ON THE THEORY OF THE GRAVITATIONAL FIELD

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Possible gravitational equations in the general theory of relativity consistent with the present empirical data are discussed. It is shown that the equations apparently must contain covariant derivatives of the curvature tensor. It is shown further that scalar gravitational waves apparently must exist in nature, and that the mass of matter must differ from its active gravitational mass (for the sun, a mass difference of ~50% is not excluded). The explicit form of the probable gravitational equations is given for a wide class of gravitational fields, and the possible region of applicability of these equations is indicated.

1. PHENOMENOLOGICAL DATA

EXPERIMENT indicates that the gravitational field reduces completely to a curvature of four-space (i.e., is a metric field; the only admissible[1] non-metric theory[2] is in contradiction with experiment[3,4]) and interacts with matter according to the principle of equivalence.[5] Therefore, the gravitational equations must be in accordance with the basic principles of Einstein's general theory of relativity. The actual form of the equations can be established only with the help of additional empirical data. Let us discuss these, starting from the principle of equivalence, according to which neutral particles move along geodesics for any admissible (phenomenologically) gravitational equation; the empirical information derived from this is free of any a priori assumptions whatsoever.

Let us consider first of all the gravitational deflection of light rays in the field of the sun. Owing to the conformal invariance of the geodesic equation of the light ray, the gravitational deflection angle β is in first approximation linear in the conformal Weyl tensor:

$$W_{lm}^{ik} = R_{lm}^{ik} - {}^1/_2(R_l^i\delta_m^k + R_m^k\delta_l^i - R_m^i\delta_l^k - R_l^k\delta_m^i) + {}^1/_6(\delta_l^i\delta_m^k - \delta_m^i\delta_l^k)R, \quad (1)$$

where R_{lm}^{ik} is the Riemann-Christoffel tensor, $R_k^i = R_{kn}^{in}$ is the Ricci tensor, and $R = R_n^n$ is the scalar curvature. Because of the poor accuracy of astronomical data one can restrict oneself to the linear approximation and regard the field as static and spherical; in this case we have, according to the geodesic equations,

$$\beta(r) = 6r\int_r^\infty \sqrt{1-\frac{r^2}{x^2}}\,W_{0r}{}^{0r}(x)\,dx, \quad (2)$$

where r is the smallest distance from the ray to the center, and the index 0 refers (as in the following) to the coordinate ct; c is the velocity of light, and t is the time. In the nonrelativistic approximation, we have outside the sun

$$W_{0r}{}^{0r}(r) = 2Gm_\odot c^{-2}r^{-3}[1+w(r)], \quad r \geqslant a_\odot, \quad (3)$$

where G is Newton's constant, $m_\odot = 1.99\times10^{33}$ g is the gravitational mass of the sun,[1)] $a_\odot = 6.96\times10^{10}$ cm is the radius of the sun, and w(r) is an empirical function. Including, for an estimate, the nonstatic, relativistic ($\lesssim Gm_\odot c^{-2}a_\odot^{-1} \sim 10^{-6}$), quadrupole ($\lesssim 10^{-5}$), and all other non-Schwarzschildian corrections, it is easy to see that according to Einstein's equations $|w(a_\odot)| \sim 10^{-5}$ (the ratio of the potentials outside and inside the solar matter). According to (2) and (3), $w(a_\odot) \sim \gamma = [\beta(a_\odot)/1.75''] - 1$. Hence, according to Einstein's equations, $|\gamma| \sim 10^{-5}$, which is also the necessary condition for their applicability. Therefore the empirical value closest to 1.75″, $\beta(a_\odot) = 1.75'' \pm 0.19''$ (Trümpler,[6] $\gamma = 0.00 \pm 0.11$) contains no information on the gravitational equations (the assertion "the result does not contradict Einstein's equations" would be incorrect), and the same refers to all such data, for example 1.72″ ± 0.11″ (Campbell and Trümpler,[6] $\gamma = -0.02 \pm 0.06$) or 1.79″ ± 0.06″ (Mitchell,[6] $\gamma = +0.02 \pm 0.03$), which do not fix the order of magnitude of the real γ effect. Let us estimate it from the averaged astronomical data; in this case[2)]

$$\gamma = +0.17 \pm 0.07\ [^6],\ +0.07 \pm 0.05\ [^7],\ +0.13 \pm 0.07\ [^7] \quad (4)$$

and since in (4) the average is larger than the error, one may assume that indeed $\gamma \gg 10^{-5}$, i.e., that Einstein's equations are incorrect. This result is consistent with the measurements[8] according to which the gravitational shift of the spectrum at the solar surface differs from the Schwarzschild value by a factor $z_1 = 1.05 \pm 0.05$ (according to Einstein's equations, $|z_1 - 1| \sim 10^{-5}$). If the data (4) reflect more or less correctly the real gravitational effect, then

$$w(a_\odot) \sim \gamma \sim +(10^{-2}-10^{-1}), \quad (5)$$

which can only correspond (according to the Biancchi identities) to the external field of the Ricci tensor

1)According to (8) and (17), $m_\odot$ is practically equal to its usual value.

2)The first value is the average of all corrected data quoted in [6] [the average of the most reliable (according to the estimate [6]) uncorrected data is equal to +0.04 ± 0.04]. Here and in (4) the errors indicate the scatter of the individual averages.

generated by the solar matter. In this case the Ricci tensor is nonlocal, i.e., in contrast to Einstein's equations,[3)] it does not vanish outside matter. According to (3) and (5), the effect should be of nonrelativistic nature; one may therefore expect that the field of the sun differs from the Newtonian field even in the nonrelativistic approximation, which is apparently confirmed by the neutrino data and the data on the perihelia. We see that the assumption (5) about the nonlocality of the Ricci tensor is not only entirely probable but the only substantial consequence derived from the present (not fully reliable) data on the γ effect and the red shift. We note that in view of the very weak quadrupole deformation of the sun its non-Einsteinian field (if it exists) is mainly spherical, which serves as a justification of the original approximation (2) and analogous approximations in the following.

Let us consider the quantities

$$\rho^* = -\frac{c^2}{4\pi G}R_0^0, \quad m = \int \rho^* \sqrt{-g}\, dx^1\, dx^2\, dx^3 \tag{6}$$

for isolated static matter; R_0^0 is a component of the Ricci tensor, g is the determinant of the metric g_{ik}, and the integral will be regarded as convergent, since experiment does not contradict a Newtonian asymptotic form $\sim r^{-1}$ for the static gravitational potential. According to phenomenological calculations ([10], Sec. 99) one finds that in synchronous coordinates ($g_{0\alpha} \equiv 0$, $\alpha = 1, 2, 3$) $g_{00} = -1 + 2Gmc^{-2}r^{-1}$ at large distances from the matter, i.e., according to the geodesical principle, m is the active gravitational mass, and ρ^* (more precisely $\sqrt{-g_{00}}\rho^*$) is its volume density. Judging from the γ data, ρ^* does not at all have to be local (even in the nonrelativistic approximation) and therefore, may not be positive. According to (6), the nonrelativistic gravitational potential satisfies the phenomenological equation

$$\Delta\varphi = 4\pi G\rho^* \tag{7}$$

and differs from the Newtonian potential to the extent that ρ^* is nonlocal; in the spherical field, we have according to (7)

$$\varphi'(r) = Gmr^{-2}[1-\eta(r)], \quad \eta(r) = 4\pi m^{-1}\int_r^\infty \rho^*(x)x^2\,dx, \tag{8}$$

where the prime denotes the derivative with respect to r, $\eta(0) = 1, \eta(\infty) = 0$, where the quantity $\eta(r)$ outside the matter determines the deviation from the Newtonian field. The internal nonlocality and deviation from the Newtonian regime is naturally characterized by the parameter

$$\mu = \rho^*\rho_M^{-1} - 1, \tag{9}$$

where $\rho_M = c^{-2}(T_\alpha^\alpha - T_0^0)$ is the Tolman density of matter.

Let us now consider the distribution of the gravitational mass of the sun. We note first of all that the observable current[11] of solar neutrinos is weaker than the theoretical value when all factors[4,12] except the possible nonlocality of the gravitational mass are taken into account. If this discrepancy is indeed due to the non-Newtonian effect, then the internal non-Newtonian regime in the sun should lead to a lowering of the temperature and of the intensity of the gravitational field. In this case $\eta(a_\odot) > 0$ according to (8), and according to a crude preliminary estimate, $\eta(a_\odot) \sim 0.1$, i.e., from (8) and (9),

$$|\mu_\odot| \sim \eta(a_\odot) \sim 0.1, \quad \rho_\odot^*(r \sim a_\odot) \sim +0.1 \text{ g-cm}^{-3}, \tag{10}$$

where $\mu_\odot$ is of the order of the parameter (9) for the sun; the results are consistent with (5), which confirms to some extent the non-Newtonian interpretation of the neutrino effect. The latter is not excluded by the indicated contradiction[3,4] between theory[2] and the neutrino data, since the metric structure of ρ^* can not be the same as the non-metric one.

We note now that the μ effect has so far not be observed under laboratory conditions, while the errors in laboratory measurements are apparently much less than 10%. Hence, the assumption (10) can be replaced by the even weaker assumption

$$|\mu_\odot| \gg |\mu_{\text{lab}}|, \quad |\mu_{\text{lab}}| \ll 1, \tag{11}$$

which, however, definitely excludes Einstein's equations $\mu \equiv 0$ and rather stringently fixes the corresponding gravitational equations (cf. Sec. 2); therefore, the investigation of the neutrino emission from the sun and the experimental estimate of the accuracy of the equation $\Delta\varphi = 4\pi G\rho_M$ are most important fundamental tasks. We note further that the non-Newtonian regime may have an appreciable influence on the quadrupole moment of the sun $q_\odot$. In the absence of pressure differential rotation at the surface of the sun, $2\varphi + \omega^2 r^2 \sin^2\vartheta = \text{const}$, where ω is the angular velocity, ϑ is the angle with the axis of rotation, and the gravitational potential is $\varphi = \varphi_0(r) + (3\cos^2\vartheta - 1)\varphi_q(r)$ for $\epsilon_\odot \ll 1$; in first approximation with respect to $\epsilon_\odot$, we have, using (8),

$$q_\odot = [1-\eta(a_\odot)]\varepsilon_\odot - k_\odot, \tag{12}$$

where $q_\odot = 3a_\odot\varphi_q(a_\odot)G^{-1}m_\odot^{-1}$, $k_\odot = 0.5\omega^2 a_\odot^3 G^{-1} m_\odot^{-1} = 1.0 \times 10^{-5}$,[13] and $\epsilon_\odot$ is the relative difference between the equatorial and polar radii of the sun. According to direct measurements,[13] $\epsilon_\odot = (5.0 \pm 0.7) \times 0.7) \times 10^{-5}$. In this case the non-Newtonian part of $q_\odot$ may exceed the correction for the differential rotation. For example, if $\eta(a_\odot) \approx 0.3$ [cf. (30) and (10)], then $10^5 q_\odot \approx 2.5 \pm 0.5$ with a non-Newtonian part of ~60%.

Let us finally consider the angular shift $\alpha_\odot$ of the perihelium of a planet (per revolution) which is caused by the field of the sun. The angle $\alpha_\odot$ is phenomenologically calculated from the geodesic equations, where, within the limits of accuracy of the astronomical data, it is sufficient to restrict oneself to the first nonvanishing approximation for the spherical and quadrupole parts of the shift. Neglecting the excentricity and the inclination of the orbit [their contribution to (14) to (16) is less than the error of the measurements] we have

$$\alpha_\odot = 2\pi q_\odot a_\odot^2 r^{-2} y(r) + 3\pi r^2 W_{0r}{}^{0r}(r) - 4\pi^2 m_\odot^{-1} r^3 \rho_\odot^*(r), \tag{13}$$

3)The cosmological constant is not taken into account in the present paper, since according to the estimate [9], cosmological terms are unobservable in all cases considered by us.

where r is the radius of the orbit, $y(r) = r^3 a_\odot^{-3} \varphi_q(r) \varphi_q^{-1}(a_\odot)$, and all remaining quantities were determined above; in the Newtonian field $y(r \geq a_\odot) \equiv 1$. According to (13) one can determine phenomenologically the external gravitational density of the sun $\rho_\odot^*(r)$, which must be equal to zero if Einstein's equations are correct.

Let us consider the astronomical data.

For Mercury $r = r_☿ = 58 \times 10^6$ km, and $y(r \geq r_☿) = 1$ with an accuracy better than 10^{-6}%, according to (17). Hence, according to (13) and (3), $\rho_\odot^*(r_☿) = 0.24 \times 10^{-14} \xi$ g cm^{-3}, where

$$\xi = 10^5 q_\odot + 53 w(r_☿) + \xi_0, \tag{14}$$

and w(r) is the function in (3), and ξ_0 is proportional to the difference between the actual and the Schwarzschild shifts of the periphelium; for $\alpha_\odot(r_☿) = 43.11 \pm 0.45$[14] and 42.9 ± 0.2[15] angular seconds per century we have $\xi_0 = -0.10 \pm 0.55$ and $+0.16 \pm 0.25$, respectively, i.e., for given $q_\odot$ and w the quantity ξ is determined with an accuracy $\sim |\xi_0| \sim 0.1$. The dependence of $\alpha_\odot$ on the Weyl tensor, i.e., the relation between ξ and w, is very important, since it is the reason (taking also the coefficient 53 into account) why it is impossible in principle to confirm Einstein's equations more definitely from the data on the perihelium than from the data on the γ effect. Indeed, according to Einstein's equations not only $\xi = 0$ but also $w(r_☿) = 0$ [more precisely, $|w(r_☿)| \lesssim 10^{-4} |\xi_0|$; cf. (3)], i.e., according to (14), $q_\odot = -10^{-5}\xi_0$, which is smaller than (12) by an order of magnitude. This discrepancy can perhaps be explained by the experimental errors[13] or by the peculiarities of the internal structure of the sun, but its removal does not solve the problem of Einstein's equations which also imply the w condition; the latter does not depend on the quantity $q_\odot$,[4)] and it can be tested only by direct measurement of the γ effect, the results of which have been discussed at the beginning.

Let us now consider the numerical values. Depending on the rate of decrease of w(r), we may set in (14) either $w(r_☿) = 0$ or $w(r) = \text{const} = \gamma = +0.1 \pm 0.1$ [according to (2) to (4)], where in the latter case $\beta(r) = \text{const} \cdot r^{-1}$ [cf. (2)], which is not at all excluded by the present data. Taking the two values $10^5 q_\odot = 2.5 \pm 0.5$ (cf. below) and $q_\odot = 0$, we have according to (14), and independently of the above values of ξ_0,

$$\dot{\rho}_\odot(r_☿) = \begin{pmatrix} +0.6 \pm 0.2 \\ +1.9 \pm 1.3 \end{pmatrix}, \begin{pmatrix} 0.24\,\xi_0 \\ +1.2 \pm 1.2 \end{pmatrix} \cdot 10^{-14}\,\text{g-cm}^{-3}, \tag{15}$$

where in the second parenthesis $q_\odot = 0$, in the upper rows w = 0, and in the lower rows $w = \gamma$; in the lower rows $\xi \sim 10|\xi_0|$, and the numerical values are seen not to be determined mainly by the quadrupole deformation of the sun but by the nonlocal γ effect, as illustrated by the importance of the γ information for a correct estimate of the data on the perihelium of Mercury. For Venus ($r_♀ = 108 \times 10^6$ km) and the earth ($r_♁ = 150 \times 10^6$ km) one may take $w = q_\odot = 0$, since their contribution to((13) is smaller than the errors in the astronomical data[5)] $a_\odot(r_♀) = 8.4 \pm 4.8$ and $\alpha_\odot(r_♁) = 5.0 \pm 1.2$,[14] according to which

$$\rho_\odot^*(r_♀) \approx (-0.1 \pm 0.6) \cdot 10^{-15}\,\text{g-cm}^{-3} \tag{16}$$

$$\rho_\odot^*(r_♁) \approx (-0.9 \pm 0.9) \cdot 10^{-15}\,\text{g-cm}^{-3}.$$

It is clear that if the estimates (15) and (16) give more or less correctly the true order of magnitude of $\rho_\odot^*$ on the planetary orbits, then they cannot refer to the local density of matter (the latter is known to be less than the density of the corona = 10^{-16} g-cm^{-3}); rather, the data on the perihelium point unambiguously to a nonlocality of the gravitational mass of the sun and a nonlocality of the Ricci tensor, i.e., they exclude Einstein's equations. According to (15), (16), and (10), $\rho_\odot^*$ first decreases sharply by about 13 orders of magnitude between the surface of the sun and the orbit of Mercury, whereupon the fall-off becomes slower and $\rho_\odot^*$ possibly goes through zero and changes sign between the orbits of Mercury and the earth. Therefore, according to (8) and (15),

$$|\eta(r)| \lesssim 4\pi r^3 m_\odot^{-1} |\rho_\odot^*(r)| \lesssim 10^{-8}, \quad r \geqslant r_☿, \tag{17}$$

which is the phenomenological limit of the nonrelativistic non-Newtonian corrections to the third law of Kepler; the effect is at least two orders smaller than the errors of observation and is not in contradiction with the data of celestial mechanics.

We see that the classical gravitational effects cannot be regarded as confirming Einstein's equations; moreover, these effects indicate, phenomenologically and completely consistently, a nonlocality of the Ricci tensor and a number of quantitative characteristics. An interpretation of the empirical data will be given below.

2. BASIC ASSUMPTIONS

The basic principles of general relativity lead unambiguously[16] [6)] to the following gravitational equations:

$$R_k{}^i - \tfrac{1}{2} R \delta_k{}^i + \{X\}_k{}^i = \varkappa_1 T_k{}^i, \tag{18}$$

where κ_1 is the coupling constant, $\sqrt{-g}\{X\}_k^i = g^{im}\delta(\sqrt{-g}X)/\delta g^{km}$, and $T_k^i = -2\{M\}_k^i$ is the energy-momentum tensor of gravitational matter, where its invariant Lagrangian does not depend explicitly on the curvature tensor (equivalence principle), and X is a dynamical invariant. The latter can depend (necessarily non-linearly[16]) only on the invariant contractions of the curvature tensor and its covariant derivatives taken (according to the causality principle) in the same four-point as X; in flat four-space $X \equiv 0$ and $\{X\}_k^i \equiv 0$ (for vanishing cosmological constant),

4)The quadrupole moment is determined only by the component R_0^0, and the function w(r) depends, according to the Bianchi identities, also on the spatial components of the Ricci tensor.

5)Reference 14 apparently contains a printing error; the value 3.4 ± 4.8 for Venus must be replaced by 8.4 ± 4.8.

6)In [16] one must disregard the pseudo-Einsteinian conditions and, possibly, the R symmetry.

since otherwise the four-space would not be flat for $T^i_k \equiv 0$. Equations (18) are phenomenologically consistent with the equations $T^i_{k;i} = 0$, and with the exception of the single case $\{X\}^i_k \equiv 0$ (Einstein's equations), the tensor $\{X\}^i_k$ always contains covariant derivatives, in which case the Ricci tensor is nonlocal, according to (18); hence (according to the results of Sec. 1) the covariant derivatives must apparently be present in the true gravitational equations.

For an analysis of the results of Sec. 1 it is sufficient to use the nonrelativistic equations. Let us consider these in coordinates with the metric $g_{ik} \approx \gamma_{ik}$ ($\gamma_{00} = -1$, $\gamma_{0\alpha} = 0$, $\gamma_{\alpha\beta} = \delta_{\alpha\beta}$, $\gamma^{ik} = \gamma_{ik}$), restricting ourselves only to the first nonvanishing approximation to the gravitational quantities; in this case the variation of the dynamic invariant is

$$\delta X = {}^1/_2 X^{ik}_{\cdot\cdot lm}\, \delta R^{lm}_{\cdot\cdot ik} + \partial_n X^n, \quad \partial_n = \partial / \partial x^n, \tag{19}$$

where R^{lm}_{ik} is the curvature tensor, and the nonrelativistic gravitational equations have, according to (18) and (19), the following form:

$$R_k{}^i - {}^1/_2 R \delta_k{}^i + \gamma^{mn} \partial_i \partial_m X_{k\ n}^{\ il} = \varkappa_1 T_k{}^i. \tag{20}$$

In a static spherical field, to which we can restrict ourselves in all that follows, the curvature tensor in synchronous coordinates is determined by the four quantities R, $W = 6\, W^{or}_{or}$ (the radial component of the Weyl tensor), $P = R - 4\, R^0_0$, and $Q = 3\, R^r_r - R + R^0_0$, of which only two are independent (owing to the Biancchi identities). Therefore, only two nonrelativistic equations are needed in this case, which follow from (20) and (19):

$$R - \Delta(3\zeta + U) - 2\bar{\Delta} V = \varkappa_1 c^2 \rho_M, \tag{21a}$$

$$W + d_{-1}[2d_0(\psi - 2U) + d_3(V + 3\Sigma)] = 0, \tag{21b}$$

where

$$d_n = (d/dr) + nr^{-1}, \quad \Delta = r^{-2} d_0 r^2 d_0,$$

$$\bar{\Delta} = r^{-2} d_0 r^2 d_3, \quad \zeta = \delta X / \delta R, \quad \Sigma = \delta X / \delta W, \quad U = \delta X / \delta P$$

and $V = \delta X / \delta Q$; the potential ψ is determined by the equation $\Delta\psi = \kappa_1 c^2 \rho_M$ and we assume the usual nonrelativistic matter, so that only the component $T^0_0 = -c^2 \rho_M$ of the matter tensor is conserved. According to (9), we have in this case

$$\varkappa c^2 \rho_M \mu = (\varkappa_1 - \varkappa) c^2 \rho_M - (\Delta\zeta + 3\Delta U - 2\bar{\Delta}\Sigma), \tag{22}$$

where $\kappa = 8\pi c^{-4} G$. In Einstein's equations $\mu \equiv 0$ and $\kappa_1 = \kappa$.

Let us now consider the assumptions (11). As they are more or less qualitative, they are in principle consistent with the possible order (5) of the γ effect, with the data on the perihelium, and with the laboratory data for any intensity of the solar neutrino radiation, which by no means depends only on gravitational effects. Therefore the conditions (11) (if they correspond to reality) are the most phenomenological restrictions on the equations (18), and in this case these conditions are the starting point of the generally-relativistic theory, as is assumed in all that follows.

We note first of all that the conditions (11) exclude completely the quasi-Einsteinian structure $2R^i_k - R\delta^i_k \approx \text{const} \cdot T^i_k$ of the nonrelativistic field even under laboratory conditions ($|\mu| \ll 1$), since in the quasi-Einsteinian case the quantity μ cannot, according to (21) and (22), increase with the radius of the matter (with constant average density). Hence, according to (22), the gravitational equations must be complemented by the following additional condition for $|\mu| \ll 1$ (with an accuracy $\sim\mu$):

$$\Delta\zeta + 3\Delta U - 2\Delta\Sigma = (\varkappa_1 - \varkappa) c^2 \rho_M, \tag{23}$$

the right-hand side of this equation must be $\sim \kappa c^2 \rho_M$, according to (11). It is clear that the density must drop out of (23), since the gravitational equations would otherwise lead to the standard equation for ρ_M; it is therefore necessary to assume that

$$\Delta\zeta = \sigma_1 c^2 \rho_M, \quad \Delta U = \sigma_2 c^2 \rho_M, \quad \bar{\Delta}\Sigma = \sigma_3 c^2 \rho_M, \quad |\mu| \ll 1, \tag{24}$$

where $\sigma_{1,2,3}$ are universal constants, and according to (23),

$$\sigma_1 + 3\sigma_2 - 2\sigma_3 = \varkappa_1 - \varkappa \neq 0. \tag{25}$$

We note now that according to (24) the structure components (which enter in the dynamic invariant) of the curvature tensor are nonlocal; for $|\mu| \ll 1$ they must therefore be $\sim \kappa c^2 \rho_M$ in order to avoid a "standardization" of matter. Since, according to (6) and (9), $2\, R^0_0 = -\kappa c^2 \rho_M$ for $|\mu| \ll 1$, the quantities R^0_0, P, and Q (cf. below) must in this case evidently not be structural, i.e., $U \approx 0$, $V \approx 0$, and $\sigma_2 = 0$ [cf. (24)]; the dynamic invariant can contain either R or W for $|\mu| \ll 1$, but not both simultaneously [this would contradict (25)], where the W case is excluded since it would lead to the instability of the gravitational vacuum [according to (25), $\kappa_1 = -3\kappa$ in the W case].

Thus it follows unambiguously from (11) and (18) that

I. In the nonrelativistic field the scalar curvature for $|\mu| \ll 1$ is small ($\sim\mu$) compared with the components of the Ricci tensor, i.e., the nonrelativistic field cannot be approximately Einsteinian under any conditions whatsoever.

II. The dynamic invariant of the nonrelativistic field for $|\mu| \ll 1$ must be (with an accuracy not lower than μ) a pure R structure, i.e., must depend only on the scalar curvature and its covariant derivatives, where the dependence must not be quadratic (to avoid contradiction with (26); cf.[18]).

Conditions I and II fix the coupling constant phenomenologically, since according to these conditions and (21a) and (24), $\sigma_1 = -3\kappa_1$, $\sigma_2 = \sigma_3 = 0$, and according to (25),

$$\varkappa_1 = {}^3/_4 \varkappa = 6\pi c^{-4} G; \tag{26}$$

in the following, we call conditions I and II, (26), and the corresponding invariants fundamental. They completely exclude Einstein's equations, but under laboratory conditions ($|\mu| \ll 1$) they lead of course, according to (7) and (9), to the inescapable Newtonian equation $\Delta\varphi = 4\pi G \rho_M$. We see that it is of fundamental importance for the gravitational equations and for the coupling constant what the reason for the laboratory result $\mu = 0$ is—the identity $\mu \equiv 0$ or the nonvanishing (in our case) smallness of the finite μ effect.

Let us consider the general consequences of the fundamental conditions. We note first of all that in the fundamental case the invariant $\zeta = \delta X/\delta R \neq 0$, so that there must exist scalar gravitational ζ waves; their parameters for $|\mu| \ll 1$ can be calculated completely phenomenologically, so that the fundamental conditions can be experimentally verified in the most general form. Furthermore, we have for the fundamental dynamic invariants, according to (21a), (22), and (26),

$$\mu = -\frac{R}{3\varkappa c^2 \rho_M} + \delta\mu, \quad \delta\mu = \frac{2}{3\varkappa c^2 \rho_M}[\Delta(3\Sigma + V) - 4\Delta U], \tag{27}$$

where $\delta\mu$ can be regarded as a measure of the violation of the R structure. Depending on the type of the dynamic invariant, $\delta\mu$ can either decrease (type A) or increase (type B) with increasing matter radius a, and it is clear that in the case A, for any matter,

$$|\delta\mu| \lesssim |\mu_{\text{lab}}|(a_0/a)^\nu, \quad \nu > 0, \tag{28}$$

where a_0 is the radius of the smallest laboratory bodies for which $|\mu| \gg 1$; apparently one can assume that $a_0 \lesssim 10^2$ cm. Let us now consider the gravitational deflection of light rays in the field of the sun. If the fundamental conditions are fulfilled, it is determined, according to (21b), (2), and (26), by the phenomenological relation

$$\beta(r) = 1'',31(m_{M\odot}/m_\odot)a_\odot r^{-1}[1 + \delta\mu_\odot(r)], \tag{29}$$

where $m_{M_\odot}$ is the material mass of the sun (the volume integral over ρ_M), $m_\odot = 1.99 \times 10^{33}$ g is its gravitational mass, $1.31'' = (\kappa_1/\kappa) \times 1.75''$ [cf. (26)], and $\delta\mu_\odot(r) \lesssim \delta\mu_\odot$, where $\delta\mu_\odot$ is of the order of the parameter (27) for the sun. The theoretical accuracy of (29) is better than $10^{-3}\%$ [cf. (3)]. If the dynamic invariant X has the structure A, then $\delta\mu_\odot(r)$ is not observable according to (28), and $\beta(r) = \text{const} \cdot r^{-1}$, which in any case is not excluded by the present data. Hence, one can assign the structure A to X at least as a starting assumption; in this case, (28) implies that for all $a \gg a_0$ and in particular in the field of the sun, X must have an R structure, as we shall assume in the following. This assumption (which completely corresponds to the fundamental condition II) is to some extent confirmed by the considerations of Sec. 3 in connection with the inequality (36), and can be directly verified when exact data on the function $\beta(r)$ are available. We note now that in the case of an R structure, (29) implies

$$\frac{m_{M\odot}}{m_\odot} = \frac{\beta(a_\odot)}{1'',31} \tag{30}$$

and judging from the astronomical data, the identity $m_M \equiv m$ (which does not follow from experiment[17]) is excluded phenomenologically for the fundamental A structures. According to (30), the difference between $m_{M_\odot}$ and $m_\odot$ is not small. For example, if $\beta(a_\odot) = 1.87'' \pm 0.08''$ (the smallest average in[7] then $m_{M_\odot} = (1.43 \pm 0.06) m_\odot = (2.85 \pm 0.12) \times 10^{33}$ g, i.e., $\mu_\odot \approx m_\odot m_{M_\odot}^{-1} - 1 \approx -0.3$, and even for $\beta(a_\odot) = 1.75''$ formula (30) leads to $\mu_\odot \approx -0.25$ and contradicts Einstein's equations (this must be taken into account in estimating their empirical justification). We see that the fundamental conditions directly connect the degree of the violation of the Newtonian regime inside the sun $\mu_\odot$ with the observable γ effect. Therefore the assumption (11) could be tested when exact data on the γ effect, (including its radial dependence) and on the neutrino emission are available.

3. GRAVITATIONAL EQUATIONS

Let us now discuss additional data on the structure of the dynamic invariant. We note first of all that for an arbitrary (not necessarily fundamental) R structure, we have in the field of the sun, according to (21a), (27), and (9),

$$\Delta\zeta = \varkappa_1 c^2 \rho_{M\odot} - \varkappa c^2 \rho_\odot^*, \quad (r\zeta)_\infty = -\gamma r_0, \tag{31}$$

where $r_0 = 2\,Gm_\odot c^{-2}$, γ is the parameter (4), and the boundary condition follows from (31) and (30); according to (31) and (7),

$$\zeta(a_\odot) = -r_0 a_\odot^{-1}\left[\frac{\beta(a_\odot)}{1''75} - z_1\right] \tag{32}$$

where z_1 is the ratio of the gravitational shift of the spectrum at the solar surface over its Einsteinian value 2.12×10^{-6}. According to (31), (8), (17), (4), and (10), $\zeta(r \geq 2a_\odot) = -\gamma r_0 r^{-1} < 0$ with an accuracy of no less than 10%, where the sign is determined by the empirical condition $\gamma > 0$. Furthermore, according to (27) and (9), we have outside the sun $R = -3\kappa c^2 \rho_\odot^*$ for the fundamental dynamic R structures; hence, in the fundamental R case $\zeta < 0$ corresponds to possible zeros of R(r) outside the sun (cf. Sec. 1). Moreover, judging from the neutrino estimate (10) of the sign of $\rho_\odot^*(a_\odot)$, we have in the R case

$$R(a_\odot) < 0, \tag{33}$$

and if $\gamma > 0$ and $\eta(a_\odot) > 0$, (31), (33), (27), and (9) imply that the functions R(r) and $\mu_\odot^*(r)$ must have zeros and change sign inside the sun in the R case; the latter is important for the regime of generating neutrinos corresponding to (7).

Let us now consider the theoretical connection between R and $\zeta = \delta X/\delta R$. In the R case, the dynamic invariant can contain covariant derivatives, but only with factors $L_1, L_2, \ldots$ of the dimension of a length (to compensate for the dimensionality of the derivatives); one may therefore assume that in the R case, X depends only on R (structure RO) if L_{max} is small compared with the geometric parameters of the matter. In the last case R is a function (not a functional) of ζ, where the dependence of $R(\zeta)$ is unique [otherwise the equations (31) and (38) would not connect the field with the matter uniquely] and satisfies the causality condition

$$R(\zeta) > 0 \quad (0 < \zeta \ll 1), \tag{34}$$

to avoid a group velocity larger than that of light for weak perturbations of a homogeneous nonrelativistic medium.[7)] Hence $R(\zeta \to 0) = \text{const} \cdot \zeta^n$, where const > 0 according to (34), and $n \geq 4$ (integer) owing to the convergence of the integral (6) (it converges even for $n = 1$, but this value is evidently excluded, cf.[18]).

Let us compare our theory with experiment. Accord-

[7)] According to the equation $3\Box\zeta - R = \kappa_1 T_n^n$, which in the R case replaces (21a) in a nonstatic field.

ing to (34), the function $R(\zeta)$ can have real nonrelativistic roots even for $\zeta < 0$, which agrees with the possible property of the solar field (cf. below) for which, therefore, the fundamental RO conditions are not excluded (they would be definitely excluded only if simultaneously $\rho^*_\odot$ had zeros outside the sun and the γ effect were negative). Furthermore, the condition for $R(\zeta \to 0)$ does not at all contradict the possible rate of decrease of $\rho^*_\odot$ outside the sun (cf. Sec. 1), and it follows from the assumption (33) that in the case RO, according to (34) and (32),

$$\beta(a_\odot) > 1'',75 z_1; \tag{35}$$

the observed value $z_1 = 1.05 \pm 0.05$[8] corresponds, according to (35), to $\beta(a_\odot) > 1.84'' \pm 0.09''$, which is completely consistent with the astronomical data. One can therefore assume that in the field of the sun, the fundamental conditions of Sec. 2 are realized in the form RO, i.e., in their simplest form. In this case

$$L_{max} \ll a_\odot \approx 7\cdot 10^{10}\ \text{cm}, \tag{36}$$

i.e., the fundamental RO theory should be applicable to all objects with linear dimensions $b \gtrsim a_\odot$ and in particular, to the entire cosmological process, with the possible exception of the first second of it.

If all our assumptions are valid, then at least for $b \gtrsim a_\odot$, the gravitational equations must be of the simplest of the fundamental types, i.e., of the fundamental type RO. Without loss of information, one can take for the function (37) the most general expression corresponding to the conditions indicated, i.e.,

$$R(\zeta) = l^{-2}\zeta^n f(\zeta) \prod_i (1 + \zeta\zeta_i^{-1})^{n_i}, \tag{37}$$

where l is a constant of the dimension of a length, $n \geq 4$, $n_i \geq 1$ (integer), ζ_i is dimensionless 'the small ζ_i are positive), the products contains all real roots, and $f(\zeta) > 0$ for all real ζ, where $f(0) = 1$. Judging from the data on the field of the sun (cf. below), one may have $\zeta_i \sim 10^{-7}$ and 10^{-9} in (37), which values correspond to the internal and external roots of $R(r)$; the remaining parameters of the function (37) are so far unknown. Equations (18) and (37) lead (without approximations) to the desired gravitational equations[16]

$$(1+\zeta)R_k{}^i - \tfrac{1}{2}(R+X)\delta_k{}^i + \delta_k{}^i\zeta_n{}^n - \zeta_k{}^i = \varkappa_1 T_k{}^i, \tag{38}$$

which contain covariant derivatives of no higher than second order; here $\kappa_1 = 6\pi c^{-4} G$ [cf. (26)], $\zeta^i_k = g^{in}\zeta_{;n;k}$, $R = R^n_n$ is the function (37) and X is determined by the conditions $dX = \zeta\, dR$, $X(\zeta = 0) = 0$ [if the cosmological constant must be included, $\lambda\delta^i_k$ must be added to the left-hand side of (38)]. The limits of application of (38) in the region $b < a_\odot$ and the lower limit of the constant L_{max} are unknown; it is possible that they can be established with the help of the background radiation or the μ effect in small bodies [according to (38), the vanishingly small μ effect is sure to be negative].

CONCLUSION

We see that at present there is no substantial empirical evidence which speaks in favor of Einstein's equations; moreover, there are grounds for regarding the Ricci tensor as nonlocal, and it seems quite probable that in the nonrelativistic case the nonlocality corresponds to the assumptions (11). The latter are very important since if indeed, for example, $|\mu_\odot| \sim |\mu_{lab}|$ [which is not probable judging from the data (4)], then all fundamental results of Secs. 2 and 3 are without foundation. The assumptions (11) fix uniquely the coupling constant and the boundary conditions, with which the gravitational equations must be in accord (for $\mu \to 0$); the equations of this the considered class do not allow for a continuous transition to Einstein's equations. Less justified (but not lacking any empirical foundation whatsoever) is the assumption that the simplest equations of the class (11), i.e., the equations (38) are indeed realized. For a definite conclusion one needs reliable data on the neutrino emission and on the three classic effects; of the latter, the most important are the gravitational deflection of light rays and the red shift, since no additional data are required for their phenomenological interpretation. For a test of the assumptions (11), an experimental search for scalar gravitational waves and direct (at present nonexistent) experimental estimates of the possible nonrelativistic violations of the equation $\Delta\varphi = 4\pi G\rho_M$ are needed.

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