contact relative to the sample axis), special investigations were performed, similar to those performed by Borovik and co-workers[3]. It was confirmed, that these effects are insignificant in aluminum, at least in our measurements.

Measurement results. For all the samples, simultaneous measurements were made of the dependence of the transverse magnetoresistance on the angle of rotation of the magnetic field H in weak and strong fields, the dependence on the field intensity (100–20,000 Oe), and the temperature dependence (12.2–20°K).

Sample	R (300° K)/ /R (4,2° K)	Sample axis orientation*
Al-22	13550	[110]
Al-23	15850	[100] **
Al-24	15900	[100]
Al-25	11200	[111]
Al-26	12730	[110]
Al-28	21130	[111]

*Accuracy ± 2°.
**Accuracy ± 4° in the {110} plane.

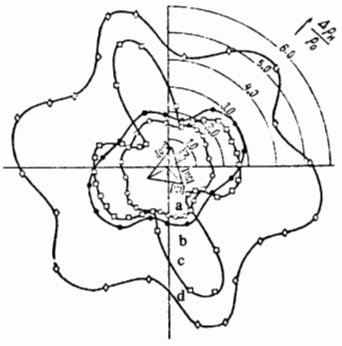

FIG. 2

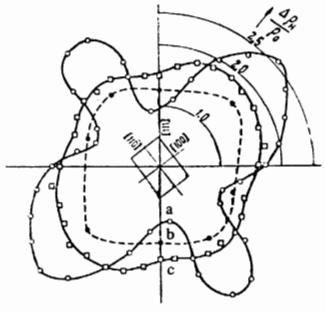

FIG. 3

FIG. 2. Rotation diagrams (j ∥ [111]). Sample Al-28: a – T = 4.2°K, H = 2 kOe; c – T = 4.2°K, H = 20 kOe; d – T = 20.4°K, H = 20 kOe; b – sample Al-25, T = 4.2°K, H = 20 kOe.

FIG. 3. Rotation diagrams (j ∥ [110]). Sample Al-26: a – T = 4.2°K, H = 2 kOe; c – T = 4.2°K, H = 20 kOe; b – sample Al-22, T = 4.2°K, H = 2 kOe.

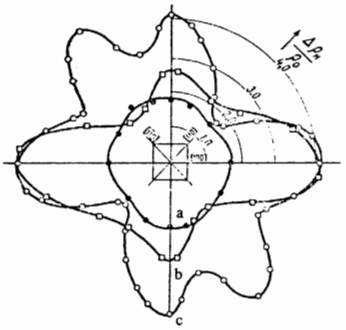

FIG. 4. Rotation diagrams (j ∥ [100]). Sample Al-23; a – T = 4.2°K, H = 2 kOe; c – T = 4.2°K, H = 20 kOe; b – sample Al-24, T = 4.2°K, H = 20 kOe.

1. Figures 2–4 show rotation diagrams of the investigated samples with axes parallel to the threefold (Fig. 2), twofold (Fig. 3), and four-fold axes (Fig. 4) in fields of 2 and 20 kOe at a temperature 4.2°K. It was observed that the form of the rotation diagrams is quite sensitive to the relative orientation of the vector H and of the crystallographic axes (especially for the sample Al-23), and this is not a consequence of the geometrical shape effect. This will be discussed in greater detail in the discussion section.

2. Figure 5 shows typical plots of the relative variation of the transverse resistance $\Delta\rho_H/\rho_0 = (\rho_H - \rho_0)/\rho_0$ as a function of the magnetic field intensity, obtained for different samples at the most characteristic directions of the vector H. In previously published papers,

(for example[3]), there are no hints of such a difference in the character of the field dependences of $\Delta\rho_H/\rho_0$ in large effective fields. A dependence with a distinct saturation (curve 2) has been observed here for the first time for pure samples. We call attention also to a plot showing a decrease of the magnetoresistance at certain directions of H (curve 1). Such a plot might have been observed also by others (see, for example, Fig. 40 of[10], which shows rotation diagrams for a sample with $\mathbf{j} \parallel [111]$ at two values of the field; compare with the values of $\Delta\rho_H/\rho_0$ at $\mathbf{H} \parallel [112]$). The anomalous dependence of the type represented by curve 6 is qualitatively similar to curve 2 of Fig. 4 in[3], and reduces to the plot 2 when the axis of the Al-23 sample deviates somewhat from the rotation axis (the shape of the rotation diagrams changes simultaneously).

3. Figure 6 shows a typical variation of the field dependence with increasing temperature from 4.2 to 20°K. The behavior of the magnetoresistance is unusual: when the temperature increases to approximately 16–18°K, the $\Delta\rho_H/\rho_0 = f(H)$ curves lie higher than the curves corresponding to the lower temperatures. With further increase of the temperature, the normal arrangement of the curves is restored, although at T = 20°K the value of $\Delta\rho_H/\rho_0$ still remains larger than at T = 4.2°K. The latter circumstance was observed also in[1].

For a detailed illustration of the character of the anomaly and for the comparison with the results of other authors, Fig. 7 shows typical plots of $\Delta\rho_H/\rho_0$ against the temperature in the entire investigated temperature interval for the directions of H corresponding to the field dependences with different asymptotic forms at T = 2.4°K.

4. We investigated the temperature dependence of the transverse resistance $\delta_H(T) = \rho_H(T)/\rho_0(300°K)$ at different values of the magnetic field intensity in the interval 4.2–20°K for the following directions of the vector H: for Al-23—$\mathbf{H} \parallel [100], [110]$; for Al-26—$\mathbf{H} \parallel [100], [110], [111]$; for Al-28—$\mathbf{H} \parallel [110], [112], [121]$. The values of the field were chosen in such a way as to be able to trace the behavior of the magnetoresistance with changing temperature in the characteristic sections of the field dependences of the type shown in Fig. 5. For comparison with the temperature dependences of the resistance in the absence of the magnetic field, the $\delta_0(T)$ curves were plotted for each of the investigated samples, in view of the fact that the Matthiessen rule may not be satisfied in aluminum. It was observed that the exponents n of the temperature dependences of the transverse resistance are essentially determined by the magnitude and direction of the field and by the temperature interval. The following regularities were revealed upon classification of the curves:

a) For a given orientation of the vector H and a fixed value of the field, i.e., for each curve separately, the exponent n in the temperature intervals above 10°K and below this value are generally not the same.

b) The character of the $\delta_H(T)$ dependence at comparable values of the field for directions of H corresponding at T = 4.2°K to an unlimited growth of $\Delta\rho_H/\rho_0$ as a function of the field, differs from the character of the $\delta_H(T)$ dependence for the direction of H corresponding to the $\Delta\rho_H/\rho_0 = f(H)$ curves with saturation at the same temperature. In the former case n is somewhat higher than in the latter, particularly for the interval T > 10°K. For the first of the indicated directions of H, the exponent n changes in larger limits with changing field intensity.

c) In the temperature interval below 10°K, n is lar-

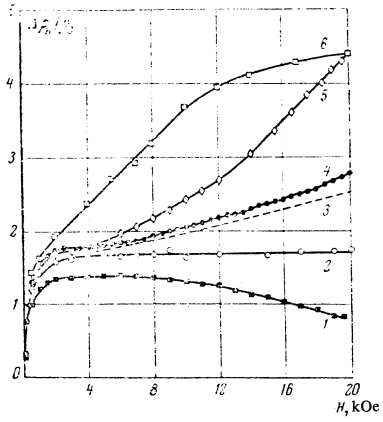

FIG. 5. Dependence of transverse magnetoresistance on the magnetic field intensity at T = 4.2°K: 1 – $\mathbf{j} \parallel [110]$, $\mathbf{H} \parallel [111]$; 2 – $\mathbf{j} \parallel [100]$, $\mathbf{H} \parallel [110]$; 3 – $\mathbf{j} \parallel [100]$, $\mathbf{H} \parallel [100]$ from [4]); 4 – $\mathbf{j} \parallel [110]$, $\mathbf{H} \parallel [100]$; 5 – $\mathbf{j} \parallel [110]$, $\mathbf{H} \parallel [110]$; 6 – $\mathbf{j} \parallel [100]$, $\mathbf{H} \parallel [110]$.

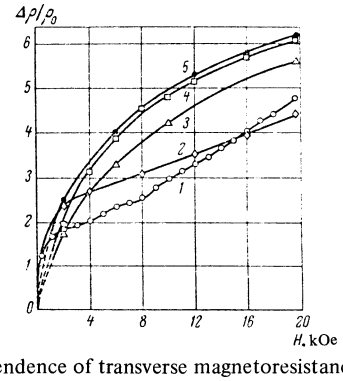

FIG. 6. Dependence of transverse magnetoresistance on the intensity of the magnetic field at different temperatures for the sample Al-28, $\mathbf{H} \parallel [110]$: 1 – T = 4.2°K, 2 – T = 9.3°K, 3 – T = 22°K, 4 – T = 20°K, 5 – T = 18°K.

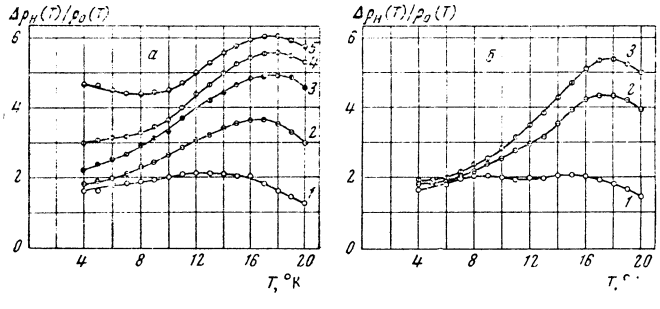

FIG. 7. Dependence of the relative magnetoresistance on the temperature for H directions corresponding at T = 4.2°K in case a – to a growth of $\Delta\rho H/\rho_0 = f(H)$: curve 1 – H = 1 kOe, 2 – H = 3 kOe, 3 – H = 7 kOe, 4 – H = 15 kOe, 5 – H = 20 kOe (for a dependence of the type 5 in Fig. 5); in case b – to saturation of $\Delta\rho H/\rho_0 = f(H)$: curve 1 – 2 kOe, 2 – 10 kOe, 3 – 20 kOe (for dependences of type 1 and 2 in Fig. 5).

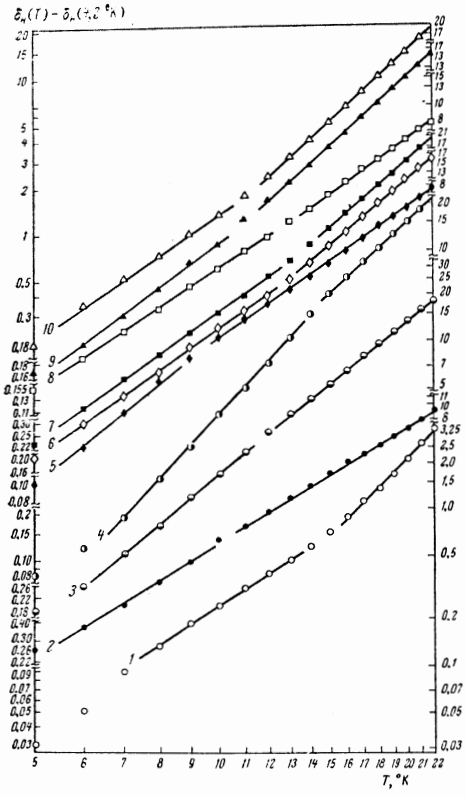

FIG. 8. Temperature dependence of the transverse resistance of the Al-28 sample for different directions and intensitites of the magnetic field. Case H ∥ [110]: curve 1 – H = 0, 2 – 2 kOe, 3 – 10 kOe, 4 – 20 kOe; case H ∥ [112]: curve 5 – 2 kOe, 6 – 10 kOe, 7 – 20 kOe; case H ∥ [121]: curve 8 – 2 kOe, 9 – 10 kOe, 10 – 20 kOe. (The ordinates represent $[\delta_H(T) - \delta_H(4.2°K)] \times 10^4$).

ger in a field than in the absence of a field, for directions of H corresponding to an unlimited growth of the resistance at T = 4.2°K. This difference is practically unobservable for other directions of H. When T > 10°K, the exponent is much smaller (n < 3) in fields 1–2 kOe and is close to the exponent for the case H = 0 in the same temperature interval (n ≈ 4) in strong fields.

These regularities are general and are illustrated by the data for one sample, plotted logarithmically in Fig. 8 with the values of $\delta_H(4.2°K)$ subtracted.

DISCUSSION

One of the main results obtained by us is the observation, in strong fields, of a different asymptotic behavior of the transverse components of resistance of aluminum. In the discussion of the topology of the Fermi surface of aluminum, it was customary earlier to start from the concept that $\Delta\rho_H/\rho_0$ does not depend on the field with saturation, and that the transverse resistance increases with increasing field for most directions of H. These notions, which are difficult to explain within the framework of the known Fermi-surface models, served as the basis for a model with open trajectories at the corners W of the Brillouin zone[2]. However, attempts to reconcile the different experimental data within the framework of this model encounter considerable difficulties. Inasmuch as one can speak only of narrow layers of open trajectories in the corners W[2,4] (it is immaterial whether they are the consequence of the presence of bridges or magnetic breakdown), there should be observed an anomalously sharp anisotropy of the resistance, the existence of which is not indicated in any of the known experiments. In addition, at directions of H close to the principal crystallographic axes, but inclined to a sufficient degree as to make it impossible to realize open or extended orbits (for example by an angle larger than 1°), it is necessary to expect in this model either the absence of a field dependence of the resistance, or a weak field dependence, whereas something entirely different was observed, namely an increase of the resistance, and furthermore in very pure samples ([3] and our measurements). In addition, our data indicates that the directions of H at which the situation of the resistance takes place do not agree with the directions of H where this saturation should be observed in a model with open trajectories only in the corners of the zone. Finally, the most significant difficulties arise when it comes to explain the data on the temperature dependence of the magnetoresistance, as will be discussed later on.

These contradictions can be reconciled by postulating the possibility of formation of open trajectories not only at the W corners, thereby eliminating the limitations connected with the need for considering only very narrow layers of such trajectories. The formation of open trajectories, which do not pass through the W corners, can be due only to magnetic breakdown. The reality of such a breakdown becomes clear when certain data on band calculations for aluminum are considered. In the almost-free-electron model, the energy gap is equal to twice the value of the matrix element of the lattice potential. Calculations of the matrix elements in the OPW model, carried out, in particular, by Animalu[11], show that for different Bragg planes they can differ by one order of magnitude. Thus, for a local pseudopotential, Animalu obtained $\overline{V}_{111} = 0.004$ and $\overline{V}_{200} = 0.04$ Ry (the reciprocal-lattice vectors are given in units of $2\pi/a$). In accordance with the expression for the magnitude of the interband-breakdown field[12]

$$H_0 = K\frac{mc(\Delta\varepsilon_g)^2}{h|e|v_F}$$

(H_0—from the expression for the breakdown probability $P = \exp(-H_0/H)$; $K \sim 1$, $\Delta\epsilon_g$—energy gap at the boundary of the band, and ϵ_F—Fermi energy, equal to 0.86 Ry in the free-electron model), this yields $H_0 \sim 5 \times 10^3$ Oe and $H_0 \sim 5 \times 10^5$ Oe respectively for the faces {111} and {100}. Thus, the possibility of observing breakdown in the investigated field interval is real. The breakdown probability is large here only for the faces {111}, whereas for the faces {100} the probability of Bragg reflection is large.

Within the framework of these concepts, we investigated the geometry of the possible open trajectories. To this end, for the experimentally encountered characteristic directions of the vector H we plotted grids of coupled orbits[13,14] and the corresponding trajectories, with allowance for breakdown only through the {111} plane. For certain directions of the field open trajectories of the type shown in Fig. 9 then become possible (this corresponds to the case of transition III → IV in Fig. 8 of[15]), and for others only closed orbits (transi-

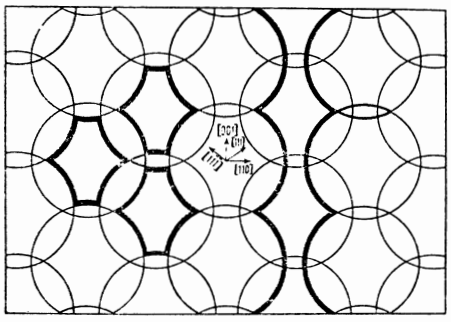

FIG. 9. Grid of coupled orbits at H || [100].

tion III — II in Fig. 11 of[15]). In each of these cases, within the framework of the semiclassical theory of galvanomagnetic phenomena[15-17], we analyze the structure of the conductivity tensors, the components of which were compared with the experimental data[18]. When the vector H coincides exactly with the symmetry axis, the open trajectories with a single openness direction are possible only in the case H || [110] (Fig. 9), whereas when H || [111], [112], [100] the most probable are closed electron orbits. This, in general, is in good agreement with the observed character of the anisotropy and the field dependences for all the investigated sample, explaining both the absence of a sharp anisotropy and the presence of saturation of the magneto resistance for those directions of H, where it is predicted by the proposed breakdown picture.

The inevitable inaccuracy of the orientation of the sample, and consequently also of H, relative to the crystallographic axes, leads to deviations from the ideal picture. This is most significant for the region near the singular direction [100] (in the ideal case H || [100], owing to breakdown at all the W corners, the single openness direction vanishes, which, as is well known, excludes the possibility of an increase in the resistance[16]) and is responsible for the anomaly of the type shown in Fig. 4 (the distortion of the shape of the rotation diagram at T = 4.2°K, curve c). In particular, by inclining the axis of the Al-23 sample away from the vertical direction, within a solid angle of 4°, we observed a strong change in the shape of the rotation diagram (up to a reconstruction of the characteristic symmetry). A similar effect took place also in the experiments of Borovik and co-workers, who obtained, using practically identical samples with a disorientation of the axes relative to each other by only 1—2°, rotation diagrams that differed greatly in shape, with anomalies precisely when the H direction was close to [100] (for example, curves 2 and 4 of Fig. 1 of[3]). Unfortunately, no experiments, with which to compare our results, have been performed with samples that are well oriented relative to the crystallographic directions.

The character of the anomalies of the anisotropy and of the field dependences does not contradict the assumption that they are simultaneously the result of different breakdown conditions at the corners and on the boundaries of the zone, and of inexact coincide of H with the crystallographic directions, and the overall picture is determined by the breakdown in the cross sections which do not pass through the corners of the zone, while the role of the breakdown at the corners reduces to non-characteristic distortions of the overall picture. This is clearly illustrated, for example, by the form of the field dependences for the case of the orientation H shown in Fig. 9 (Fig. 5, curve 5, and also Fig. 4, curve 1 of[3]), and for the direction of H near [100] (Fig. 5, curve 4 and curve 3, constructed in accordance with the data of Parker and Balcombe[4] for samples with orientation analogous to the orientation of Al-23): in the second case, the dependence is weaker and is closer to the dependence represented by curve 2 of Fig. 5. It is impossible to discuss curve 3 in greater detail, owing to the lack of data in[4] concerning the accuracy with which the sample is oriented.

With Fig. 9 as an example, it is seen that in the case of breakdown on the boundaries of the zone no small orbits are realized, in contrast to the corner of the zone, where precisely such orbits (β orbits[19]) occur. From the theory of magnetic breakdown it is known[14,20] that the effects of phase coherence on large orbits are readily violated owing to the large sensitivity to small-angle scattering (by phonons and imperfections of the dislocation type). It is therefore not surprising that in aluminum at most directions of H there are no coherent effects of the type of giant oscillations of the magnetoresistance; these effects are observed only when H || [100][4], although it is possible that traces of these effects are present also in curve 1 of Fig. 6.

The anomalous decrease of the magnetoresistance with increasing field, at those directions of H where saturation should be observed (Fig. 5, curve 1), can apparently be explained by assuming a change of the difference in the carrier densities in breakdown, although this question remains open.

Further evidence favoring the proposed picture of the magnetic breakdown in aluminum is afforded by the results of measurements of the temperature dependence of the transverse resistance. The latter make it possible also to clarify the nature of the anomalous growth of the magnetoresistance with increasing temperature, which has not yet been explained to date.

Using the results of the theoretical papers[5,17], we can show that the transverse resistance in strong fields, in the presence of open trajectories (both a narrow layer and one that is not too narrow) in the isotropic-relaxation-time approximation has in the general case the form

$$\rho_{H\perp} \approx \rho_0 + \rho(\omega\tau_H) = \rho_0 + Af(\omega)\tau_H^n, \quad \rho_{yy} \approx \rho_0, \qquad (1)$$

ρ_{yy} is the component of the resistance in the direction perpendicular to the openness; $\rho_{H\perp}$ is the transverse resistance in the plane perpendicular to the magnetic field, with the exception of the case j || y; A is a constant; $f(\omega)$ is a function that depends only on the magnetic field intensity; τ_H is the relaxation time in the magnetic field; n = 0 for not too narrow a layer of open trajectories, and n = 1 for a narrow layer of any origin.

Consequently, the relative change of the resistance can be written in the following manner:

$$\Delta\rho_{H\perp}/\rho_0 = Cf(\omega)\tau_0\tau_H^n, \qquad (2)$$

where $\tau_{0,H}^{-1} = \tau_i^{-1} + \tau_{0,H}^{ep\,-1}$, τ_i and τ^{ep} are the electron-impurity and electron-phonon relaxation times, and C is a constant.

The relaxation time can be different in the absence of a magnetic field (τ_0) and in the presence of a field (τ_H)[5]. In spite of this, in the approximation of isotropic relaxation time, one cannot understand the reason for the increase of $\Delta\rho_H/\rho_0$ with increasing temperature. Before we discuss another approximation, it is necessary to verify that the experimental results on the temperature dependences of the transverse resistance do not contradict the idea of the existence of open trajectories, and to identify the trajectories that we are dealing with.

Examining the curves of Fig. 7a, we see that the behavior of $\Delta\rho_{H\perp}/\rho_0$ with changing temperature agrees with the behavior expected on the basis of expression (2) for the case of open trajectories, namely, if $f(\omega)$ is small (fields that are not too strong), then the change of $\Delta\rho_{H\perp}/\rho_0$ becomes noticeable only following an appreciable change of τ_0 and τ_H, i.e., at sufficiently high temperatures, when τ_0^{-1}, $\tau_H^{-1} > \tau_i^{-1}$ (Fig. 7a, curve 1); in strong fields, when $f(\omega)$ is large, it is natural to expect a noticeable decrease of $\Delta\rho_{H\perp}/\rho_0$ at lower temperatures (curve 5, start of the interval). Were we to deal with narrow layers of open trajectories, then in accordance with (1) we should observe, if not a decrease (which is possible), at any rate a very weak temperature dependence of $\rho_H(T)$ in strong fields. With decreasing field, the character of the $\rho_H(T)$ dependence should approach the character of $\rho_0(T)$ ($\tau_H^{-1} \to \tau_0^{-1}$). Actually, the opposite is observed: at temperatures exceeding 13°K, the character of the dependences of $\rho_H(T)$ and $\rho_0(T)$ differs greatly in weaker fields, and is closer in stronger fields (Fig. 8, curves 1–4).

Thus, narrow layers of open trajectories cannot be responsible for the observed effects—a conclusion drawn above on the basis of data on the anisotropy and field dependences. At the same time, the assumption of not too narrow layers of open trajectories makes it possible to understand some of the indicated effects. Thus, for example, it follows from (1) in this case that $\rho(\omega\tau) = \rho(\omega)$ does not depend on the temperature, i.e., $\Delta\rho_{H\perp}/\rho_0$ can be large at sufficiently high temperatures. In this case $\rho_H \propto \tau_0^{-1}$, as is observed for the $\delta_H(T)$ plots at H = 20 kOe when T > 13°K (curve 4).

A very interesting circumstance is the presence of a steeper $\delta_H(T)$ dependence in a field than in the absence of a field, for the temperature interval below 13° (Fig. 8, curves 3 and 4). An acceptable explanation, although not the only one, might be here the assumption of the "temperature breakdown"[5], to be sure, of a somewhat different character than that considered in[5]. We are dealing with allowance for the smearing of the Fermi distribution, characterized by the value of the thermal energy kT, which can become appreciable under conditions close to intraband breakdown (interband breakdown in the case of a band with very small dimensions). The qualitative formula is similar in this case to the formula for the intraband breakdown[5], which takes into account the fact that the relative number of electrons taking part in the breakdown is $\sim\sqrt{kT/\epsilon_F}$. For fixed H we have here $\Delta\rho_H/\rho_0 \propto T^{1/2}e^{-\Delta\epsilon/T}$, which can lead to a growth of $\Delta\rho_H/\rho_0$ as a function of T with increasing temperature (so long as τ_0^{-1} is not too large compared with τ_i^{-1}), and in the case of a steeper power-law dependence of $\rho_H(T)$ then $\rho_0(T)$, as is indeed the case for curves 3 and 4 of Fig. 8.

In spite of the fact that at sufficiently high temperatures the individual features possessed by the different resistance components are still retained, by virtue of the specific geometry of the open trajectories, all these components have a common singularity, namely, an anomalous increase of $\Delta\rho_H/\rho_0$ with increasing temperature. It is impossible to explain this singularity and still remain within the framework of the concepts of the isotropic character of the scattering of the conduction electrons.

The question of the anisotropic scattering of electrons in a magnetic field arose first in an analysis of narrow layers of open trajectories[5]. It was shown that an important factor for the existence of such trajectories is small-angle scattering, which violates the infinite motion in the narrow layer. If N-processes prevail, then this leads to a strong difference between τ_H and τ_0, making it possible to identify narrow layers of open trajectories by means of the temperature dependences of the magneto-resistance. The influence of U-processes was not considered in[5]. In aluminum, however, they play an important role[7]. It is clear that in this case the difference between τ_H and τ_0 should not be so characteristic ($\tau_0 \propto (\vartheta/T)^4$ and $\tau_H \propto (\vartheta/T)^3$, where ϑ is the characteristic temperature). But even in this case the contribution of the narrow layers cannot exceed several percent of ρ_0 in a field of H = 20 kOe at T = 4.2°K. It is therefore reasonable to assume that in aluminum the principal role is played by not too narrow layers of open trajectories. The role of small-angle scattering does not become less significant here. Pippard has shown[6] that the influence of such scattering on the electric conductivity of metals in a magnetic field has a rather general character and is accompanied by an increase of the efficiency of electron-phonon interaction.

Let us use the estimates made in[6] for a hexagonal orbit (realized in aluminum at H ∥ [111]) in the case when there is no magnetic breakdown, and let us take also into account the contribution of U-processes, which were not considered by Pippard. The effective relaxation time due to scattering by small angles in the limit of strong fields is as follows: $\tau_H \sim \tau_c/\Phi$, where τ_c is the time between two electron collisions, and is proportional to $(\vartheta/T)^3$, and Φ is the scattering angle ($\Phi \sim T/\vartheta$). If it is assumed that τ_0 is determined only by N processes, i.e., $\tau_0 \sim \tau_c/\Phi^2$, then small-angle scattering leads to an increase of the transverse resistance and to saturation compared with the resistance without the field, by a factor $\tau_0/\tau_H = 2/\Phi$, from which we can expect $\rho > 10\,\rho_0$ as $H \to \infty$. However, owing to the presence of U-processes, the electron-phonon collisions are sufficiently effective also in the absence of a field. Since in this case τ_0 and τ_H are proportional to $1/T^4$, the contribution of scattering through small angles should not be too large, as is indeed the case for parts of $\Delta\rho_H/\rho_0 = f(H)$ with saturation (for example, for curves 1 and 2 of Fig. 5 at T ≤ 15°K we have $\rho \leq 5\,\rho_0$ as $H \to \infty$). This explains apparently why scattering through small angles does not mask completely the effects connected with the magnetic breakdown, up to

sufficiently high temperatures, although it does exert a definite influence on these effects. In particular, it becomes possible to explain such an anomaly as the growth of $\Delta\rho_H/\rho_0$ with increasing temperature.

Let us consider weak fields, when the effect of magnetic breakdown can be disregarded and the trajectories can be regarded closed for all directions of the vector H. We can expect here the small-angle scattering to be equally important for all the directions of H, although not necessarily to the same degree. It is known that in the single-band model there should be no magnetoresistance effect in the case of isotropic scattering. However, if it is assumed that the scattering efficiency in a magnetic field changes, then magnetoresistance also appears. The latter is connected with the increase of the number of effective collisions, and consequently with the change in the dependence of the relaxation time on the temperature, since the decisive factor here is the scattering angle $\Phi \sim T/\vartheta$. Taking this account at fixed values of H, we obtain the following qualitative relation for the ratio of the resistances in the field and without the field, within the framework of the single-band model:

$$\frac{\rho_H}{\rho_0} = \frac{1+\alpha(T/\vartheta)^{n_H}}{1+\beta(T/\vartheta)^{n_0}}. \qquad (3)$$

Here α and β are the coefficients of the same order, which depend on the parameters of the metal; $n_H < n_0$ in view of the efficiency of small-angle scattering (the value $n_H = 4$ obtained in the determination of τ_H and given above corresponds to the limiting case of strong fields, where τ_H ceases to depend on the field). We do not take into account the effect of ordinary magnetoresistance, which appears, as is well known, as a result of the difference, say, between the kinematic velocities of different carriers, in view of its smallness: in isotropic scattering $\rho_{H\to\infty}/\rho_0 = 1.62$, in the case of a hexagonal hole orbit[6], and small-angle scattering should decrease this value by another factor τ_0/τ_H.

The condition $n_H < n_0$ signifies that the contribution of the small-angle scattering may turn out to be important already at those temperatures, at which in the absence of the field the electron-phonon scattering is still insignificant compared with impurity scattering. In this case, in a certain temperature interval, the second term of the denominator of (3) can be neglected, and ρ_H/ρ_0 increases with increasing temperature. For pure samples, this interval should not be large. At temperatures for which the scattering by the phonons becomes noticeable also in the absence of the field, the ratio ρ_H/ρ_0 will decrease, tending to $\rho_H/\rho_0 \infty (\vartheta/T)^{n_0-n_H}$, where $(n_0 - n_H) > 0$. For not very pure samples, mainly as a result of the isotropic scattering by impurities in a sufficiently wide temperature interval, there may be no increase of ρ_H/ρ_0 with increasing temperature, so that the Kohler rule is satisfied for such samples, as was indeed observed in[1,2].

Examining curves 1 of Fig. 7, we can deduce that although the small-angle scattering in weak fields is small, since $n_H \approx n_0$ in the interval 4.2–10°K (Fig. 8, curves 2 and 8), it is responsible for the fact that the decrease of $\Delta\rho_H/\rho_0$ occurs at temperatures higher than might be expected. The smoother temperature dependence of $\delta_H(T)$ compared with $\delta_0(T)$ can be attributed to the usual decrease of the magnetoresistance in terms of the Kohler rule for the two-band model[21] at temperatures when $\tau_{ep}^{-1} \gg \tau_i^{-1}$. The fact that at these temperatures $n_H < 3$ indicates that the latter effect plays the principal role in weak fields, since the magnetic breakdown is still insignificant.

In strong fields, when the contribution of small-angle scattering should be large, we are justified in assuming that the temperature dependence of $\rho_H(T)$ will differ noticeably from that in the case H = 0. Then n_H should be much smaller than n_0, particularly for the temperature interval when $\tau_{ep}^{-1} \gg \tau_i^{-1}$. However, first, this effect is small even at the orientations of H corresponding at T = 4.2°K to saturation of the magnetoresistance (Fig. 8, curve 7). Second, for directions of H with different asymptotic values of the resistance of T = 4.2°K, a different effect of increase of $\Delta\rho_H/\rho_0$ with increasing temperature is observed (Fig. 7a, curve 5; Fig. 7b, curve 3). In addition, as already noted, in strong fields the exponent of the temperature dependence of $\delta_H(T)$ for curves with increasing resistance is close to that for the case H = 0, and when T < 10°K it is even higher (curve 4, Fig. 8). This makes it possible to conclude that in strong fields the singularities of the magnetoresistance of aluminum in the investigated temperature interval are due to an equal degree to magnetic breakdown as well as to the influence of small scattering. In weak fields, the latter is more important.

Thus, in the investigated temperature interval, it is necessary to consider the entire aggregate of effects simultaneously, both magnetic breakdown and small-angle scattering, whereas previously only the former was taken into account.

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