ELECTRIC BREAKDOWN IN A SEMICONDUCTOR IN CROSSED ELECTRIC AND MAGNETIC FIELDS

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The behavior of semiconductors in crossed electric and magnetic fields is investigated. Formulas for electric breakdown are obtained in this case for an arbitrary dispersion law in the quasiclassical approximation.

IN this paper, the behavior of semiconductors in crossed electric and magnetic fields with arbitrary dispersion law is investigated in the two-band model. To this end, we first determine the nature of the classical motion in this case. In the sequel, we shall use the following choice of fields and potentials:

$$\mathbf{E} = (E, 0, 0), \quad \mathbf{H} = (0, 0, H), \quad \mathbf{A} = (0, Hx, 0).$$

Then for band zone there are conservation laws:

$$\varepsilon_n\left(p_x,\ p_y - \frac{eHx}{c},\ p_z\right) - eEx = \varepsilon_0 = \text{const},$$
$$p_y = \text{const}, \quad p_z = \text{const}.$$

We determine the dependence of the energy ϵ_0 of the turning points, where $v_x = \partial\epsilon(\mathbf{p})/\partial p_x = 0$. If momenta are reckoned relative to the bottom or top of the corresponding band, we have both $v_x = 0$ and $p_x = 0$. If, in addition, x is measured from $x_0 = cp_y/eH$, and energy from $-eEx_0$, then we obtain for the energy dependence of the turning point:

$$\varepsilon_n\left(0,\ \frac{eHx}{c},\ p_z\right) - eEx = \varepsilon_0 \tag{1}$$

(an irrelevant minus sign in front of H has been omitted). We investigate the nature of this dependence.

Differentiation of Eq. (1) gives (primes denote differentiation with respect to x)

$$\varepsilon_0' = -eE + \frac{eH}{c}v_y.$$

We notice that the velocity v_y inside each band has at least one maximum v_{max}, and the values of the ratios v/v_{max} are not less than two.

If $\epsilon_0' = 0$, then $v_y = cE/H = v_H$, where v_H is the Hall velocity. Now let $v_H > v_{max}$. In this case, $\epsilon_0(x)$ is a monotonic function with negative derivative (Fig. 1; the parallel curves are for the different bands). For some $v_H = v_{max}$, the value x_1 is reached, where $\epsilon_0' = \epsilon_0'' = 0$ ($\epsilon_0'' = 0$ is obligatory at the point with $v = v_{max}$), but the curve $\epsilon_0(x)$ still has a monotonic character. For $v_H < v_{max}$, there are two points where $\epsilon_0' = 0$, and the curves have the form depicted in Fig. 2. With further reduction of v_H, the curves attain forms like those shown in Fig. 3. We note here the very special case of a square-root dispersion

$$\varepsilon_n(\mathbf{p}) = \pm(\Delta^2/4 + \Delta p^2/2m)^{1/2}.$$

Then $0 \le v_y \le v_0$, $v_0 = \sqrt{\Delta/2m}$. In this case, for $v_H > v_0$

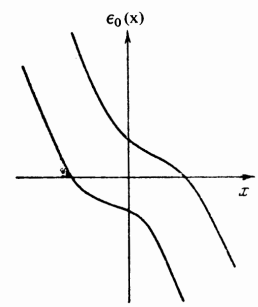

FIG. 1

the curves $\epsilon_0(x)$ are monotonic, and for $v_H < v_0$ both curves have extrema, the minimum of one being situated higher than the maximum of the other.

What are the consequences of this analysis? First, electric breakdown is formally possible at all values of the magnetic field, since there is degeneracy (i.e., there are states of the same energy in both bands). The situation represented in Figs. 2 and 3 is specially interesting. Thus, in the case corresponding to Fig. 2, two types of motion are possible within each band: discrete magnetic states in region I, and infinite states elsewhere (restricted, of course, by the boundaries of the band). There are evidently singularities in the density of states on the boundaries of region I. In the case depicted in Fig. 3, magnetic breakdown (tunneling I—II) is possible, distorted, however, by the electric field.

It is natural to enquire concerning the magnitude of the momentum at which the maximum velocity is attained. To this end, we observe that in the case of a dispersion law close to the square-root law, the maximal velocity is reached very quickly indeed. Clearly, deviations from the square-root dispersion law commence when $\partial^2\epsilon/\partial p^2 \sim 1/m_0$ (m_0 is the mass of a free electron). This gives for the magnitude of the momenta

$$p \sim (m_0/m)^{1/3}(m\Delta/2)^{1/2}.$$

The quantity $(m_0/m)^{1/3}$ is usually near to unity (~ 3).

We can be somewhat more detailed about electric breakdown. We mention that there is a body of works in which electric breakdown is discussed in the case of the square-root dispersion law[1-3]. In this context, we remark that, as follows from the analysis above, the proof of the vanishing of the tunnel current, given for an arbitrary dispersion law in the work of Aronov and Pikus[1],

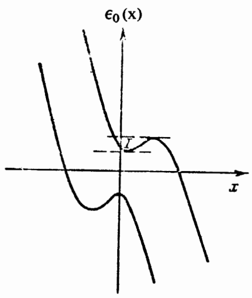

FIG. 2

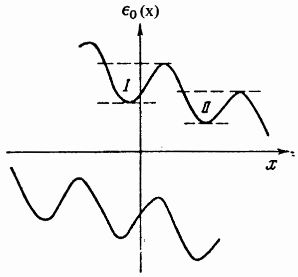

FIG. 3

is false, since it was assumed that tunneling goes on between magnetic states, whereas we see that it is between electric and magnetic states.

We next obtain a formula, at an arbitrary dispersion law, for the probability of transitions between bands. We represent the wave function in the form

$$\psi(\mathbf{r}) = \sum_n \int d\mathbf{p}\, c_n(\mathbf{p})\psi_{n\mathbf{p}}(\mathbf{r}).$$

Here, $\psi_{n\mathbf{p}}(\mathbf{r})$ are Bloch functions. In the quasiclassical approximation, the functions $c_n(\mathbf{p})$ reduce to the form

$$c_n(\mathbf{p}) = \delta(p_y - p_{y0})\delta(p_z - p_{z0})\exp\left\{\frac{ic}{\hbar eH}\left[-p_{y0}p_x + \int^{p_x} P(p_x')\,dp_x'\right]\right\}.$$

Then, clearly, $P(p_x)$ satisfies the equation

$$\varepsilon_n(p_x, P(p_x), p_{z0}) + v_H P(p_x) = \varepsilon_0 + eEx_0. \quad (2)$$

It is now easy to write an expression for the probability R_{vc} (see[4]) of tunneling from one zone to the other (the indices v and c refer to the valence and conduction bands respectively):

$$R_{vc} = \exp\left\{-\frac{2c}{eH}\,\mathrm{Im}\int_{p_{x1}}^{p_{x0}} [P_c(p_x') - P_v(p_x')]\,dp_x'\right\}. \quad (3)$$

Here, p_{x0} is the point where $P_v(p_{x0}) = P_c(p_{x0})$, and p_{x1} is an arbitrary point on the real axis.

We remark that the factor before the exponential in Eq. (3) is unity. Therefore, all the formulas in the works[1-3], obtained from perturbation theory and inapplicable in quasiclassics, are valid only to exponential accuracy.

To obtain an expression for the tunnel current, we must integrate over ϵ_0, p_{y0} and p_{z0}, which in the quasiclassical situation can be done by the saddle-point method, i.e., the expression in the exponent must be taken for those p_{z0} and $\epsilon = \epsilon_0 + eEx_0$ for which

$$\frac{\partial(P_c - P_v)}{\partial p_{y0}} = \frac{\partial(P_c - P_v)}{\partial p_{z0}} = \frac{\partial(P_c - P_v)}{\partial \varepsilon} = 0.$$

For the case of symmetric bands, when

$$\varepsilon_c = -\varepsilon_v = \varepsilon(\mathbf{p}) = \varepsilon(-\mathbf{p}),$$

it is easy to show that $P_c - P_v$ is an even function of ϵ and p_{z0}, i.e., we may set $\epsilon = p_{z0} = 0$ in the exponent. Moreover, $P_c = -P_v$, and p_{x0} is found from the condition

$$\varepsilon(p_{x0}, 0, 0) = 0$$

which does not involve the external fields. If $P_c(P_x)$ is estimated in some interval

$$P_c(0) \equiv P_c(p_x = 0),$$

and $p_x = i|p_{x0}|x$, ($P_c = P_c(0)y(x)$, $0 \le y(x) \le 1$), then we obtain for the tunnel current (we do not write out the factor multiplying the exponential, which comes from the integration over ϵ_0, p_{y0}, p_{z0})

$$j(E, H) \sim \exp\left\{-\frac{4v_H}{\hbar eE}P_c(0)\,|p_{x0}|\int_0^1 y(x)\,dx\right\}. \quad (4)$$

In the case of the square-root dispersion law, the expression in the exponent in Eq. (4) is the same as that in[1].

We remark that $P_c(0)$, up to a factor eH/c agrees with the value of x at the turning point of the classical motion with $\epsilon_0 = 0$, and, as can be seen from Figs. 1–3, is never infinite. In this connection, we estimate the possibility of observing deviation from the square-root dispersion law, using the tunnel current. For PbTe with fields $E \sim 10^5$ V/cm, the reduction of the tunnel current by one order of magnitude starts, according to Eq. (4), at $P_c(0) \sim 1.4(m\Delta/2)^{1/2}$. Deviations from the square-root law, according to our estimate, start with $P_c(0) < (m_0/m)^{1/3}(m\Delta/2)^{1/2}$. The coefficient in the last estimate is indeterminate, and if we set is equal to $\sim 1/2$, then observation of deviations is completely realistic at the present day. We note that indirect transitions will also undoubtedly overestimate the tunnel current.

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